PHYS 1443 – Section 002
Lecture #4

Monday, September 8, 2008
Dr. Jaehoon Yu

- One Dimensional Motion
- Instantaneous Speed and Velocity
- Acceleration
- 1D Motion under constant acceleration
- Free Fall

Today's homework is homework #3, due 9pm, Monday, Sept. 15!!

Announcements

- Homework
  - 63 out of 67 registered so far.
  - 25% of the total
  - Homework site is https://quest.cns.utexas.edu/student/
  - How many of you had trouble with HW#1?
  - Due for homework#1 extended till 9pm tonight, Monday, Sept. 8
    - Last chance!
- 57 out of 67 subscribed to the class e-mail list
  - Thank you for the confirmation!!
- The first term exam is to be next Wednesday, Sept. 17
  - Will cover CH1 – what we finish this Wednesday + Appendices A and B
  - Mixture of multiple choices and essay problems
  - Jason will conduct a review in the class Monday, Sept. 15
- Quiz results
  - Average: 7.8/15 → 52
  - Top score: 14/15
  - Quiz takes up 10% of the final grade!!

LHC Related Links

- LHC Rap: http://www.youtube.com/watch?v=j50ZssEojtM
- Web cast of the Sept. 10 first collisions is at http://webcast.cern.ch
- Further details of the first collisions: http://www.cern.ch/lhc-first-beam
- Some photos:
  - http://multimedia-gallery.web.cern.ch/multimedia-gallery/PhotoGallery_Main.aspx
  - http://www.uslhc.us/Images.
- CERN is 7 hours ahead of us. So if you happen to be awake at around 3am on Wednesday, Sept. 10, you might be able to follow the exciting event of the turn on of the LHC.

Special Problems for Extra Credit

- Derive the quadratic equation for $Bx^2-Cx+A=0$ ➔ 5 points
- Derive the kinematic equation $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$ from first principles and the known kinematic equations ➔ 10 points
- You must show your work in detail to obtain full credit
- Due at the start of the class, Monday, Sept. 22

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

Displacement is the difference between initial and final positions of motion and is a vector quantity. How is this different than distance?

Average velocity is defined as: $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Displacement per unit time in the period throughout the motion

Average speed is defined as:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Can someone tell me what the difference between speed and velocity is?

Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion?
- Instantaneous velocity is defined as:

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

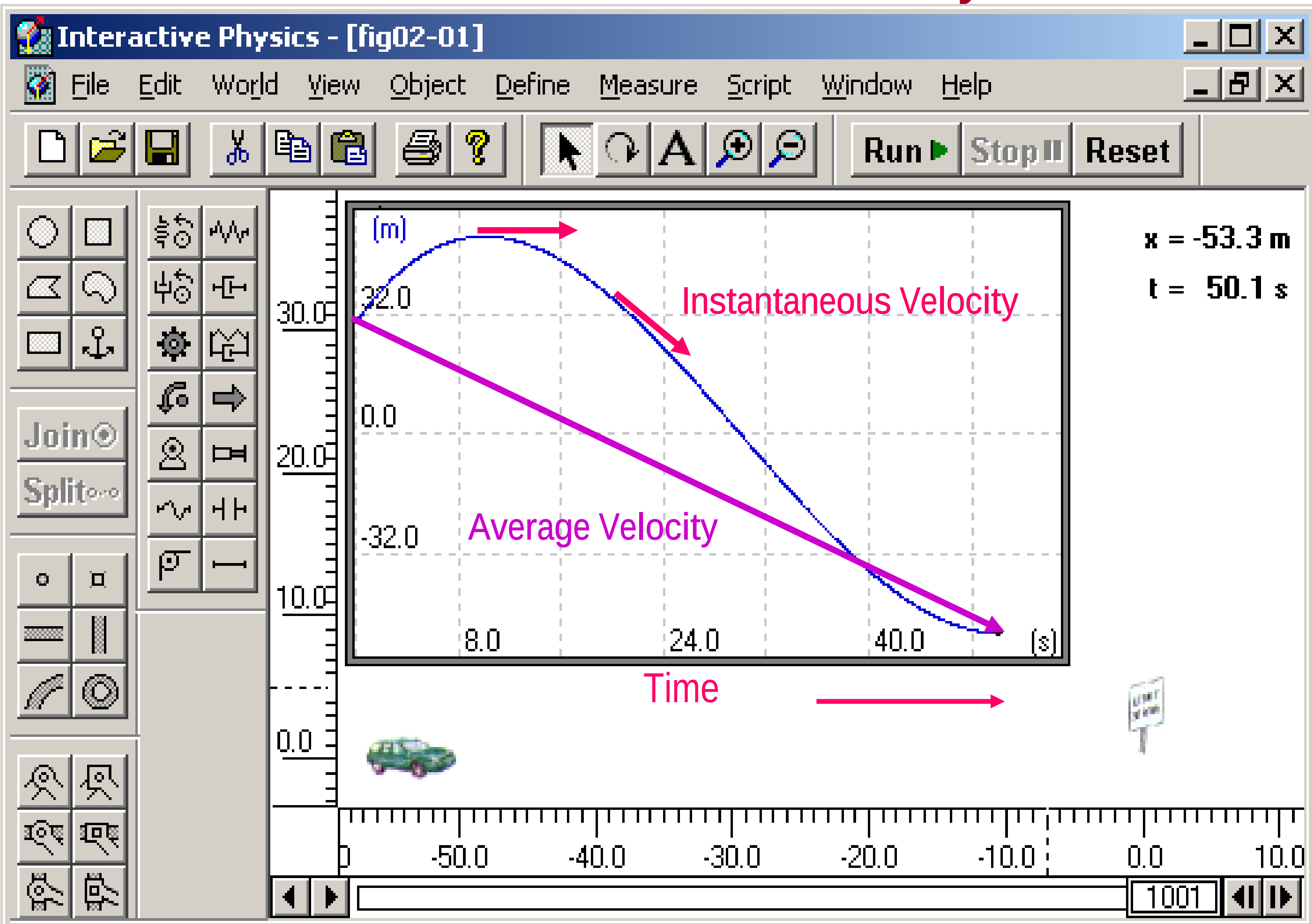
  – What does this mean?
    - Displacement in an infinitesimal time interval
    - Mathematically: Slope of the position variation as a function of time

•Instantaneous speed is the size (magnitude) of the velocity vector:

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right| = \left| \frac{dx}{dt} \right|$$

