

PHYS 1443 – Section 002

Lecture #19

Monday, Nov. 12, 2007

Dr. Jae Yu

- Rolling Motion of a Rigid Body
- Torque
- Vector Product
- Moment of Inertia
 - Parallel Axis Theorem

Today's homework is HW #12, due 9pm, Monday, Nov. 19!!



Announcements

- Two other problems with homework system
 - In the middle of last week, the answers you have submitted got reset
 - Most of you were ok since you haven't submitted HW 11 yet but some of you who submitted earlier need to double check and might have to re-enter your answers
 - On Nov. 7 and over the weekend, another ghost submission problem occurred
 - Will give you full credit as long as you got the answers right
 - Please be honest with the multiple choice problems
 - Please be sure to send me the problem numbers you had ghost submission problems
- Quiz graded but will be released Wednesday
- Today is the evaluation day



Rolling Motion of a Rigid Body

What is a rolling motion?

A more generalized case of a motion where the rotational axis moves together with an object

A rotational motion about a moving axis

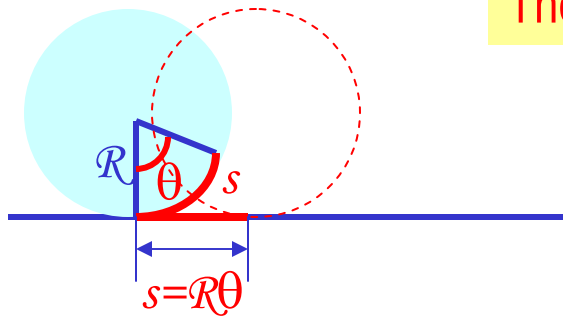
To simplify the discussion, let's make a few assumptions

1. Limit our discussion on very symmetric objects, such as cylinders, spheres, etc
2. The object rolls on a flat surface

Let's consider a cylinder rolling on a flat surface, without slipping.

Under what condition does this “Pure Rolling” happen?

The total linear distance the CM of the cylinder moved is $s = R\theta$



Thus the linear speed of the CM is

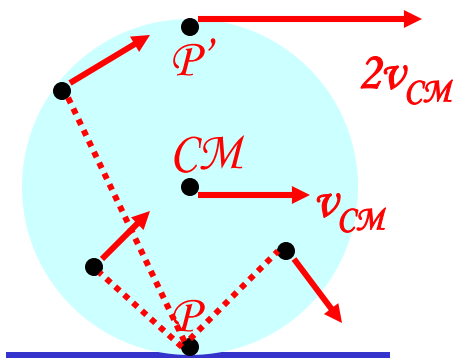
$$v_{CM} = \frac{ds}{dt} = R \frac{d\theta}{dt} = R\omega$$

The condition for a “Pure Rolling motion”

More Rolling Motion of a Rigid Body

The magnitude of the linear acceleration of the CM is

$$a_{CM} = \frac{dv_{CM}}{dt} = R \frac{d\omega}{dt} = R\alpha$$



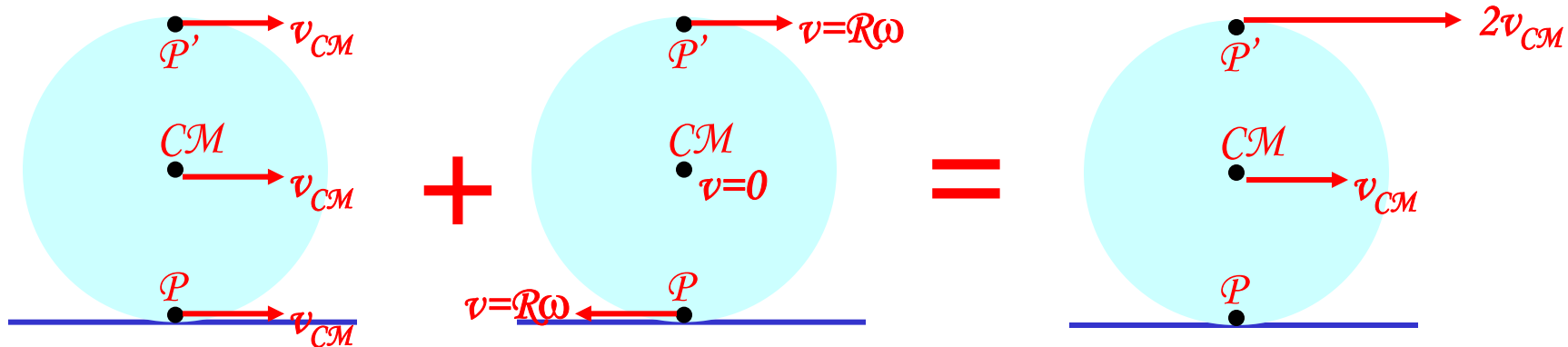
As we learned in rotational motion, all points in a rigid body moves at the same angular speed but at different linear speeds.

