PHYS 1443 – Section 002
Lecture #4

Monday, Sept. 10, 2007
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- Acceleration
- 1D Motion under constant acceleration
- Free Fall
- Coordinate System
- Vectors and Scalars
- Motion in Two Dimensions

Today's homework is homework #3, due 7pm, Monday, Sept. 17!!

Announcements

- E-mail distribution list: 59 of you subscribed to the list so far
    - 42 of the 59 confirmed
    - Please reply if you haven't yet done so
    - Also please subscribe to the class distribution list since this is the primary communication tool for this class
- If you have not registered to homework system yet, please do so ASAP
    - Homework is 25% of the total grade!!
- Quiz results
    - Average: 7.7/15 ➔ 51.3
    - Top score: 15/15
    - Quiz takes up 10% of the final grade!!

Displacement, Velocity and Speed

Displacement

$$\boxed{\Delta x \equiv x_f - x_i}$$

Average velocity

$$\boxed{v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}}$$

Average speed

$$\boxed{v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}}$$

Instantaneous velocity

$$\boxed{v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

Instantaneous speed

$$\boxed{|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right| = \left| \frac{dx}{dt} \right|}$$

Acceleration

Change of velocity in time (what kind of quantity is this?)

- Average acceleration:

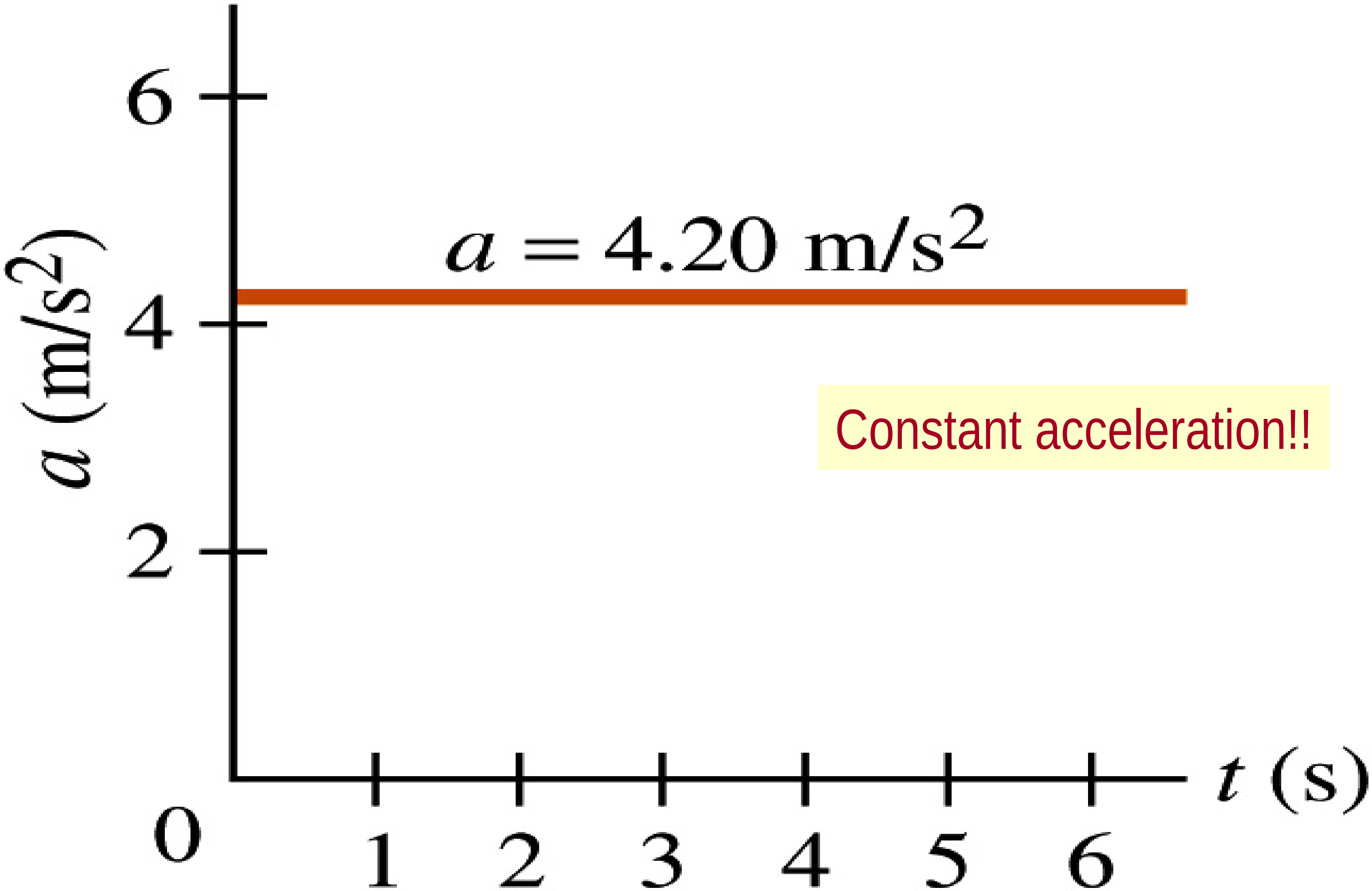
$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$$ analogous to $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$ analogous to $$\boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

