# PHYS 1443 – Section 002
# Lecture #6

*Monday, Sept. 17, 2007*
*Dr. Jaehoon Yu*

- Motion in Two Dimensions
  - Motion under constant acceleration
  - Projectile Motion
  - Maximum ranges and heights

Today's homework is homework #4, due 7pm, Monday, Sept. 24!!

# Announcements

- E-mail distribution list: 66 of you subscribed to the list so far
- There was a brief problem with the homework system last night
  - 3 of you noticed, yeah!!
  - It is fixed now.  So please submit your homework!!
- First term exam next Wednesday, Sept. 26
  - Will be in class from 1pm
  - Will cover Ch 1 to what we cover next Monday (~Ch4)
  - Mixture of multiple choice and numeric essay problems

# Kinematic Quantities in 1D and 2D

| Quantities | 1 Dimension | 2 Dimension |
|---|---|---|
| Displacement | $\Delta x = x_f - x_i$ | $\Delta \vec{r} = \vec{r}_f - \vec{r}_i$ |
| Average Velocity | $v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ | $\vec{v} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$ |
| Inst. Velocity | $v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ | $\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$ |
| Average Acc. | $a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$ | $\vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$ |
| Inst. Acc. | $a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2 x}{dt^2}$ | $\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}$ |

What is the difference between 1D and 2D quantities?

# 2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane:

$$\vec{r}_i = x_i\vec{i} + y_i\vec{j} \qquad \vec{r}_f = x_f\vec{i} + y_f\vec{j}$$

- Velocity vectors in x-y plane:

$$\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j} \qquad \vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$$

Velocity vectors in terms of the acceleration vector

X-comp $v_{xf} = v_{xi} + a_x t$ Y-comp $v_{yf} = v_{yi} + a_y t$

$$\vec{v}_f = (v_{xi} + a_x t)\vec{i} + (v_{yi} + a_y t)\vec{j} = (v_{xi}\vec{i} + v_{yi}\vec{j}) + (a_x\vec{i} + a_y\vec{j})t =$$

$$= \vec{v}_i + \vec{a}t$$

# 2-dim Motion Under Constant Acceleration

- How are the 2D position vectors written in acceleration vectors?

Position vector components

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Putting them together in a vector form

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} =$$

$$= \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

Regrouping the above

$$= \left(x_i\vec{i} + y_i\vec{j}\right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right)t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j}\right)t^2$$

$$= \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

2D problems can be interpreted as two 1D problems in x and y

# Example for 2-D Kinematic Equations

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x = 4.0m/s^2$. Determine the components of the velocity vector at any time t.

$$v_{xf} = v_{xi} + a_x t = 20 + 4.0t\,(m/s) \qquad v_{yf} = v_{yi} + a_y t = -15 + 0t = -15\,(m/s)$$

Velocity vector

$$\vec{v}(t) = v_x(t)\vec{i} + v_y(t)\vec{j} = (20 + 4.0t)\vec{i} - 15\vec{j}\,(m/s)$$

Compute the velocity and the speed of the particle at t=5.0 s.

$$\vec{v}_{t=5} = v_{x,t=5}\vec{i} + v_{y,t=5}\vec{j} = (20 + 4.0 \times 5.0)\vec{i} - 15\vec{j} = \left(40\vec{i} - 15\vec{j}\right)\,m/s$$

$$speed = \left|\vec{v}\right| = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(40)^2 + (-15)^2} = 43m/s$$

# Example for 2-D Kinematic Eq. Cnt'd

Angle of the Velocity vector

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^\circ$$

Determine the $x$ and $y$ components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times 5 + \frac{1}{2}\times 4\times 5^2 = 150(m)$$

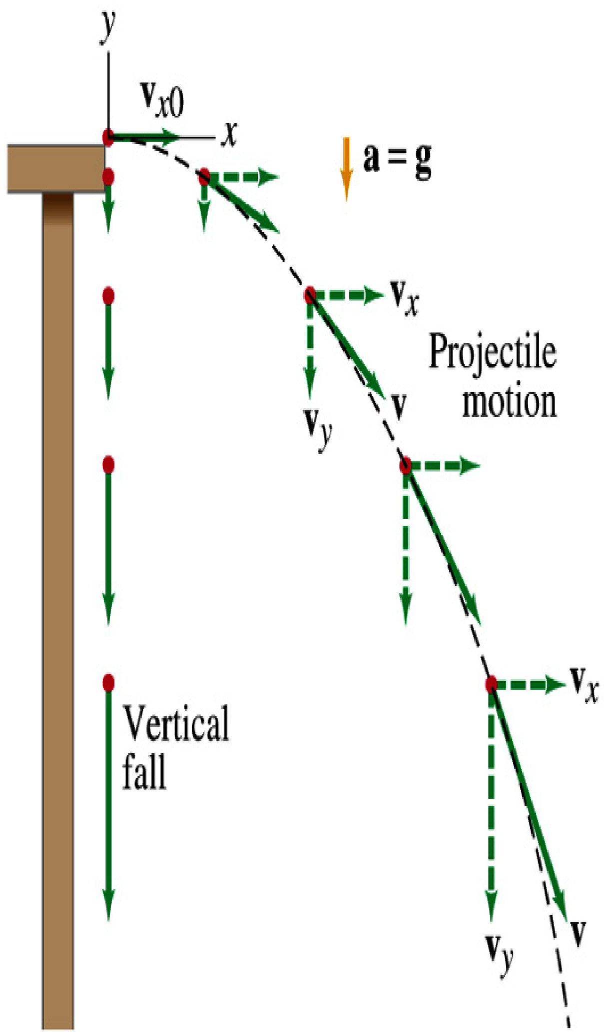
$$y_f = v_{yi}t = -15\times 5 = -75\,(m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = 150\vec{i} - 75\vec{j}\,(m)$$

# Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, $g$, is constant over the range of the motion
    - $\vec{g} = -9.8\vec{j}\left(m/s^2\right)$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity ( no acceleration )
  - Vertical motion under constant acceleration ( $g$ )



# *Show that a projectile motion is a parabola!!!*

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

$a_x=0$

$$x_f = v_{xi}t = v_i \cos\theta_i t$$

$$\boxed{t = \frac{x_f}{v_i \cos\theta_i}}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)\,t^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

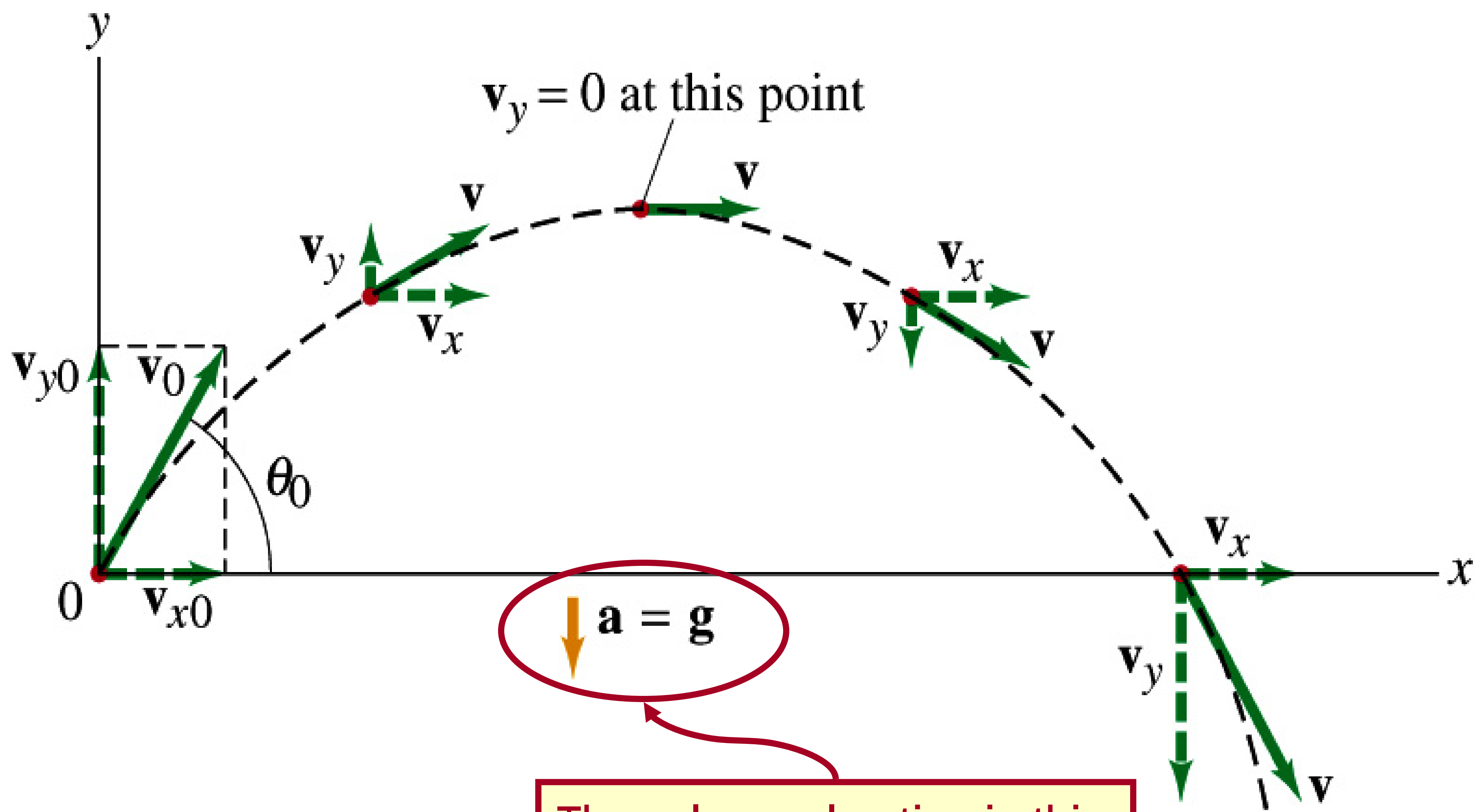
Plug t into the above

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$\boxed{y_f = x_f \tan\theta_i - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right)x_f^2}$$

What kind of parabola is this?

# Projectile Motion




The only acceleration in this motion. It is a constant!!

# Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by the $y$ component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore \ t \approx 8\sec$$

Why isn't 0 the solution?

Distance is determined by the $x$ component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

$$x_f = v_{xi}t = 20\times 8 = 160(m)$$