CM is moving at the same speed at all times.

At any given time, the point that comes to P has 0 linear speed while the point at P' has twice the speed of CM

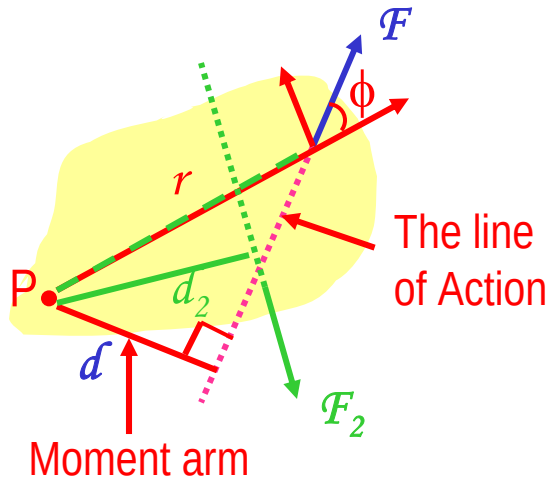
Why??

A rolling motion can be interpreted as the sum of Translation and Rotation



Torque

Torque is the tendency of a force to rotate an object about an axis.
Torque, τ , is a vector quantity.



Consider an object pivoting about the point **P** by the force **F** being exerted at a distance **r** from **P**.

The line that extends out of the tail of the force vector is called the **line of action**.

The perpendicular distance from the pivoting point **P** to the **line of action** is called the **moment arm**.

Magnitude of torque is defined as the product of the force exerted on the object to rotate it and the moment arm.

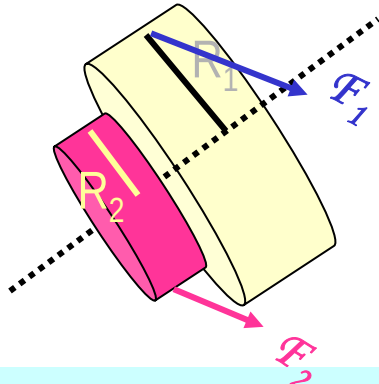
$$\tau \equiv rF \sin \phi = Fd$$

$$\begin{aligned} \sum \tau &= \tau_1 + \tau_2 \\ &= F_1 d_1 - F_2 d_2 \end{aligned}$$

When there are more than one force being exerted on certain points of the object, one can sum up the torque generated by each force vectorially. The convention for sign of the torque is positive if rotation is in counter-clockwise and negative if clockwise.

Example for Torque

A one piece cylinder is shaped as in the figure with core section protruding from the larger drum. The cylinder is free to rotate around the central axis shown in the picture. A rope wrapped around the drum whose radius is R_1 exerts force F_1 to the right on the cylinder, and another force exerts F_2 on the core whose radius is R_2 downward on the cylinder. A) What is the net torque acting on the cylinder about the rotation axis?



The torque due to F_1 $\tau_1 = -R_1 F_1$ and due to F_2 $\tau_2 = R_2 F_2$

So the total torque acting on the system by the forces is

$$\sum \tau = \tau_1 + \tau_2 = -R_1 F_1 + R_2 F_2$$

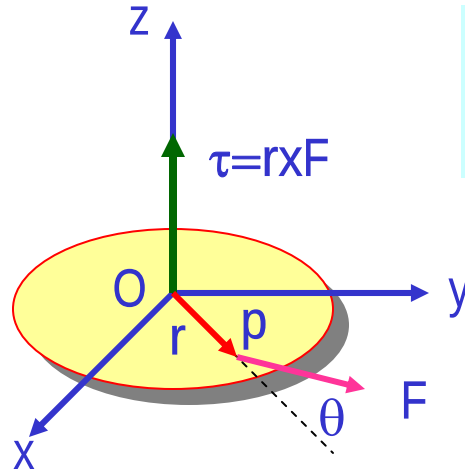
Suppose $F_1 = 5.0 \text{ N}$, $R_1 = 1.0 \text{ m}$, $F_2 = 15.0 \text{ N}$, and $R_2 = 0.50 \text{ m}$. What is the net torque about the rotation axis and which way does the cylinder rotate from the rest?

Using the above result

$$\begin{aligned} \sum \tau &= -R_1 F_1 + R_2 F_2 \\ &= -5.0 \times 1.0 + 15.0 \times 0.50 = 2.5 \text{ N} \cdot \text{m} \end{aligned}$$

The cylinder rotates in counter-clockwise.

Torque and Vector Product



Let's consider a disk fixed onto the origin O and the force $\mathcal{F}$ exerts on the point p. What happens?

The disk will start rotating counter clockwise about the Z axis
The magnitude of torque given to the disk by the force $\mathcal{F}$ is

$$\tau = Fr \sin \theta$$

But torque is a vector quantity, what is the direction?
How is torque expressed mathematically?

$$\vec{\tau} \equiv \vec{r} \times \vec{F}$$

What is the direction?

