PHYS 1443 – Section 002
Lecture #14

Monday, Oct. 22, 2007

Dr. Mark Sosebee

- Energy Diagram and Equilibrium
- Generalized Gravitational Potential Energy
- Power
- Linear Momentum
- Conservation of Momentum
- Impulse and Momentum Change

Today's homework is HW #9, due 7pm, Monday, Oct. 29!!

Announcements

- 2^{nd} term exam this Wednesday, Oct. 24
  - Covers from Ch. 4.1 to what we cover today
  - Time: 1 – 2:20pm in class
  - Location: SH103
- Please do NOT miss the exam

How is the conservative force related to the potential energy?

Work done by a force component on an object through the displacement Δx is

$$W = F_x \Delta x = -\Delta U$$

For an infinitesimal displacement Δx

$$\lim_{\Delta x \to 0} \Delta U = -\lim_{\Delta x \to 0} F_x \Delta x$$

$$dU = -F_x dx$$

Results in the conservative force-potential relationship

$$F_x = -\frac{dU}{dx}$$

This relationship says that any conservative force acting on an object within a given system is the same as the negative derivative of the potential energy of the system with respect to the position.

Does this statement make sense?

1. spring-ball system:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

2. Earth-ball system:

$$F_g = -\frac{dU_g}{dy} = -\frac{d}{dy}(mgy) = -mg$$

The relationship works in both the conservative force cases we have learned!!!

Energy Diagram and the Equilibrium of a System

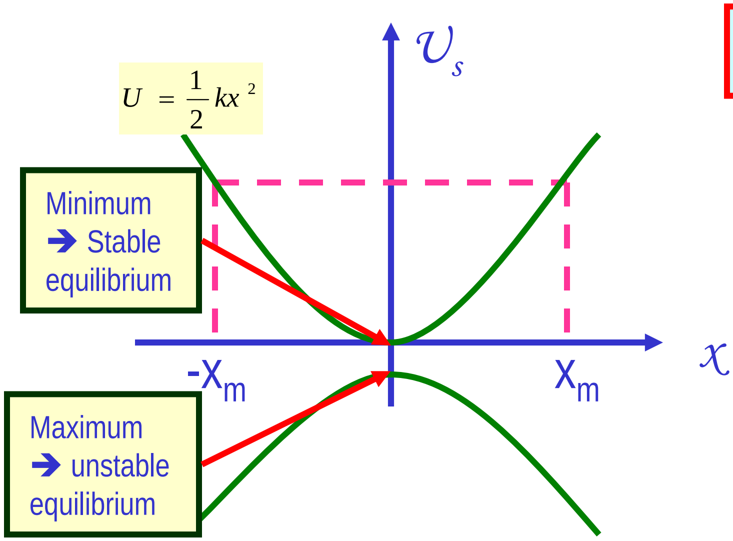
One can draw potential energy as a function of position ➔ Energy Diagram

Let's consider potential energy of a spring-ball system

$$U_s = \frac{1}{2}kx^2$$

What shape is this diagram?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 at the position.*
3. *x=0 is the stable equilibrium position of this system where the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new forms of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one to another. The total energy of universe is constant as a function of time!! The total energy of the universe is conserved!

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$E_R = mc^2$$

How many joules does your body correspond to?

The Gravitational Field

The gravitational force is a field force. The force exists everywhere in the universe.

If one were to place a test object of mass m at any point in the space in the existence of another object of mass M, the test object will feel the gravitational force exerted by M, $\vec{F}_g = m\vec{g}$.

Therefore the gravitational field g is defined as $\vec{g} \equiv \frac{\vec{F}_g}{m}$

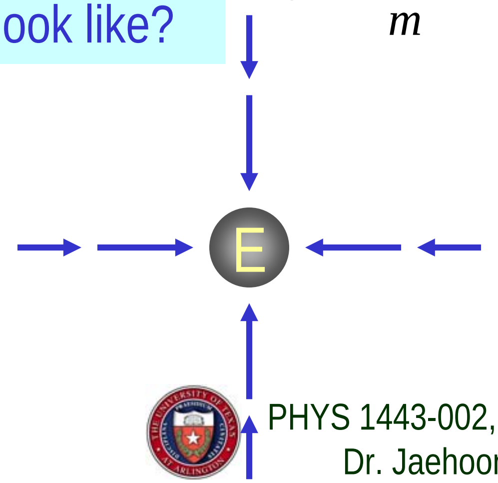
In other words, the gravitational field at a point in the space is the gravitational force experienced by a test particle placed at the point divided by the mass of the test particle.

