PHYS 1443 – Section 002
Lecture #10

Monday, Oct. 8, 2007

Dr. Jaehoon Yu

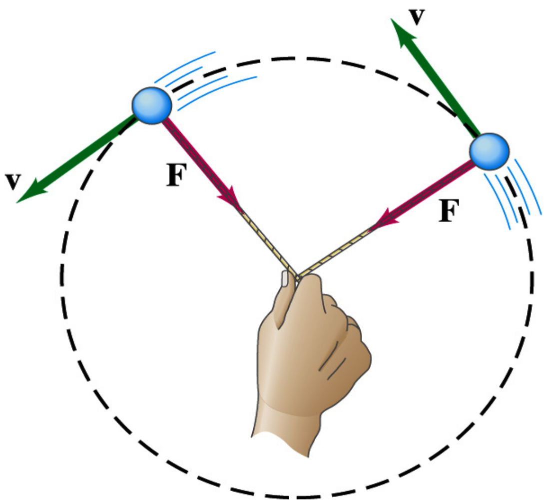
- Uniform and Non-uniform Circular Motions
- Motion Under Resistive Forces
- Newton's Law of Universal Gravitation
- Free Fall Acceleration

Today's homework is homework #7, due 7pm, Monday, Oct. 15!!

Announcements

- Homework #6 due extended to 7pm, Wednesday, Oct. 10

Newton's Second Law & Uniform Circular Motion



The ***centripetal**** *acceleration is always perpendicular to the velocity vector,* ***v****, and points to the center of the axis (radial direction) in a uniform circular motion.*

$$\boxed{a_r = \frac{v^2}{r}}$$

Are there forces in this motion? If so, what do they do?

The force that causes the centripetal acceleration acts toward the center of the circular path and causes the change in the direction of the velocity vector. This force is called the centripetal force.

$$\sum F_r = ma_r = m\frac{v^2}{r}$$

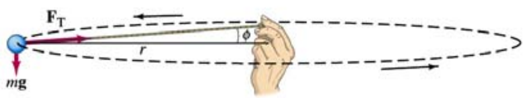
What do you think will happen to the ball if the string that holds the ball breaks?

The external force no longer exist. Therefore, based on Newton's 1st law, the ball will continue its motion without changing its velocity and will fly away following the tangential direction to the circle.

*Mirriam Webster: Proceeding or acting in a direction toward a center or axis

Example of Uniform Circular Motion

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?



Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

When does the string break?

$$\sum F_r = ma_r = m\frac{v^2}{r} > T$$

when the required centripetal force is greater than the sustainable tension.

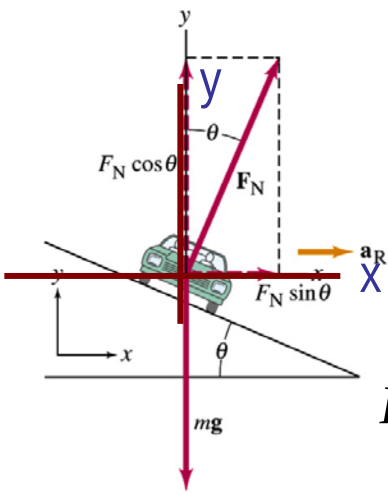
$$m\frac{v^2}{r} = T \qquad v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2\,(m/s)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33\,(N)$$

Example of Banked Highway

(a) For a car traveling with speed v around a curve of radius r, determine the formula for the angle at which the road should be banked so that no friction is required to keep the car from skidding.



x comp. $\sum F_x = F_N \sin\theta - ma_r = F_N \sin\theta - \frac{mv^2}{r} = 0$

$$F_N \sin\theta = \frac{mv^2}{r}$$

y comp. $\sum F_y = F_N \cos\theta - mg = 0 \qquad F_N \cos\theta = mg$

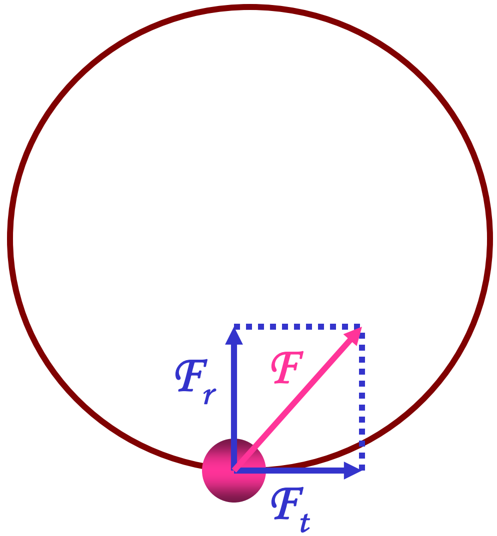
$$F_N = \frac{mg}{\cos\theta} \Rightarrow F_N \sin\theta = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = \frac{mv^2}{r}$$

