# PHYS 1443 – Section 002
# Lecture #20

*Wednesday, Nov. 14, 2007*
*Dr. Jae Yu*

- Moment of Inertia
  - Parallel Axis Theorem
- Torque and Angular Acceleration
- Rotational Kinetic Energy
- Work, Power and Energy in Rotation

# Announcements

- Problem #14 in HW#11
  - The answer key in the homework system is incorrect many cases
  - I gave all of you full credit for the problem.
  - Please send me the list of problem numbers that you experienced the ghost submission issues
    - I will fix these by hand
- Quiz results
  - Class Average: 4/8 (50/100)
    - Previous quizzes: 50/100, 49/100
  - Top score: 8/8
- There will be a quiz next Monday, Nov. 19, in the beginning of the class
- There will be a cloud chamber dedication ceremony at 12:30pm, Friday, Nov. 16, in CPB 303
  - You are welcome to come and see the chamber in action

# Physics Department
# The University of Texas at Arlington
# COLLOQUIUM

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# Photonic Crystals and Quantum Dots for Infrared Imaging and Sensing

**Dr. Weidong Zhou**
**Department of Engineering**
**University of Texas at Arlington**

**4:00 pm Wednesday, November 14, 2007**
**Room 101 SH**

**Abstract**

Infrared (IR) photodetectors with wide spectral coverage (1 to 20mm) and controllable spectral resolution are highly desirable for absorption spectroscopy gas sensing and hyper-spectral imaging applications. Owing to the light-matter interaction modification, spectrally selective absorption can be achieved in photonic crystal defect cavities, making it a promising nanophotonic platform for the spectrally selective infrared sensing and hyper-spectral imaging. We present here the research results on the proposed photonic crystal quantum dot infrared photodetectors (PC-QDIPs or PCIPs). We show that significantly enhanced absorption at the defect mode can be obtained at surface-normal direction in a dielectric single-defect photonic crystal slab, with an absorption enhancement factor greater than 4,000, based on three-dimensional finite-difference time-domain technique. Complete absorption suppression within the photonic bandgap region can also be observed in defect-free photonic crystal cavities. The dot-in-a-well quantum dot heterostructure was designed and grown by Molecular Beam Epitaxy technology, with center absorption wavelength of 11um. The design and fabrication process will be discussed, along with the experimental results. A slight dark current increase was measured in the over-temperature dark-current measurement, largely due to the increased surface area and large surface recombination velocity. The spectrally selective enhancement in the PCIP devices was also depending on the spectral overlap of the QDIP absorption peak and the PC defect mode. The work is in collaboration with groups within and outside UTA, including AFRL, Duke University and University of New Mexico, etc. The work is supported by SPRING, AFOSR, AFRL CONTACT, TSGC, and NSF.

**Refreshments will be served in the Physics Library at 3:30 pm**

# Calculation of Moments of Inertia

Moments of inertia for large objects can be computed, if we assume that the object consists of small volume elements with mass, $\Delta m_i$.

The moment of inertia for the large rigid object is

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$$

It is sometimes easier to compute moments of inertia in terms of volume of the elements rather than their mass

How do we do this?

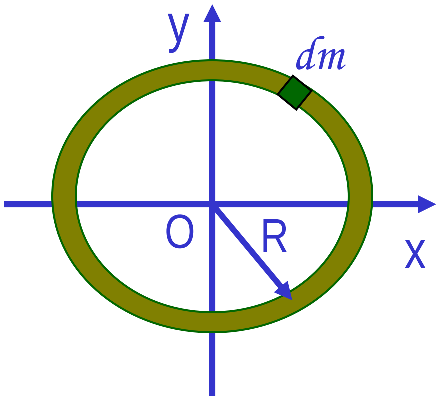
Using the volume density, $\rho$, replace $dm$ in the above equation with dV.

$$\rho = \frac{dm}{dV} \Rightarrow dm = \rho dV$$

The moments of inertia becomes

$$\boxed{I = \int \rho r^2 dV}$$

Example: Find the moment of inertia of a uniform hoop of mass M and radius R about an axis perpendicular to the plane of the hoop and passing through its center.




