PHYS 1443 – Section 002
Lecture #18

Wednesday, Nov. 7, 2007
Dr. Jae Yu

- Rolling Motion of a Rigid Body
- Relationship between angular and linear quantities

Announcements

- Midterm grade discussions for last names, M - Z

"Frontiers in Science"

- a series of informal presentations on popular science topics -

"50 Years of Space Plasma Observations: What Next?"

Dr. Tom Moore

NASA Goddard Space Center

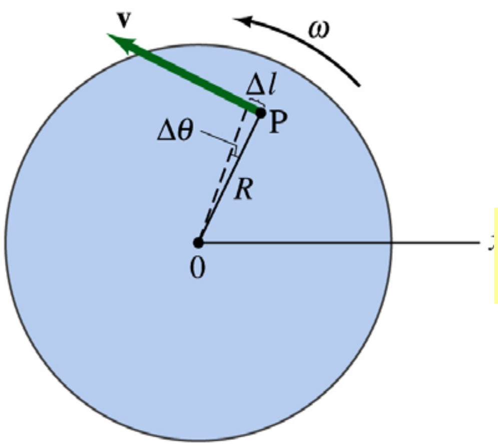
Before the space age of orbiting spacecraft, we thought the ionosphere was a thin layer confined to altitudes below 1000 - 2000 km altitude, based on sounding rocket probes. The magnetosphere appeared to be a cavity into which leaked a small amount of solar wind plasma. Beginning with the Russian Lunik 2, K. Gringauz found that the ionosphere extends out to about 4 Earth radii before dropping steeply in density, and this was confirmed by remote sensing using radio waves from lightning strokes by D. Carpenter. Since this pioneering work at the beginning of the space age, we have known that ionospheric plasmas expand into the magnetosphere at high altitudes inside and beyond the plasmapause. Dr. Moore will present the science of interesting phenomena based on simulation and observations that explains how the magnetosphere acts as a pressure vessel that is filled by the heated expanding ionosphere, inflating it substantially as it seeks escape from both the gravity of Earth and its geomagnetic plasma trap.

Refreshments will be served at 2:00 p.m. in the atrium.
For more information, please call 817-272-3491.

Relationship Between Angular and Linear Quantities

What do we know about a rigid object that rotates about a fixed axis of rotation?

Every particle (or masslet) in the object moves in a circle centered at the same axis of rotation.



When a point rotates, it has both the linear and angular components in its motion.

What is the linear component of the motion you see?

Linear velocity along the tangential direction.

How do we related this linear component of the motion with angular component?

The direction of ω follows a right-hand rule.

The arc-length is $\boxed{l = r\theta}$ So the tangential speed v is

$$v = \frac{dl}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt} = r\omega$$

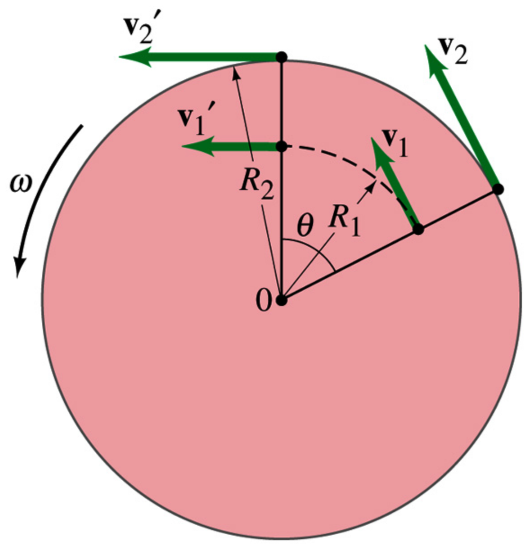
What does this relationship tell you about the tangential speed of the points in the object and their angular speed?:

Although every particle in the object has the same angular speed, its tangential speed differs and is proportional to its distance from the axis of rotation.

The farther away the particle is from the center of rotation, the higher the tangential speed.

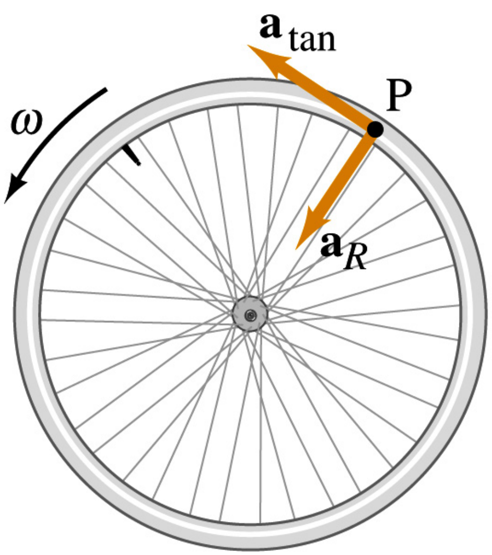
Is the lion faster than the horse?