The direction of the torque follows the right-hand rule!!

The above operation is called the
Vector product or Cross product

$$\vec{C} \equiv \vec{A} \times \vec{B}$$

$$|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$$

What is the result of a vector product?

Another vector

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What is another vector operation we've learned?

Scalar product

$$C \equiv \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

Result? A scalar

Properties of Vector Product

Vector Product is Non-commutative

What does this mean?

If the order of operation changes the result changes

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

Following the right-hand rule, the direction changes

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

Vector Product of two parallel vectors is 0.

$$|\vec{C}| = |\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin \theta = |\vec{A}||\vec{B}|\sin 0 = 0$$

Thus,

$$\vec{A} \times \vec{A} = 0$$

If two vectors are perpendicular to each other

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}|\sin \theta = |\vec{A}||\vec{B}|\sin 90^\circ = |\vec{A}||\vec{B}| = AB$$

Vector product follows distribution law

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

The derivative of a Vector product with respect to a scalar variable is

$$\frac{d(\vec{A} \times \vec{B})}{dt} = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$



More Properties of Vector Product

The relationship between unit vectors, $\vec{i}$, $\vec{j}$ and $\vec{k}$

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

$$\vec{i} \times \vec{j} = -\vec{j} \times \vec{i} = \vec{k}$$

$$\vec{j} \times \vec{k} = -\vec{k} \times \vec{j} = \vec{i}$$

$$\vec{k} \times \vec{i} = -\vec{i} \times \vec{k} = \vec{j}$$

Vector product of two vectors can be expressed in the following determinant form

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \vec{i} \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \vec{j} \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \vec{k} \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$= (A_y B_z - A_z B_y) \vec{i} - (A_x B_z - A_z B_x) \vec{j} + (A_x B_y - A_y B_x) \vec{k}$$



Moment of Inertia

Rotational Inertia:

Measure of resistance of an object to changes in its rotational motion.
Equivalent to mass in linear motion.

For a group
of particles

$$I \equiv \sum_i m_i r_i^2$$

For a rigid
body

$$I \equiv \int r^2 dm$$

What are the dimension and
unit of Moment of Inertia?

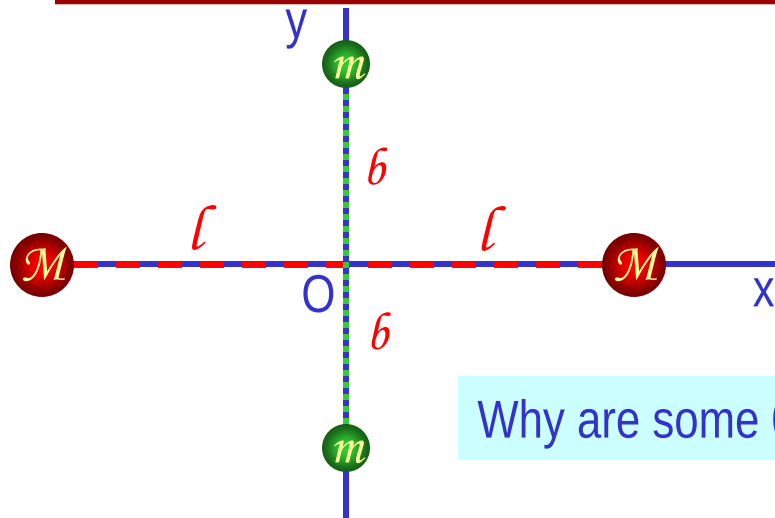
$$[ML^2] \quad kg \cdot m^2$$

Determining Moment of Inertia is extremely important for
computing equilibrium of a rigid body, such as a building.

Dependent on the axis of rotation!!!

Example for Moment of Inertia

In a system of four small spheres as shown in the figure, assuming the radii are negligible and the rods connecting the particles are massless, compute the moment of inertia and the rotational kinetic energy when the system rotates about the y-axis at angular speed ω .



Since the rotation is about y axis, the moment of inertia about y axis, I_y , is

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + m \cdot 0^2 + m \cdot 0^2 = 2Ml^2$$

Why are some 0s?

This is because the rotation is done about y axis, and the radii of the spheres are negligible.

Thus, the rotational kinetic energy is

$$K_R = \frac{1}{2} I \omega^2 = \frac{1}{2} (2Ml^2) \omega^2 = Ml^2 \omega^2$$

Find the moment of inertia and rotational kinetic energy when the system rotates on the x-y plane about the z-axis that goes through the origin O.

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + mb^2 + mb^2 = 2(Ml^2 + mb^2) \quad K_R = \frac{1}{2} I \omega^2 = \frac{1}{2} (2Ml^2 + 2mb^2) \omega^2 = (Ml^2 + mb^2) \omega^2$$