The moment of inertia is

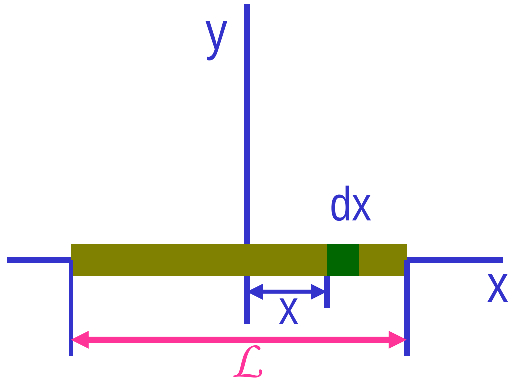
$$I = \int r^2 dm = R^2 \int dm = MR^2$$

What do you notice from this result?

The moment of inertia for this object is the same as that of a point of mass M at the distance R.

# Example for Rigid Body Moment of Inertia

Calculate the moment of inertia of a uniform rigid rod of length L and mass M about an axis perpendicular to the rod and passing through its center of mass.




The line density of the rod is $\lambda = \frac{M}{L}$

so the masslet is $dm = \lambda dx = \frac{M}{L} dx$

The moment of inertia is

$$I = \int r^2 dm = \int_{-L/2}^{L/2} \frac{x^2 M}{L} dx = \frac{M}{L}\left[\frac{1}{3}x^3\right]_{-L/2}^{L/2}$$

$$= \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right] = \frac{M}{3L}\left(\frac{L^3}{4}\right) = \frac{ML^2}{12}$$

What is the moment of inertia when the rotational axis is at one end of the rod.

$$I = \int r^2 dm = \int_0^L \frac{x^2 M}{L} dx = \frac{M}{L}\left[\frac{1}{3}x^3\right]_0^L$$

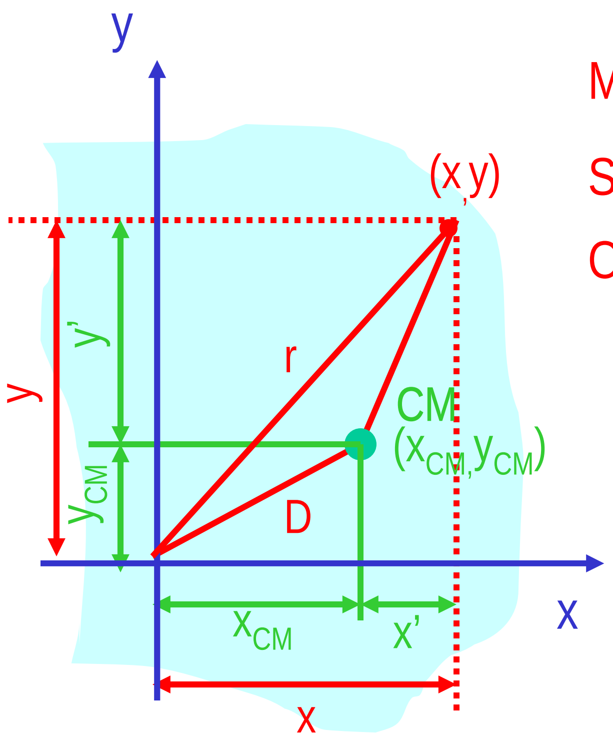
$$= \frac{M}{3L}\left[(L)^3 - 0\right] = \frac{M}{3L}\left(L^3\right) = \frac{ML^2}{3}$$

Will this be the same as the above. Why or why not?

Since the moment of inertia is resistance to motion, it makes perfect sense for it to be harder to move when it is rotating about the axis at one end.