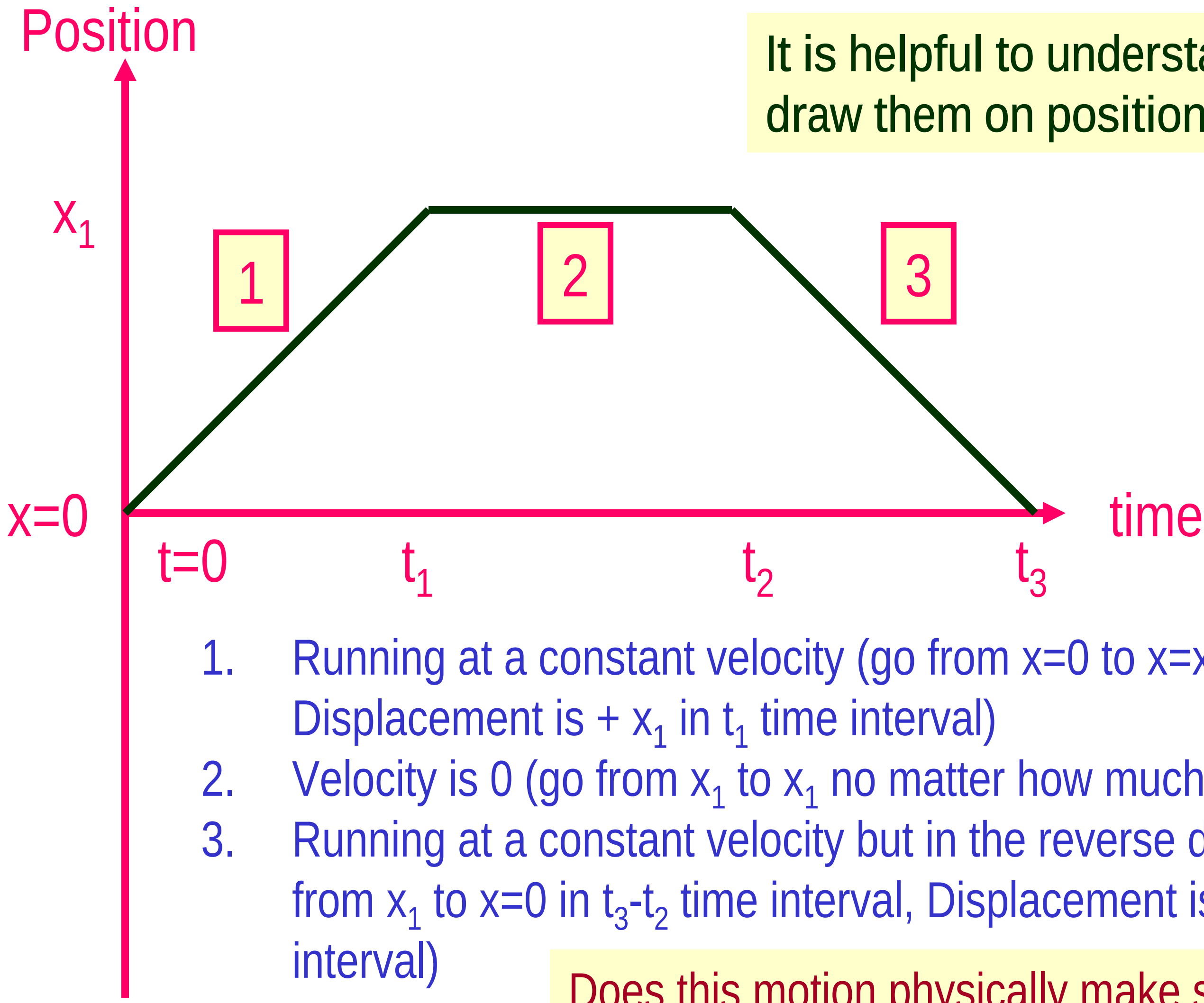
*Magnitude of Vectors are expressed in absolute values

Instantaneous Velocity



Position vs Time Plot

It is helpful to understand motions to draw them on position vs time plots.

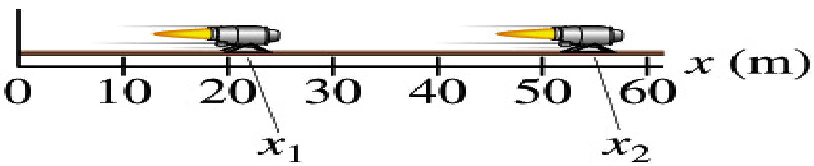


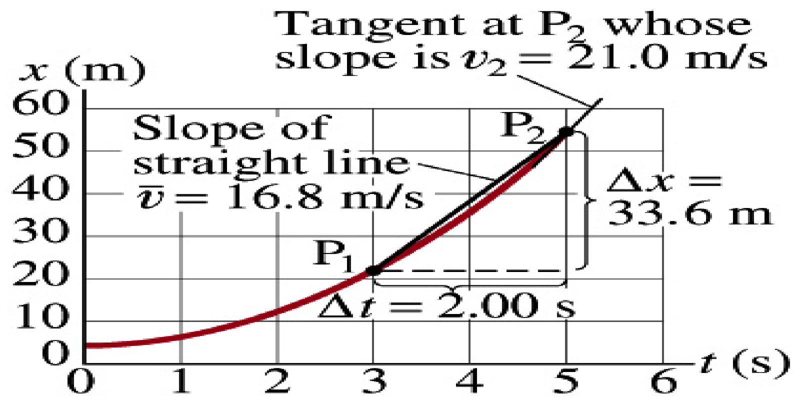
1. Running at a constant velocity (go from $x=0$ to $x=x_1$ in t_1, Displacement is + x_1 in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to $x=0$ in t_3-t_2 time interval, Displacement is - x_1 in t_3-t_2 time interval)

Does this motion physically make sense?