A rotating carousel has one child sitting on a horse near the outer edge and another child on a lion halfway out from the center. (a) Which child has the greater linear speed? (b) Which child has the greater angular speed?



(a) Linear speed is the distance traveled divided by the time interval. So the child sitting at the outer edge travels more distance within the given time than the child sitting closer to the center. Thus, the horse is faster than the lion.

(b) Angular speed is the angle traveled divided by the time interval. The angle both the children travel in the given time interval is the same. Thus, both the horse and the lion have the same angular speed.



How about the acceleration?

How many different linear acceleration components do you see in a circular motion and what are they? Two

Tangential, a_t, and the radial acceleration, a_r.

Since the tangential speed v is $$v = r\omega$$

The magnitude of tangential acceleration a_t is $$a_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$

What does this relationship tell you?

Although every particle in the object has the same angular acceleration, its tangential acceleration differs proportional to its distance from the axis of rotation.

The radial or centripetal acceleration a_r is $$a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

What does this tell you?

The father away the particle is from the rotation axis, the more radial acceleration it receives. In other words, it receives more centripetal force.

Total linear acceleration is $$a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + \left(r\omega^2\right)^2} = r\sqrt{\alpha^2 + \omega^4}$$

Example

(a) What is the linear speed of a child seated 1.2m from the center of a steadily rotating merry-go-around that makes one complete revolution in 4.0s? (b) What is her total linear acceleration?

First, figure out what the angular speed of the merry-go-around is.

$$\varpi = \frac{1rev}{4.0s} = \frac{2\pi}{4.0s} = 1.6rad/s$$

Using the formula for linear speed

$$v = r\omega = 1.2m \times 1.6rad/s = 1.9m/s$$

Since the angular speed is constant, there is no angular acceleration.

Tangential acceleration is

$$a_t = r\alpha = 1.2m \times 0rad/s^2 = 0m/s^2$$

Radial acceleration is

$$a_r = r\varpi^2 = 1.2m \times (1.6rad/s)^2 = 3.1m/s^2$$

Thus the total acceleration is

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{0 + (3.1)^2} = 3.1m/s^2$$

Example for Rotational Motion

Audio information on compact discs are transmitted digitally through the readout system consisting of laser and lenses. The digital information on the disc are stored by the pits and flat areas on the track. Since the speed of readout system is constant, it reads out the same number of pits and flats in the same time interval. In other words, the linear speed is the same no matter which track is played. a) Assuming the linear speed is 1.3 m/s, find the angular speed of the disc in revolutions per minute when the inner most (r=23mm) and outer most tracks (r=58mm) are read.

Using the relationship between angular and tangential speed $v = r\omega$

$$r = 23mm \qquad \omega = \frac{v}{r} = \frac{1.3m/s}{23mm} = \frac{1.3}{23\times10^{-3}} = 56.5rad/s$$

$$= 9.00rev/s = 5.4\times10^{2}rev/\min$$

$$r = 58mm \qquad \omega = \frac{1.3m/s}{58mm} = \frac{1.3}{58\times10^{-3}} = 22.4rad/s$$

$$= 2.1\times10^{2}rev/\min$$

b) The maximum playing time of a standard music CD is 74 minutes and 33 seconds. How many revolutions does the disk make during that time?

$$\bar{\omega} = \frac{(\omega_i + \omega_f)}{2} = \frac{(540+210)rev/\min}{2} = 375 rev/\min$$

$$\theta_f = \theta_i + \bar{\omega} t = 0 + \frac{375}{60} rev/s \times 4473\ s = 2.8 \times 10^4 rev$$

c) What is the total length of the track past through the readout mechanism?

$$l = v_t \Delta t = 1.3 m/s \times 4473 s = 5.8 \times 10^3 m$$

d) What is the angular acceleration of the CD over the 4473s time interval, assuming constant α?

$$\alpha = \frac{(\omega_f - \omega_i)}{\Delta t} = \frac{(22.4 - 56.5) rad/s}{4473 s} = 7.6 \times 10^{-3} rad/s^2$$