1. Rutherford Scattering with Coulomb force
2. Scattering Cross Section
3. Differential Cross Section of Rutherford Scattering
4. Measurement of Cross Sections
5. A few measurements of differential cross sections
Announcements

- I have eight subscribed the distribution list. You still have time for 3 extra credit points…
- Three of you sent me proofs for the other extra credit points…Anyone else?
- A cloud chamber workshop tentatively on Saturday, Sept. 30
  - I would like each group to do the following
    - Elect a group leader
    - Prepare a list of tasks to accomplish at the workshop
    - Prepare a list items to purchase for the goals ➔ Need this by Monday, Sept. 18, so that we can place an order
Special Project Assignments

• Chamber Structural Design
  – Matt Block, Cassie Dobbs, James Foley

• TEM & Refrigeration
  – Heather Brown, Shane Spivey, Pierce Weatherly

• Liquid Supply and Recirculation System
  – Daniel Thomas, Jessica Jette, Layne Price

• Light and Display
  – John Henderson, Lauren Gilbert, Justin Pickering
Rutherford Scattering

\[ b = \frac{ZZ' e^2}{2E} \cot \frac{\theta}{2} \]

- From the solution for \( b \), we can learn the following

1. For fixed \( b \), \( E \) and \( Z' \)
   - The scattering is larger for a larger value of \( Z \).
     - Since Coulomb potential is stronger with larger \( Z \).

2. For fixed \( b \), \( Z \) and \( Z' \)
   - The scattering angle is larger when \( E \) is smaller.
     - Since the speed of the low energy particle is smaller
     - The particle spends more time in the potential, suffering greater deflection

3. For fixed \( Z \), \( Z' \), and \( E \)
   - The scattering angle is larger for smaller impact parameter \( b \)
     - Since the closer the incident particle is to the nucleus, the stronger Coulomb force it feels
What do we learn from scattering?

• Scattering of a particle in a potential is completely determined when we know both
  – The impact parameter, b, and
  – The energy of the incident particle, E

\[ b = \frac{Z Z' e^2}{2E} \cot \frac{\theta}{2} \]

• For a fixed energy, the deflection is defined by
  – The impact parameter, b.

• What do we need to perform a scattering experiment?
  – Incident flux of beam particles with known E
  – Device that can measure number of scattered particles at various angle, \( \theta \).
  – Measurements of the number of scattered particles reflect
    • Impact parameters of the incident particles
    • The effective size of the scattering center

• By measuring the scattering angle \( \theta \), we can learn about the potential or the forces between the target and the projectile.
Scattering Cross Section

- $N_0$: The number of particles incident on the target foil per unit area per unit time.
- Any incident particles entering with impact parameter $b$ and $b+db$ will scatter to the angle $\theta$ and $\theta-d\theta$.
- In other words, they scatter into the solid angle $d\Omega$ ($=2\pi\sin\theta d\theta$).
- So the number of particles scattered into the solid angle $d\Omega$ per unit time is $2\pi N_0 bdb$. 
Scattering Cross Section

• For a central potential
  – Such as Coulomb potential
  – Which has spherical symmetry
• The scattering center presents an effective transverse x-sectional area of

\[ \Delta \sigma = 2\pi bdb \]

• For the particles to scatter into \( \theta \) and \( \theta + d\theta \)
Scattering Cross Section

- In more generalized cases, $\Delta \sigma$ depends on both $\theta$ & $\phi$.

\[ \Delta \sigma(\theta, \phi) = bdbd\phi = -\frac{d\sigma}{d\Omega}(\theta, \phi)\ d\Omega = -\frac{d\sigma}{d\Omega}(\theta, \phi)\sin \theta\ d\theta\ d\phi \]

Why negative? Since the deflection and change of $b$ are in opposite direction!!

- With a spherical symmetry, $\phi$ can be integrated out:

\[ \Delta \sigma(\theta) = -\frac{d\sigma}{d\Omega}(\theta)\ 2\pi \sin \theta\ d\theta = 2\pi bdb \]

Differential Cross Section

What is the dimension of the differential cross section?

Area!!
Scattering Cross Section

- For a central potential, measuring the yield as a function of $\theta$, or differential cross section, is equivalent to measuring the entire effect of the scattering.
- So what is the physical meaning of the differential cross section?
  - Measurement of yield as a function of specific experimental variable
  - This is equivalent to measuring the probability of occurrence of a physical process in a specific kinematic phase space.
- Cross sections are measured in the unit of barns:
  \[ 1 \text{ barn} \equiv 10^{-24} \text{ cm}^2 \]

Cross sectional area of a typical nucleus!

Where does this come from?
Total Cross Section

• Total cross section is the integration of the differential cross section over the entire solid angle, $\Omega$:

\[
\sigma_{Total} = \int_0^{4\pi} \frac{d\sigma}{d\Omega}(\theta, \phi) \, d\Omega = 2\pi \int_0^\pi d\theta \sin \theta \frac{d\sigma}{d\Omega}(\theta)
\]

• Total cross section represents the effective size of the scattering center at all possible impact parameter.
Cross Section of Rutherford Scattering

• The impact parameter in Rutherford scattering is

\[ b = \frac{ZZ' e^2}{2E} \cot \frac{\theta}{2} \]

• Thus,

\[ \frac{db}{d\theta} = -\frac{1}{2} \frac{Z Z' e^2}{2E} \csc \frac{\theta}{2} \]

• Differential cross section of Rutherford scattering is

\[ \frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin \theta} \frac{db}{d\theta} = \left( \frac{Z Z' e^2}{2E} \right)^2 \csc^4 \frac{\theta}{2} = \left( \frac{Z Z' e^2}{2E} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} \]
Rutherford Scattering Cross Section