So how does the Earth's gravitational field look like?

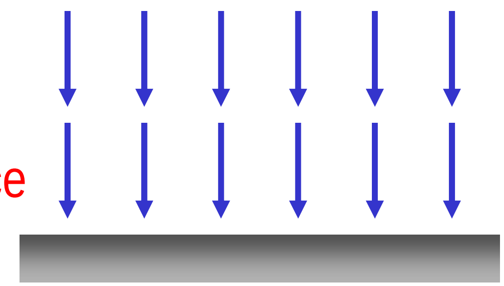
$$\vec{g} = \frac{\vec{F}_g}{m} = -\frac{GM_E}{R_E^2}\hat{r}$$

Where $\hat{r}$ is the unit vector pointing outward from the center of the Earth

Far away from the Earth's surface



Close to the Earth's surface



The Gravitational Potential Energy

What is the potential energy of an object at the height y from the surface of the Earth?

$$U = mgy$$

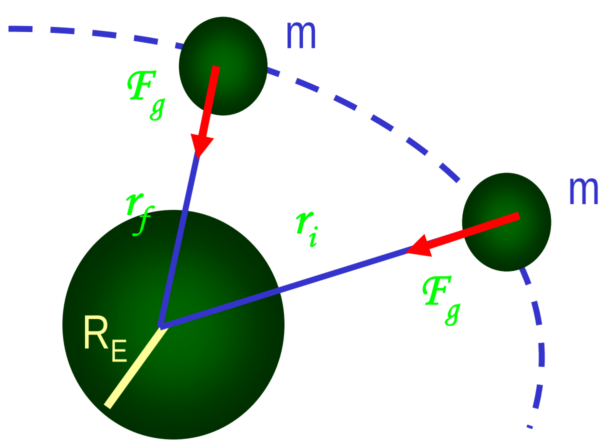
Do you think this would work in general cases?

No, it would not.

Why not?

Because this formula is only valid for the case where the gravitational force is constant, near the surface of the Earth, and the generalized gravitational force is inversely proportional to the square of the distance.

OK. Then how would we generalize the potential energy in the gravitational field?



Since the gravitational force is a central force, and a central force is a conservative force, the work done by the gravitational force is independent of the path.

The path can be considered as consisting of many tangential and radial motions.
Tangential motions do not contribute to work!!!

More on The Gravitational Potential Energy

Since the gravitational force is a radial force, it performs work only when the path has component in radial direction. Therefore, the work performed by the gravitational force that depends on the position becomes:

$$dW = \vec{F} \cdot d\vec{r} = F(r)dr$$

For the whole path

$$W = \int_{r_i}^{r_f} F(r)dr$$

Potential energy is the negative change of the work done through the path

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$$

Since the Earth's gravitational force is

$$F(r) = -\frac{GM_E m}{r^2}$$

Thus the potential energy function becomes

$$U_f - U_i = \int_{r_i}^{r_f} \frac{GM_E m}{r^2} dr = -GM_E m\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

Since only the difference of potential energy matters, by taking the infinite distance as the initial point of the potential energy, we obtain

$$U = -\frac{GM_E m}{r}$$

For any two particles?

$$U = -\frac{Gm_1 m_2}{r}$$

The energy needed to take the particles infinitely apart.

For many particles?

$$U = \sum_{i,j} U_{i,j}$$

Example of Gravitational Potential Energy

A particle of mass m is displaced through a small vertical distance Δy near the Earth's surface. Show that in this situation the general expression for the change in gravitational potential energy is reduced to the $\Delta U = -mg\Delta y$.

