PHYS 1443 – Section 002
Lecture #24

Monday, Dec. 3, 2007
Dr. Jae Yu

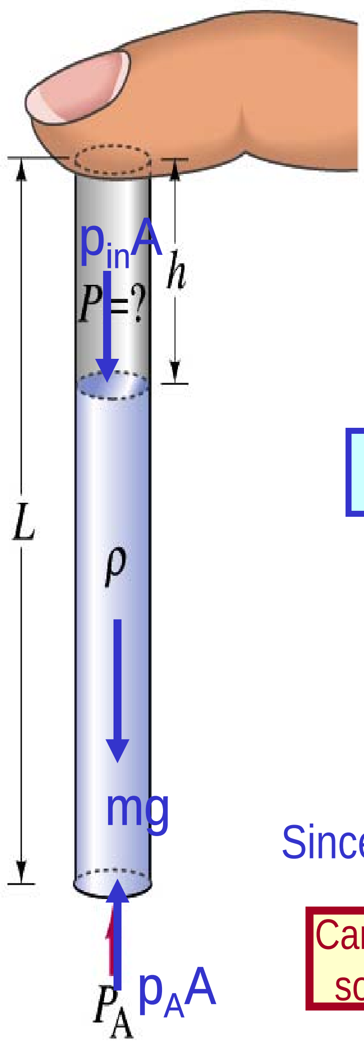
- Buoyant Force and Archimedes' Principle
- Flow Rate and Continuity Equation
- Bernoulli's Equation
- Simple Harmonic Motion
- Equation of the SHM
- Simple Block Spring System
- Energy of the SHO

Today's homework is none!!

Announcements

- Phantom submission problem arose again over the weekend.
  - Same as before, please send me the list of homework problem numbers that you had this problem
  - Will give you full credit as long as you get the answer right
- The last quiz is at the beginning of the class this Wednesday, Dec. 5
- Final exam
  - Date and time: 11am – 12:30 pm, Monday, Dec. 10
  - Location: SH103
  - Covers: CH9.1 – what we finish this Wednesday, Dec. 5 (likely to be Ch14.4)

Finger Holds Water in Straw



You insert a straw of length L into a tall glass of your favorite beverage. You place your finger over the top of the straw so that no air can get in or out, and then lift the straw from the liquid. You find that the straw strains the liquid such that the distance from the bottom of your finger to the top of the liquid is h. Does the air in the space between your finger and the top of the liquid have a pressure P that is (a) greater than, (b) equal to, or (c) less than, the atmospheric pressure P_A outside the straw?

Less

What are the forces in this problem?

Gravitational force on the mass of the liquid $$F_g = mg = \rho A(L-h)g$$

Force exerted on the top surface of the liquid by inside air pressure $$F_{in} = p_{in}A$$

Force exerted on the bottom surface of the liquid by the outside air $$F_{out} = -p_A A$$

Since it is at equilibrium $$F_{out} + F_g + F_{in} = 0$$

$$\Rightarrow -p_A A + \rho g(L-h)A + p_{in}A = 0$$

Cancel A and solve for p_{in}

$$p_{in} = p_A - \rho g(L-h)$$

So p_{in} is less than P_A by $\rho g(L-h)$.

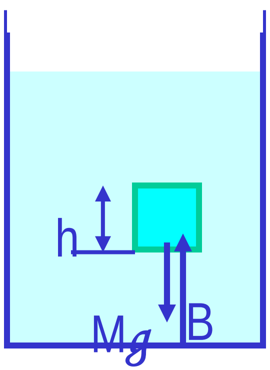
Buoyant Forces and Archimedes' Principle

Why is it so hard to put an inflated beach ball under water while a small piece of steel sinks in the water easily?

The water exerts force on an object immersed in the water.
This force is called the buoyant force.

How large is the buoyant force? The magnitude of the buoyant force always equals the weight of the fluid in the volume displaced by the submerged object.

This is called Archimedes' principle. What does this mean?



Let's consider a cube whose height is h and is filled with fluid and in its equilibrium so that its weight Mg is balanced by the buoyant force B.

$$B = F_g = Mg$$

The pressure at the bottom of the cube is larger than the top by ρgh.