# Parallel Axis Theorem

Moments of inertia for highly symmetric object is easy to compute if the rotational axis is the same as the axis of symmetry. However if the axis of rotation does not coincide with axis of symmetry, the calculation can still be done in a simple manner using **parallel-axis theorem.** $I = I_{CM} + MD^2$




Moment of inertia is defined as $I = \int r^2 dm = \int \left(x^2 + y^2\right) dm \quad (1)$

Since x and y are $\quad x = x_{CM} + x' \quad y = y_{CM} + y'$

One can substitute x and y in Eq. 1 to obtain

$$I = \int \left[ \left(x_{CM} + x'\right)^2 + \left(y_{CM} + y'\right)^2 \right] dm$$

$$= \left(x_{CM}^2 + y_{CM}^2\right)\int dm + 2x_{CM}\int x' dm + 2y_{CM}\int y' dm + \int \left(x'^2 + y'^2\right) dm$$

Since the x' and y' are the distances from CM, by definition $\quad \int x' dm = 0 \quad \int y' dm = 0$

Therefore, the parallel-axis theorem

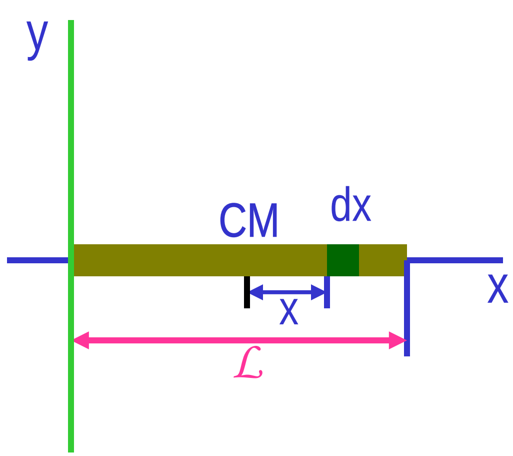
$$I = \left(x_{CM}^2 + y_{CM}^2\right)\int dm + \int \left(x'^2 + y'^2\right) dm = MD^2 + I_{CM}$$

What does this theorem tell you?

Moment of inertia of any object about any arbitrary axis is the sum of moment of inertia for a rotation about the CM and that of the CM about the rotation axis.

# Example for Parallel Axis Theorem

Calculate the moment of inertia of a uniform rigid rod of length L and mass M about an axis that goes through one end of the rod, using parallel-axis theorem.




The line density of the rod is $\lambda = \dfrac{M}{L}$

so the masslet is $dm = \lambda dx = \dfrac{M}{L} dx$

The moment of inertia about the CM

$$I_{CM} = \int r^2 dm = \int_{-L/2}^{L/2} \frac{x^2 M}{L} dx = \frac{M}{L}\left[\frac{1}{3}x^3\right]_{-L/2}^{L/2}$$

$$= \frac{M}{3L}\left[\left(\frac{L}{2}\right)^3 - \left(-\frac{L}{2}\right)^3\right] = \frac{M}{3L}\left(\frac{L^3}{4}\right) = \frac{ML^2}{12}$$

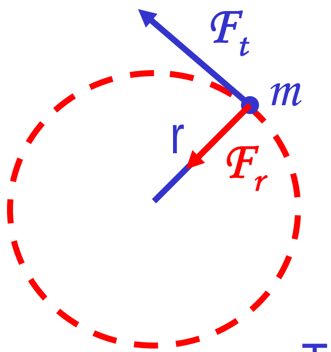
Using the parallel axis theorem

$$I = I_{CM} + D^2 M = \frac{ML^2}{12} + \left(\frac{L}{2}\right)^2 M = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3}$$

The result is the same as using the definition of moment of inertia.

Parallel-axis theorem is useful to compute moment of inertia of a rotation of a rigid object with complicated shape about an arbitrary axis

# Torque & Angular Acceleration




Let's consider a point object with mass $m$ rotating on a circle.

What forces do you see in this motion?