- In calculus terms: The slope (derivative) of the velocity vector with respect to time or the change of slopes of position as a function of time

Acceleration vs Time Plot



Meanings of Acceleration

- When an object is moving at a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any net acceleration when an object is not moving?
- When an object speeds up as time goes on, ($v=v(t)$), acceleration has the same sign as v.
- When an object slows down as time goes on, ($v=v(t)$), acceleration has the opposite sign as v.
- Is there acceleration if an object moves in a constant speed but changes direction? YES!!

One Dimensional Motion

- Let's start with the simplest case: the acceleration is constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can derive equation of motion (description of motion, position *wrt* time)

$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} \xrightarrow{\text{If } t_f=t \text{ and } t_i=0} a_x = \frac{v_{xf} - v_{xi}}{t} \xrightarrow{\text{Solve for } v_{xf}} \boxed{v_{xf} = v_{xi} + a_x t}$$

For constant acceleration, simple numeric average

$$\overline{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2} a_x t$$

$$\overline{v}_x = \frac{x_f - x_i}{t_f - t_i} \xrightarrow{\text{If } t_f=t \text{ and } t_i=0} \overline{v}_x = \frac{x_f - x_i}{t} \xrightarrow{\text{Solve for } x_f} x_f = x_i + \overline{v}_x t$$

Resulting Equation of Motion becomes

$$\boxed{x_f = x_i + \overline{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2}$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you in specific physical problems!!

How do we solve a problem using a kinematic formula under constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance, initial position or final position?
  - Time?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem wants
- Identify which kinematic formula is appropriate and easiest to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equations for the quantity or quantities wanted.

Example 2.11

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇒ As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

The acceleration is $a_x = \frac{v_{xf}^2 - v_{xi}^2}{2(x_f - x_i)} = \frac{0-(28m/s)^2}{2\times 1m} = -390m/s^2$

Thus the time for air-bag to deploy is $t = \frac{v_{xf} - v_{xi}}{a} = \frac{0-28m/s}{-390m/s^2} = 0.07s$

Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
  - A motion under constant acceleration
  - All kinematic formulae we learned can be used to solve for falling motions.

Down to the center of the Earth!

- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $|g|=9.80m/s^2$ on the surface of the Earth, most of the time.
- The direction of gravitational acceleration is <u>ALWAYS</u> toward the center of the earth, which we normally call (-y); when the vertical directions are indicated with the variable "y"
- Thus the correct denotation of the gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$

Note the negative sign!!

Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $\boxed{g=-9.80m/s^2}$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00 m/s$$ Solve for t $$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

Position

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$