$$\boxed{\tan\theta = \frac{v^2}{gr}}$$

(b) What is this angle for an expressway off-ramp curve of radius 50m at a design speed of 50km/h?

$$v = 50km/hr = 14m/s \qquad \tan\theta = \frac{(14)^2}{50 \times 9.8} = 0.4 \qquad \theta = \tan^{-1}(0.4) = 22^{\circ}$$

Forces in Non-uniform Circular Motion



The object has both tangential and radial accelerations.

What does this statement mean?

The object is moving under both tangential and radial forces.

$$\vec{F} = \vec{F}_r + \vec{F}_t$$

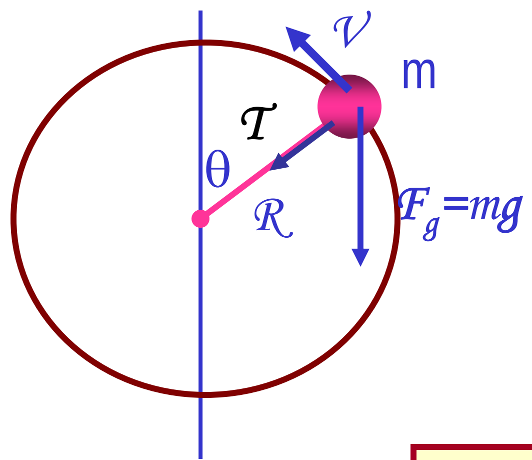
These forces cause not only the velocity but also the speed of the ball to change. The object undergoes a curved motion in the absence of constraints, such as a string.

What is the magnitude of the net acceleration?

$$a = \sqrt{a_r^2 + a_t^2}$$

Example for Non-Uniform Circular Motion

A ball of mass m is attached to the end of a cord of length R. The ball is moving in a vertical circle. Determine the tension of the cord at any instance in which the speed of the ball is v and the cord makes an angle θ with vertical.



What are the forces involved in this motion?

- The gravitational force F_g
- The radial force, T, providing the tension.

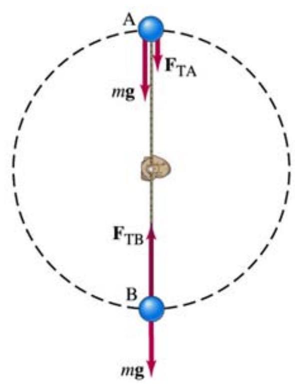
tangential comp.

$$\sum F_t = mg \sin\theta = ma_t \qquad \boxed{a_t = g \sin\theta}$$

Radial comp.

$$\sum F_r = T + mg\cos\theta = ma_r = m\frac{v^2}{R} \qquad \boxed{T = m\left(\frac{v^2}{R} - g\cos\theta\right)}$$

At what angles the tension becomes the maximum and the minimum. What are the tensions?



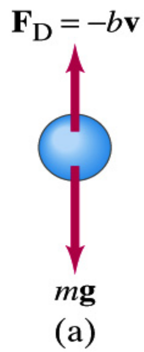
Motion in Resistive Forces

Medium can exert resistive forces on an object moving through it due to viscosity or other types frictional properties of the medium.

Some examples? Air resistance, viscous force of liquid, etc

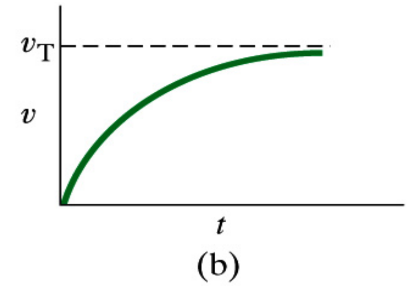
These forces are exerted on moving objects in opposite direction of the movement.

These forces are proportional to such factors as speed. They almost always increase with increasing speed.