The tangential force $\mathcal{F}_t$ and the radial force $\mathcal{F}_r$

The tangential force $\mathcal{F}_t$ is $F_t = ma_t = mr\alpha$

The torque due to tangential force $\mathcal{F}_t$ is $\tau = F_t r = ma_t r = mr^2\alpha = I\alpha$

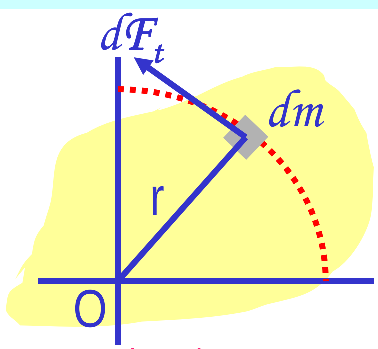
What do you see from the above relationship? $\tau = I\alpha$

What does this mean? Torque acting on a particle is proportional to the angular acceleration.

What law do you see from this relationship? Analogs to Newton's 2nd law of motion in rotation.

How about a rigid object?

The external tangential force $d\mathcal{F}_t$ is $dF_t = dma_t = dmr\alpha$




The torque due to tangential force $\mathcal{F}_t$ is $d\tau = dF_t r = \left(r^2 dm\right)\alpha$

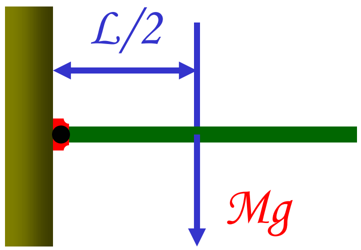
The total torque is $\sum \tau = \alpha \int r^2 dm = I\alpha$

What is the contribution due to radial force and why?

Contribution from radial force is 0, because its line of action passes through the pivoting point, making the moment arm 0.

# Example for Torque and Angular Acceleration

A uniform rod of length $\mathcal{L}$ and mass $\mathcal{M}$ is attached at one end to a frictionless pivot and is free to rotate about the pivot in the vertical plane. The rod is released from rest in the horizontal position. What are the initial angular acceleration of the rod and the initial linear acceleration of its right end?




The only force generating torque is the gravitational force $\mathcal{M}g$

$$\tau = Fd = F\frac{L}{2} = Mg\frac{L}{2} = I\alpha$$

Since the moment of inertia of the rod when it rotates about one end

$$I = \int_0^L r^2 dm = \int_0^L x^2 \lambda dx = \left(\frac{M}{L}\right)\left[\frac{x^3}{3}\right]_0^L = \frac{ML^2}{3}$$

We obtain $$\alpha = \frac{MgL}{2I} = \frac{MgL}{\frac{2ML^2}{3}} = \frac{3g}{2L}$$

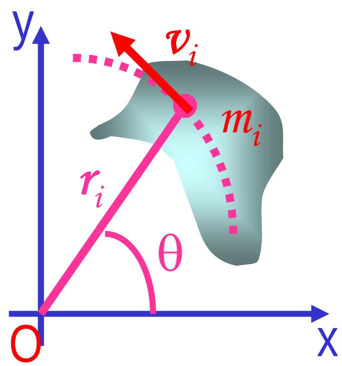
Using the relationship between tangential and angular acceleration

$$a_t = L\alpha = \frac{3g}{2}$$

What does this mean?

The tip of the rod falls faster than an object undergoing a free fall.

# Rotational Kinetic Energy




What do you think the kinetic energy of a rigid object that is undergoing a circular motion is?

Kinetic energy of a masslet, $m_i$, moving at a tangential speed, $v_i$, is

$$K_i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2$$

Since a rigid body is a collection of masslets, the total kinetic energy of the rigid object is

$$K_R = \sum_i K_i = \frac{1}{2} \sum_i m_i r_i^2 \omega^2 = \frac{1}{2} \boxed{\left( \sum_i m_i r_i^2 \right)} \omega^2$$

Since moment of Inertia, I, is defined as

$$I = \sum_i m_i r_i^2$$

The above expression is simplified as

$$K_R = \frac{1}{2} I \omega^2$$