Therefore, $\Delta P = B / A = \rho gh$

$$B = \Delta PA = \rho ghA = \rho Vg$$

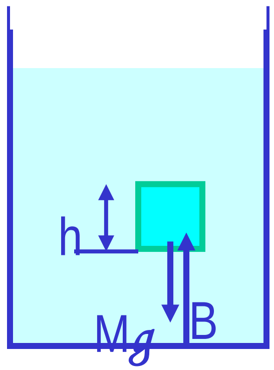
$$B = \rho Vg = Mg = F_g$$

Where Mg is the weight of the fluid in the cube.

More Archimedes' Principle

Let's consider the buoyant force in two special cases.

Case 1: Totally submerged object



Let's consider an object of mass M, with density ρ_0, is fully immersed in the fluid with density ρ_f.

The magnitude of the buoyant force is $B = \rho_f V g$

The weight of the object is $F_g = Mg = \rho_0 V g$

Therefore total force in the system is $F = B - F_g = (\rho_f - \rho_0) V g$

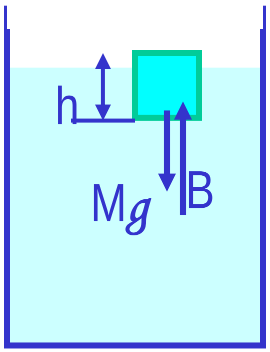
What does this tell you?

The total force applies to different directions depending on the difference of the density between the object and the fluid.

1. If the density of the object is smaller than the density of the fluid, the buoyant force will push the object up to the surface.
2. If the density of the object is larger than the fluid's, the object will sink to the bottom of the fluid.

More Archimedes' Principle

Case 2: Floating object



Let's consider an object of mass M, with density ρ_0, is in static equilibrium floating on the surface of the fluid with density ρ_f, and the volume submerged in the fluid is V_f.

The magnitude of the buoyant force is $B = \rho_f V_f g$

The weight of the object is $F_g = Mg = \rho_0 V_0 g$

Therefore total force of the system is $F = B - F_g = \rho_f V_f g - \rho_0 V_0 g = 0$

Since the system is in static equilibrium $\rho_f V_f g = \rho_0 V_0 g$

$$\frac{\rho_0}{\rho_f} = \frac{V_f}{V_0}$$

What does this tell you?

Since the object is floating, its density is smaller than that of the fluid.

The ratio of the densities between the fluid and the object determines the submerged volume under the surface.

Example for Archimedes' Principle

Archimedes was asked to determine the purity of the gold used in the crown. The legend says that he solved this problem by weighing the crown in air and in water. Suppose the scale read 7.84N in air and 6.86N in water. What should he have to tell the king about the purity of the gold in the crown?

In the air the tension exerted by the scale on the object is the weight of the crown

$$T_{air} = mg = 7.84N$$

In the water the tension exerted by the scale on the object is

$$T_{water} = mg - B = 6.86N$$

Therefore the buoyant force B is

$$B = T_{air} - T_{water} = 0.98N$$

Since the buoyant force B is

$$B = \rho_w V_w g = \rho_w V_c g = 0.98N$$

The volume of the displaced water by the crown is

$$V_c = V_w = \frac{0.98N}{\rho_w g} = \frac{0.98}{1000 \times 9.8} = 1.0 \times 10^{-4} m^3$$

Therefore the density of the crown is

$$\rho_c = \frac{m_c}{V_c} = \frac{m_c g}{V_c g} = \frac{7.84}{V_c g} = \frac{7.84}{1.0 \times 10^{-4} \times 9.8} = 8.3 \times 10^3 kg/m^3$$

Since the density of pure gold is $19.3 \times 10^3 kg/m^3$, this crown is not made of pure gold.

Example for Buoyant Force

What fraction of an iceberg is submerged in the sea water?

Let's assume that the total volume of the iceberg is V_i. Then the weight of the iceberg F_{gi} is

$$F_{gi} = \rho_i V_i g$$

Let's then assume that the volume of the iceberg submerged in the sea water is V_w. The buoyant force B caused by the displaced water becomes

$$B = \rho_w V_w g$$

Since the whole system is at its static equilibrium, we obtain

$$\rho_i V_i g = \rho_w V_w g$$

Therefore the fraction of the volume of the iceberg submerged under the surface of the sea water is

$$\frac{V_w}{V_i} = \frac{\rho_i}{\rho_w} = \frac{917\,kg/m^3}{1030\,kg/m^3} = 0.890$$

About 90% of the entire iceberg is submerged in the water!!!