- Let’s plug in the numbers
  - $Z_{\text{Au}} = 79$
  - $Z_{\text{He}} = 2$
  - For $E = 10\text{keV}$

- Differential cross section of Rutherford scattering

$$
\frac{d\sigma}{d\Omega}(\theta) = \left( \frac{Z Z' e^2}{2E} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} = -\left( \frac{79 \cdot 2 \cdot \left(1.6 \times 10^{-19}\right)^2}{2 \cdot 10 \times 10^3 \times 1.6 \times 10^{-19}} \right)^2 \frac{1}{\sin^4 \frac{\theta}{2}} =
$$

$$
= \frac{1.59 \times 10^{-42}}{\sin^4 \frac{\theta}{2}} \text{cm}^2 = \frac{1.59 \times 10^{-18}}{\sin^4 \frac{\theta}{2}} \text{barns}
$$
Total X-Section of Rutherford Scattering

- To obtain the total cross section of Rutherford scattering, one integrates the differential cross section over all $\theta$:

$$\sigma_{Total} = 2\pi \int_0^\pi \frac{d\sigma}{d\Omega}(\theta) \sin \theta d\theta = 8\pi \left(\frac{ZZ'e^2}{2E}\right)^2 \int_0^1 d\left(\sin \frac{\theta}{2}\right)\frac{1}{\sin^3 \frac{\theta}{2}}$$

- What is the result of this integration?
  - Infinity!!

- Does this make sense?
  - Yes

- Why?
  - Since the Coulomb force's range is infinite.

- Is this physically meaningful?
  - No

- What would be the sensible thing to do?
  - Integrate to a cut-off angle since after certain distance the force is too weak to impact the scattering. ($\theta = \theta_0 > 0$)
Measuring Cross Sections

- **Rutherford scattering experiment**
  - Used a collimated beam of $\alpha$ particles emitted from Radon
  - A thin Au foil target
  - A scintillating glass screen with ZnS phosphor deposit
  - Telescope to view limited area of solid angle
  - Telescope only need to move along $\theta$ not $\phi$. Why?
    - Due to the spherical symmetry, scattering only depends on $\theta$ not $\phi$. Why?
Measuring Cross Sections

• With the flux of $N_0$ per unit area per second
• Any $\alpha$ particles in $b$ and $b+db$ will be scattered into $\theta$ and $\theta-d\theta$
• The telescope aperture limits the measurable area to
  \[ A_{Tele} = R d\theta \cdot R \sin \theta d\phi = R^2 d\Omega \]

• How could they have increased the rate of measurement?
  – By constructing an annular telescope
  – By how much would it increase? \[ 2\pi/d\phi \]
Measuring Cross Sections

• Fraction of incident particles approaching the target in the small area $\Delta \sigma = b d\phi d b$ at impact parameter $b$ is $-\frac{d n}{N_0}$.
  – $d n$ particles scatter into $R^2 d\Omega$, the aperture of the telescope

• This ratio is the same as
  – The sum of $\Delta \sigma$ over all $N$ nuclear centers throughout the foil divided by the total area ($S$) of the foil.
  – Probability for incident particles to enter within the $N$ areas of the annular rings and subsequently scatter into the telescope aperture

• So this ratio can be expressed as

$$- \frac{d n}{N_0} = \frac{N}{S} \Delta \sigma(\theta, \phi) = \frac{Nbd\phi d\theta}{S}$$
Measuring Cross Sections

• For a foil with thickness $t$, mass density $\rho$, atomic weight $A$:

$$N = \frac{\rho t S}{A} A_0$$

$A_0$: Avogadro’s number of atoms per mol

• Since from what we have learned previously

$$\frac{dn}{N_0} = \frac{N bd \phi d\theta}{S}$$

• The number of $\alpha$ scattered into the detector angle $(\theta,\phi)$ is

$$dn = \frac{N_0 \rho t A_0 d\sigma(\theta,\phi)}{A} d\Omega = N_0 \frac{N}{S} \frac{d\sigma(\theta,\phi)}{d\Omega} d\Omega$$
Measuring Cross Sections

\[ dn = \frac{N_0 \rho t A_0}{A} \frac{d\sigma(\theta, \phi)}{d\Omega} d\Omega = \frac{N_0}{S} \frac{d\sigma(\theta, \phi)}{d\Omega} d\Omega \]

- This is a general expression for any scattering process, independent of the existence of theory
- This gives an observed counts per second
Some Example Cross Section Measurements

- Azimuthal angle distribution of electrons in W+2jet events

![Graph showing events vs. azimuthal angle distribution for W+2jet events.](image)
Example Cross Section: \( W(\to e\nu) + X \)

- Transverse momentum distribution of electrons in \( W + X \) events
- Mass of the \( W \) boson is 80GeV
Example Cross Section: $W(\rightarrow e\nu) + X$

- Transverse mass distribution of electrons in $W+X$ events
Example Cross Section: $Z(\rightarrow ee) + X$

- Invariant mass distribution of electrons in $Z+X$ events
- Mass of the $Z$ boson is 91 GeV
Example Cross Section: Jet +X

- Inclusive jet production cross section as a function of transverse energy
Assignments

1. Plot the differential cross section of the Rutherford scattering as a function of the scattering angle $\theta$ with some sensible lower limit of the angle.

2. Compute the total cross section of the Rutherford scattering in unit of barns to the cut-off angle of your choice above.

3. Make the list of tasks and goals to accomplish in the workshop and the list of items to be purchased, along with the cost estimate and the contact information for purchase.
   - These lists should be written up and presented at the beginning of the next class on Monday Sept. 18.
   - Assignments #1 & 2 are due Monday, Sept. 11.