PHYS 1443 – Section 002
Lecture #12

Monday, Oct. 15, 2007
Dr. Jaehoon Yu

- Work done by a Varying Force
- Work and Kinetic Energy Theorem
- Potential Energy and the Conservative Force
  - Gravitational Potential Energy
  - Elastic Potential Energy
- Conservation of Energy

Today's homework is homework #8, due 7pm, Monday, Oct. 22!!

Announcements

- 2^{nd} term exam on Wednesday, Oct. 24
  - Covers from Ch. 4.1 to what we cover Monday, Oct. 22 (Ch 8 or 9)
  - Time: 1 – 2:20pm in class
  - Location: SH103
- Special Project due this Wednesday, Oct. 17

Work Done by Varying Force

- If the force depends on the position of the object in motion,

$\longrightarrow$ one must consider the work in small segments of the displacement where the force can be considered constant

$$\Delta W = F_x \cdot \Delta x$$

– Then add all the work-segments throughout the entire motion ($x_i \rightarrow x_f$)

$$W \approx \sum_{x_i}^{x_f} F_x \cdot \Delta x$$

In the limit where $\Delta x \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \sum_{x_i}^{x_f} F_x \cdot \Delta x = \int_{x_i}^{x_f} F_x dx = W$$

– If more than one force is acting, the net work done by the net force is

$$W(net) = \int_{x_i}^{x_f} \left(\sum F_{ix} \right) dx$$

One of the position dependent forces is the force by the spring $\boxed{F_s = -kx}$

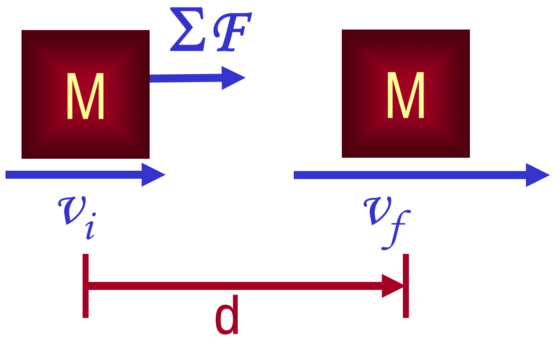
Hooke's Law

The work done by the spring force is

$$W = \int_{-x_{max}}^{0} F_s dx = \int_{-x_{max}}^{0} (-kx)\, dx = \frac{1}{2} k x_{max}^2$$

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on an object during the motion are complicated
  - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force $\Sigma\mathcal{F}$ was exerted on an object for displacement d to increase its speed from v_i to v_f.

The work on the object by the net force $\Sigma\mathcal{F}$ is

$$W = \left(\sum \vec{F}\right)\cdot\vec{d} = (ma)d\cos 0 = (ma)d$$

Displacement $d = \frac{1}{2}\left(v_f + v_i\right)t$ Acceleration $a = \frac{v_f - v_i}{t}$

Work $W = (ma)d = \left[m\left(\frac{v_f - v_i}{\cancel{t}}\right)\right]\frac{1}{2}\left(v_f + v_i\right)\cancel{t} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Kinetic Energy $\boxed{KE \equiv \frac{1}{2}mv^2}$

Haven't we seen this equation somewhere?

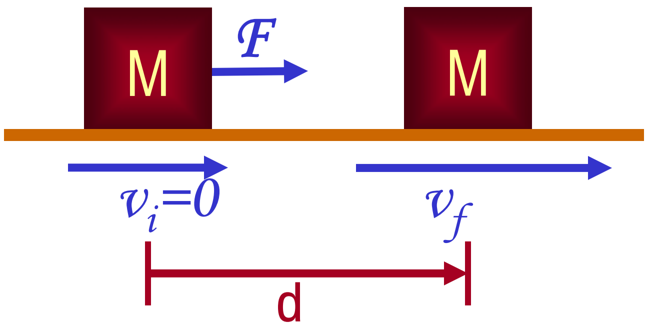
Work $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$

Work done by the net force causes change of the object's kinetic energy.