Flow Rate and the Equation of Continuity

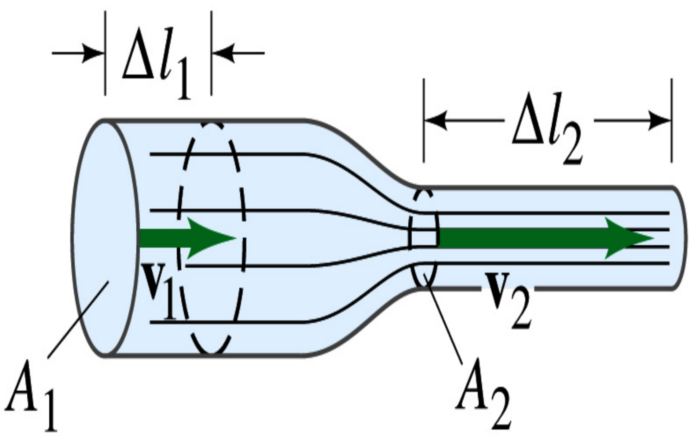
Study of fluid in motion: Fluid Dynamics

If the fluid is water: Water dynamics?? Hydro-dynamics

Two main types of flow

- **Streamline or Laminar flow**: Each particle of the fluid follows a smooth path, a streamline
- **Turbulent flow**: Erratic, small, whirlpool-like circles called eddy current or eddies which absorbs a lot of energy

Flow rate: the mass of fluid that passes the given point per unit time $\Delta m / \Delta t$



$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1$$

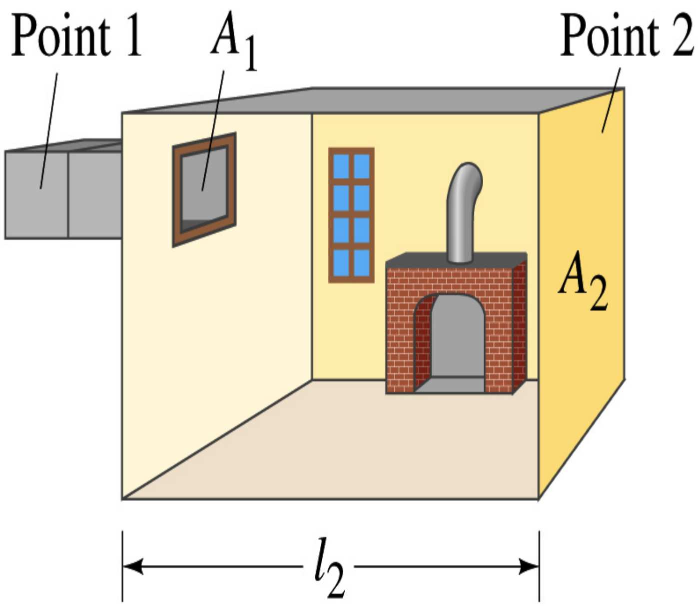
since the total flow must be conserved

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t} \Rightarrow \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Equation of Continuity

Example for Equation of Continuity

How large must a heating duct be if air moving at 3.0m/s through it can replenish the air in a room of 300m^3 volume every 15 minutes? Assume the air's density remains constant.



