PHYS 1443 – Section 002
Lecture #17

Monday, Nov. 5, 2007
Dr. Jae Yu

- Fundamentals of Rotational Motions
- Rotational Kinematics
- Rolling Motion of a Rigid Body
- Relationship between angular and linear quantities

Today's homework is HW #11, due 9pm, Monday, Nov. 12!!

Announcements

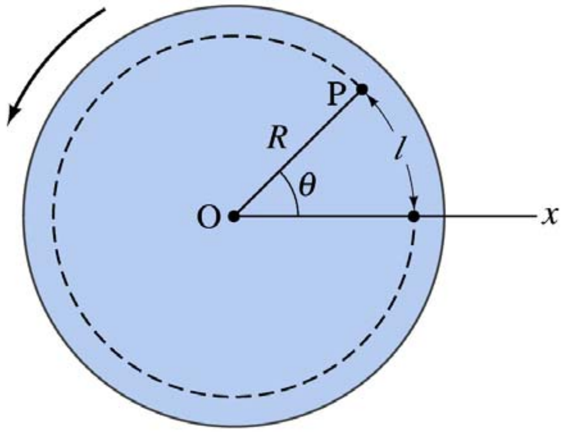
- There were 36 of you at Dr. Seo's colloquium → Impressive!!
- There will be a Ph.D. defense right after this class today!!
  - You all are welcome!!
- Will have to have two midterm grade discussions
  - Today: Last name, A - L
  - Wednesay: Last name, M - Z
- 2nd term exam results
  - Class average:51.8/100
    - Term 1: 58/100
  - Top Score: 90
- Evaluation policy
  - Exams: 22.5/exam → 45%: Will take two best
  - Homework: 25%
  - Lab: 20%
  - Quizzes: 10%
  - Extra credit: 10%
- Quiz on Wednesday, Nov. 7

Fundamentals of Rotational Motions

Linear or translational motions can be described as the motion of the center of mass with all the mass of the object concentrated on it.

Is this still true for rotational motions?

No, because different parts of the object have different linear velocities and accelerations.



Consider a motion of a rigid body – an object that does not change its shape – rotating about the axis protruding out of the slide.

The arc length is $l = R\theta$

Therefore the angle, θ, is $\theta = \frac{l}{R}$. *And the unit of the angle is in **radian**. It is **dimensionless**!!*

One radian is the angle swept by an arc length equal to the radius of the arc.

Since the circumference of a circle is 2πr, $360^\circ = 2\pi r / r = 2\pi$

The relationship between radian and degrees is $1\,\text{rad} = 360^\circ / 2\pi = 180^\circ / \pi$

$$\cong 180^\circ / 3.14 \cong 57.3^\circ$$

Rotational Kinematics

The first type of motion we have learned in linear kinematics was under a constant acceleration. We will learn about the rotational motion under constant angular acceleration, because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular Speed under constant angular acceleration:

$$\omega_f = \omega_i + \alpha t$$

Angular displacement under constant angular acceleration:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

One can also obtain

$$\omega_f^2 = \omega_i^2 + 2\alpha\left(\theta_f - \theta_i\right)$$

Angular Displacement, Velocity, and Acceleration

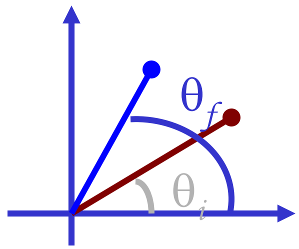
Using what we have learned in the previous slide, how would you define the angular displacement?

$$\Delta\theta = \theta_f - \theta_i$$

How about the average angular speed?

Unit? rad/s

$$\bar{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$



And the instantaneous angular speed?

Unit? rad/s

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

By the same token, the average angular acceleration is defined as…

Unit? rad/s^2

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

And the instantaneous angular acceleration? Unit? rad/s^2

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.

Example for Rotational Kinematics

A wheel rotates with a constant angular acceleration of 3.50 rad/s^2. If the angular speed of the wheel is 2.00 rad/s at t_i=0, a) through what angle does the wheel rotate in 2.00s?

Using the angular displacement formula in the previous slide, one gets

$$\theta_f - \theta_i = \omega t + \frac{1}{2}\alpha t^2$$

$$= 2.00 \times 2.00 + \frac{1}{2} 3.50 \times (2.00)^2 = 11.0 rad$$

$$= \frac{11.0}{2\pi} rev. = 1.75 rev.$$

Example for Rotational Kinematics cnt'd

What is the angular speed at t=2.00s?

Using the angular speed and acceleration relationship

$$\omega_f = \omega_i + \alpha t = 2.00 + 3.50 \times 2.00 = 9.00 rad/s$$

Find the angle through which the wheel rotates between t=2.00s and t=3.00s.

Using the angular kinematic formula

$$\theta_f - \theta_i = \omega t + \frac{1}{2}\alpha t^2$$

At t=2.00s

$$\theta_{t=2} = 2.00 \times 2.00 + \frac{1}{2} 3.50 \times 2.00 = 11.0 rad$$

At t=3.00s

$$\theta_{t=3} = 2.00 \times 3.00 + \frac{1}{2} 3.50 \times (3.00)^2 = 21.8 rad$$

Angular displacement

$$\Delta\theta = \theta_3 - \theta_2 = 10.8 rad = \frac{10.8}{2\pi} rev. = 1.72 rev.$$