Taking the general expression of gravitational potential energy

$$\Delta U = -GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Reorganizing the terms w/ the common denominator

$$= -GM_E m\frac{(r_f - r_i)}{r_f r_i} = -GM_E m\frac{\Delta y}{r_f r_i}$$

Since the situation is close to the surface of the Earth

$r_i \approx R_E$ and $r_f \approx R_E$

Therefore, ΔU becomes

$$\Delta U = -GM_E m\frac{\Delta y}{R_E^2}$$

Since on the surface of the Earth the gravitational field is

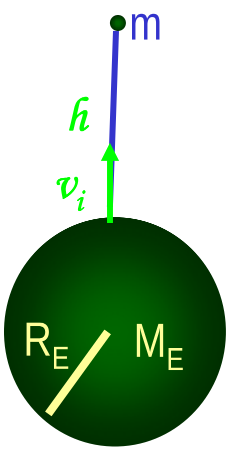
$$g = \frac{GM_E}{R_E^2}$$

The potential energy becomes

$$\Delta U = -mg\Delta y$$

Escape Speed

v_f=0 at $h=r_{max}$



Consider an object of mass m is projected vertically from the surface of the Earth with an initial speed v_i and eventually comes to stop v_f=0 at the distance r_{max}.

Since the total mechanical energy is conserved

$$ME = K + U = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{max}}$$

Solving the above equation for v_i, one obtains

$$v_i = \sqrt{2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{max}}\right)}$$

Therefore if the initial speed v_i is known, one can use this formula to compute the final height *h* of the object.

$$h = r_{max} - R_E = \frac{v_i^2 R_E^2}{2GM_E - v_i^2 R_E}$$

In order for an object to escape Earth's gravitational field completely, the initial speed needs to be

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.37 \times 10^{6}}}$$

$$= 1.12 \times 10^4 m/s = 11.2 km/s$$

This is called the escape speed. This formula is valid for any planet or large mass objects.

How does this depend on the mass of the escaping object?

Independent of the mass of the escaping object

Power

- Rate at which the work is done or the energy is transferred
  - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill?
  - ➔ The time… 8 cylinder car climbs up the hill faster!

Is the total amount of work done by the engines different? NO

Then what is different? The rate at which the same amount of work performed is higher for 8 cylinders than 4.

Average power $\bar{P} \equiv \frac{\Delta W}{\Delta t}$

Instantaneous power $P \equiv \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \lim_{\Delta t \to 0} \left(\sum \vec{F}\right) \cdot \frac{\Delta \vec{s}}{\Delta t} = \left(\sum \vec{F}\right) \cdot \vec{v} = \left|\sum \vec{F}\right| \left|\vec{v}\right| \cos\theta$

Unit? $J/s = Watts$ $1HP \equiv 746Watts$

What do power companies sell? $1kWH = 1000Watts \times 3600s = 3.6 \times 10^6 J$

Energy

Energy Loss in Automobile

Automobile uses only 13% of its fuel to propel the vehicle.

Why?

67% in the engine:

- Incomplete burning
- Heat
- Sound

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

$$m_{car} = 1450kg \quad Weight = mg = 14200N$$

Coefficient of Rolling Friction; μ=0.016

$$\mu n = \mu mg = 227\,N$$

Air Drag

$$f_a = \frac{1}{2} D\rho A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2v^2 = 0.647v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed v=26.8m/s=60mi/h

$$P = f_t v = (691N) \cdot 26.8 = 18.5kW$$

Power to overcome each component of resistance

$$P_r = f_r v = (227) \cdot 26.8 = 6.08kW$$

$$P_a = f_a v = (464.7) \cdot 26.8 = 12.5kW$$

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

*Linear momentum of an object whose mass is m and is moving at a velocity of **v** is defined as*

$$\vec{p} \equiv m\vec{v}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m\left(\vec{v} - \vec{v}_0\right)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$

Linear Momentum and Forces

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

What can we learn from this Force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time.
- If a particle is isolated, the particle experiences no net force. Therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

The relationship can be used to study the case where the mass changes as a function of time.

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d\left(m\vec{v}\right)}{dt} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

Can you think of a few cases like this?

Motion of a meteorite

Motion of a rocket

Conservation of Linear Momentum in a Two Particle System

Consider an isolated system with two particles that do not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

*If particle #1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces, and the net force in the entire SYSTEM is **still 0**.*

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum $\mathbf{p}_1$ *and #2 has* $\mathbf{p}_2$ *at some point of time.*

Using momentum-force relationship

$$\vec{F}_{21} = \frac{d\vec{p}_1}{dt} \quad \text{and} \quad \vec{F}_{12} = \frac{d\vec{p}_2}{dt}$$

And since net force of this system is 0

$$\sum \vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_1}{dt} = \frac{d}{dt}\left(\vec{p}_2 + \vec{p}_1\right) = 0$$

Therefore $\vec{p}_2 + \vec{p}_1 = const$ *The total linear momentum of the system is conserved!!!*