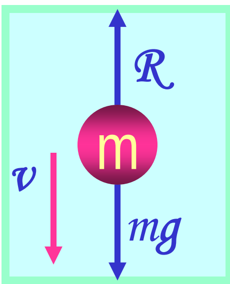
Two different cases of proportionality:

1. Forces linearly proportional to speed: Slowly moving or very small objects
2. Forces proportional to square of speed: Large objects w/ reasonable speed



Resistive Force Proportional to Speed

Since the resistive force is proportional to speed, we can write R=bv.



Let's consider that a ball of mass m is falling through a liquid.

$$\sum \vec{F} = \vec{F}_g + \vec{R} = m\vec{a} \qquad \sum F_x = 0$$

In other words

$$\sum F_y = mg - bv = ma = m\frac{dv}{dt} \qquad \frac{dv}{dt} = g - \frac{b}{m}v \qquad \frac{dv}{dt} = g - \frac{b}{m}v = g\text{, when } v = 0$$

The above equation also tells us that as time goes on the speed increases and the acceleration decreases, eventually reaching 0.

What does this mean?

An object moving in a viscous medium will obtain speed to a certain speed (terminal speed) and then maintain the same speed without any more acceleration.

What is the terminal speed in above case?

$$\frac{dv}{dt} = g - \frac{b}{m}v = 0; \quad v_t = \frac{mg}{b}$$

How do the speed and acceleration depend on time?

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right); \quad v = 0 \text{ when } t = 0;$$

$$a = \frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-bt/m} = ge^{-t/\tau}; \quad a = g \text{ when } t = 0;$$

$$\frac{dv}{dt} = \frac{mg}{b}\frac{b}{m}e^{-t/\tau} = \frac{mg}{b}\frac{b}{m}\left(1 - 1 + e^{-t/\tau}\right) = g - \frac{b}{m}v$$

The time needed to reach 63.2% of the terminal speed is defined as the time constant, $\tau = m/b$.

Newton's Law of Universal Gravitation

People have been very curious about the stars in the sky, making observations for a long time. The data people collected, however, have not been explained until Newton has discovered the law of gravitation.

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

How would you write this law mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2} \quad \xrightarrow{\text{With } G} \quad F_g = G\frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

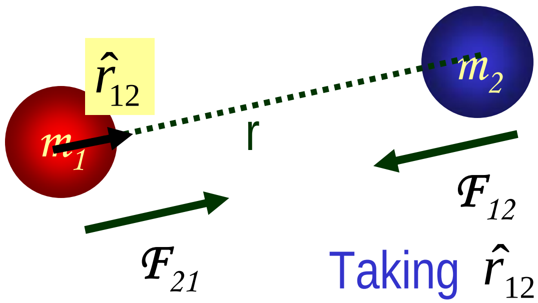
Unit? $N \cdot m^2 / kg^2$

This constant is not given by the theory but must be measured by experiments.

This form of forces is known as the inverse-square law, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

More on Law of Universal Gravitation

Consider two particles exerting gravitational forces to each other.



The two objects exert gravitational force on each other following Newton's 3rd law.

Taking $\hat{r}_{12}$ as the unit vector, we can write the force m_2 experiences as

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$

What do you think the negative sign means?

It means that the force exerted on the particle 2 by particle 1 is an attractive force, pulling #2 toward #1.

Gravitational force is a field force: Forces act on object without a physical contact between the objects at all times, independent of medium between them.

The gravitational force exerted by a finite size, spherically symmetric mass distribution on an object outside of it is the same as when the entire mass of the distributions is concentrated at the center of the object.

What do you think the gravitational force on the surface of the earth looks?

$$F_g = G\frac{M_E m}{R_E^2} = mg$$

Example for Universal Gravitation

Using the fact that g=9.80m/s² on the Earth's surface, find the average density of the Earth.

Since the gravitational acceleration is

$$F_g = G\frac{M_E m}{R_E^2} = mg$$

Solving for g

$$g = G\frac{M_E}{R_E^2} = 6.67\times 10^{-11}\frac{M_E}{R_E^2}$$

Solving for M_E

$$M_E = \frac{R_E^2 g}{G}$$

Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\frac{R_E^2 g}{G}}{\frac{4\pi}{3}R_E^3} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3\times 9.80}{4\pi\times 6.67\times 10^{-11}\times 6.37\times 10^6} = 5.50\times 10^3 kg/m^3$$