Using equation of continuity

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

Since the air density is constant

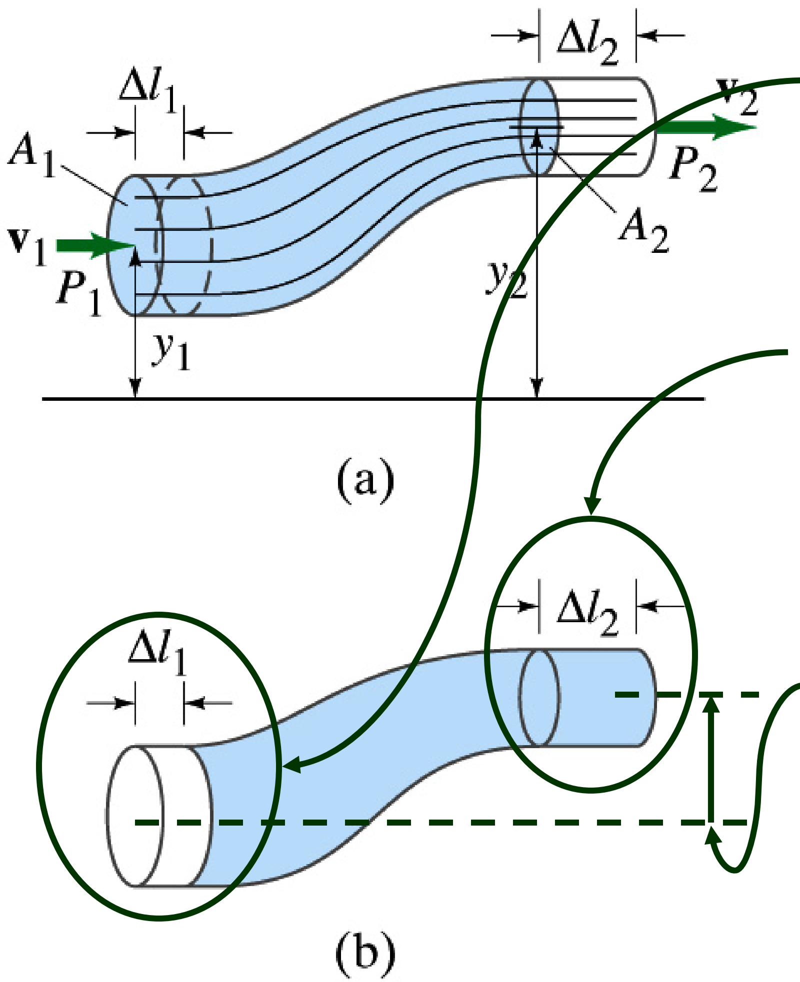
$$A_1 v_1 = A_2 v_2$$

Now let's imagine the room as the large section of the duct

$$A_1 = \frac{A_2 v_2}{v_1} = \frac{A_2 l_2 / t}{v_1} = \frac{V_2}{v_1 \cdot t} = \frac{300}{3.0 \times 900} = 0.11 m^2$$

Bernoulli's Principle

Bernoulli's Principle: Where the velocity of fluid is high, the pressure is low, and where the velocity is low, the pressure is high.



Amount of the work done by the force, F_1, that exerts pressure, P_1, at point 1

$$W_1 = F_1 \Delta l_1 = P_1 A_1 \Delta l_1$$

Amount of the work done by the force in the other section of the fluid is

$$W_2 = -P_2 A_2 \Delta l_2$$

Work done by the gravitational force to move the fluid mass, m, from y_1 to y_2 is

$$W_3 = -mg\left(y_2 - y_1\right)$$

Bernoulli's Equation cont'd

The total amount of the work done on the fluid is

$$W = W_1 + W_2 + W_3 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

From the work-energy principle

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mgy_2 + mgy_1$$

Since the mass m is contained in the volume that flowed in the motion

$$A_1 \Delta l_1 = A_2 \Delta l_2$$ and $$m = \rho A_1 \Delta l_1 = \rho A_2 \Delta l_2$$

Thus,

$$\frac{1}{2}\rho A_2 \Delta l_2 v_2^2 - \frac{1}{2}\rho A_1 \Delta l_1 v_1^2$$

$$= P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - \rho A_2 \Delta l_2 g y_2 + \rho A_1 \Delta l_1 g y_1$$

Bernoulli's Equation cont'd

Since

$$\frac{1}{2}\rho A_2 \Delta l_2 v_2^2 - \frac{1}{2}\rho A_1 \Delta l_1 v_1^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - \rho A_2 \Delta l_2 g y_2 + \rho A_1 \Delta l_1 g y_1$$

We obtain

$$\frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1$$

Re-organize

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Bernoulli's Equation

Thus, for any two points in the flow

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \ const.$$

Result of Energy conservation!

For static fluid

$$P_2 = P_1 + \rho g \left(y_1 - y_2\right) = P_1 + \rho g h$$

Pascal's Law

For the same heights

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

The pressure at the faster section of the fluid is smaller than slower section.

Example for Bernoulli's Equation

Water circulates throughout a house in a hot-water heating system. If the water is pumped at the speed of 0.5m/s through a 4.0cm diameter pipe in the basement under a pressure of 3.0atm, what will be the flow speed and pressure in a 2.6cm diameter pipe on the second 5.0m above? Assume the pipes do not divide into branches.

Using the equation of continuity, flow speed on the second floor is

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{\pi r_1^2 v_1}{\pi r_2^2} = 0.5 \times \left(\frac{0.020}{0.013}\right)^2 = 1.2 m/s$$

Using Bernoulli's equation, the pressure in the pipe on the second floor is

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(y_1 - y_2\right)$$

$$= 3.0\times 10^5 + \frac{1}{2}1\times 10^3\left(0.5^2 - 1.2^2\right) + 1\times 10^3 \times 9.8 \times (-5)$$

$$= 2.5\times 10^5 N/m^2$$