Work-Kinetic Energy Theorem

Example for Work-KE Theorem

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta = 12 \times 3.0 \cos 0 = 36(J)$$

From the work-kinetic energy theorem, we know $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Since initial speed is 0, the above equation becomes $W = \frac{1}{2}mv_f^2$

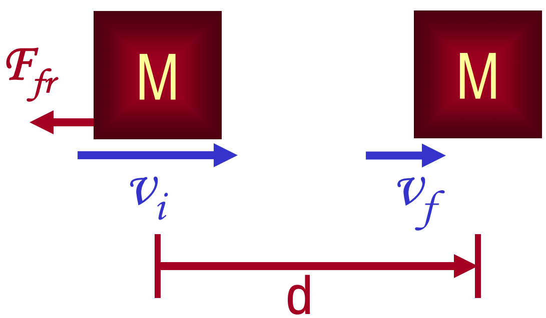
Solving the equation for v_f, we obtain $v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 36}{6.0}} = 3.5m/s$

Work and Energy Involving Kinetic Friction

- What do you think the work looks like if there is friction?
  - Static friction does not matter! Why?

    It isn't there when the object is moving.
  - Then which friction matters?

    Kinetic Friction



Friction force $\mathcal{F}_{fr}$ works on the object to slow down

The work on the object by the friction $\mathcal{F}_{fr}$ is

$$W_{fr} = F_{fr} d\cos(180) = -F_{fr} d \quad \Delta KE = -F_{fr} d$$

The negative sign means that the work is done on the friction!!

The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and other source of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

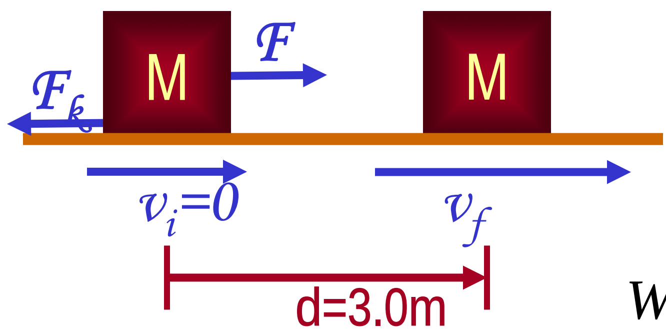
t=0, KE_i

Friction, Engine work

t=T, KE_f

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\,(J)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k \bullet \vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k m g\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26\,(J)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$

Work and Kinetic Energy

A meaningful work in physics is done only when the sum of the forces exerted on an object made a motion to the object.

What does this mean?

However much tired your arms feel, if you were just holding an object without moving it you have not done any physical work to the object.

Mathematically, the work is written as the product of magnitudes of the net force vector, the magnitude of the displacement vector and the angle between them.

$$W = \sum\left(\vec{F}_i\right)\bullet\vec{d} = \left|\sum\left(\vec{F}_i\right)\right|\left|\vec{d}\right|\cos\theta$$

Kinetic Energy is the energy associated with the motion and capacity to perform work. Work causes change of energy after the completion ← Work-Kinetic energy theorem

$$K = \frac{1}{2}mv^2$$

$$\sum W = K_f - K_i = \Delta K$$

Nm=Joule

Potential Energy

Energy associated with a system of objects → Stored energy which has the potential or the possibility to work or to convert to kinetic energy

What does this mean?

In order to describe potential energy, U, a system must be defined.

The concept of potential energy can only be used under the special class of forces called the conservative force which results in the principle of <u>conservation of mechanical energy.</u>

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

What are other forms of energies in the universe?

Mechanical Energy *Chemical Energy* *Biological Energy*

Electromagnetic Energy *Nuclear Energy*

These different types of energies are stored in the universe in many different forms!!!

If one takes into account ALL forms of energy, the total energy in the entire universe is conserved. It just transforms from one form to another.