Example 2.3

A jet engine moves along a track. Its position as a function of time is given by the equation $x = At^2 + B$ where A=2.10m/s^2 and B=2.80m.





(a) Determine the displacement of the engine during the interval from t_1=3.00s to t_2=5.00s.

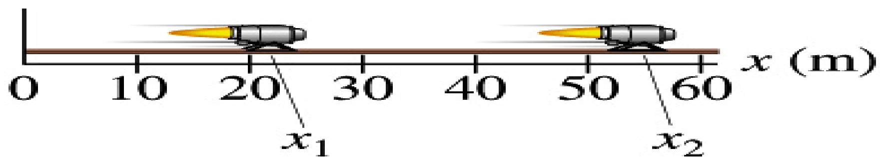
$$x_1 = x_{t_1=3.00} = 2.10\times(3.00)^2 + 2.80 = 21.7m \qquad x_2 = x_{t_2=5.00} = 2.10\times(5.00)^2 + 2.80 = 55.3m$$

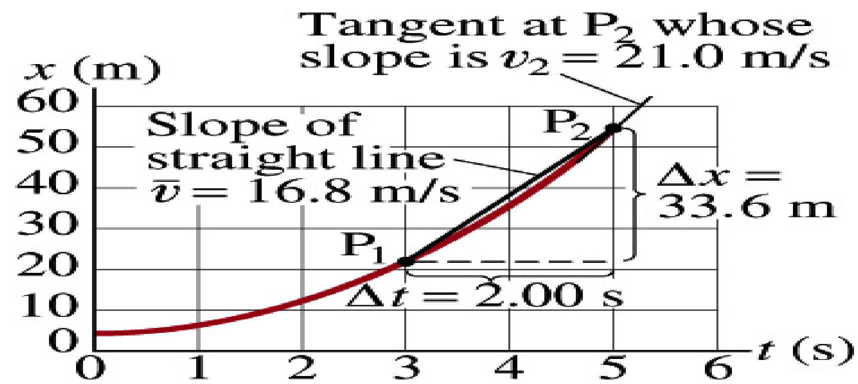
Displacement is, therefore: $\Delta x = x_2 - x_1 = 55.3 - 21.7 = +33.6(m)$

(b) Determine the average velocity during this time interval.

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{33.6}{5.00 - 3.00} = \frac{33.6}{2.00} = 16.8(m/s)$$

Example 2.3 cont'd





(c) Determine the instantaneous velocity at $t=t_2=5.00s$.

Calculus formula for derivative $\boxed{\frac{d}{dt}\left(Ct^n\right)=nCt^{n-1}}$ and $\boxed{\frac{d}{dt}\left(C\right)=0}$

The derivative of the engine's equation of motion is

$$v_x = \lim_{\Delta t \to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}\left(At^2 + B\right) = 2At$$

The instantaneous velocity at t=5.00s is

$$v_x\left(t = 5.00s\right) = 2A \times 5.00 = 2.10 \times 10.0 = 21.0\left(m/s\right)$$

Displacement, Velocity and Speed

Displacement
$$\boxed{\Delta x \equiv x_f - x_i}$$

Average velocity
$$\boxed{v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}}$$

Average speed
$$\boxed{v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}}$$

Instantaneous velocity
$$\boxed{v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

Instantaneous speed
$$\boxed{\left|v_x\right| = \left|\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\right| = \left|\frac{dx}{dt}\right|}$$

Acceleration

Change of velocity in time (what kind of quantity is this?) $\boxed{\text{Vector}}$

- Definition of the average acceleration:

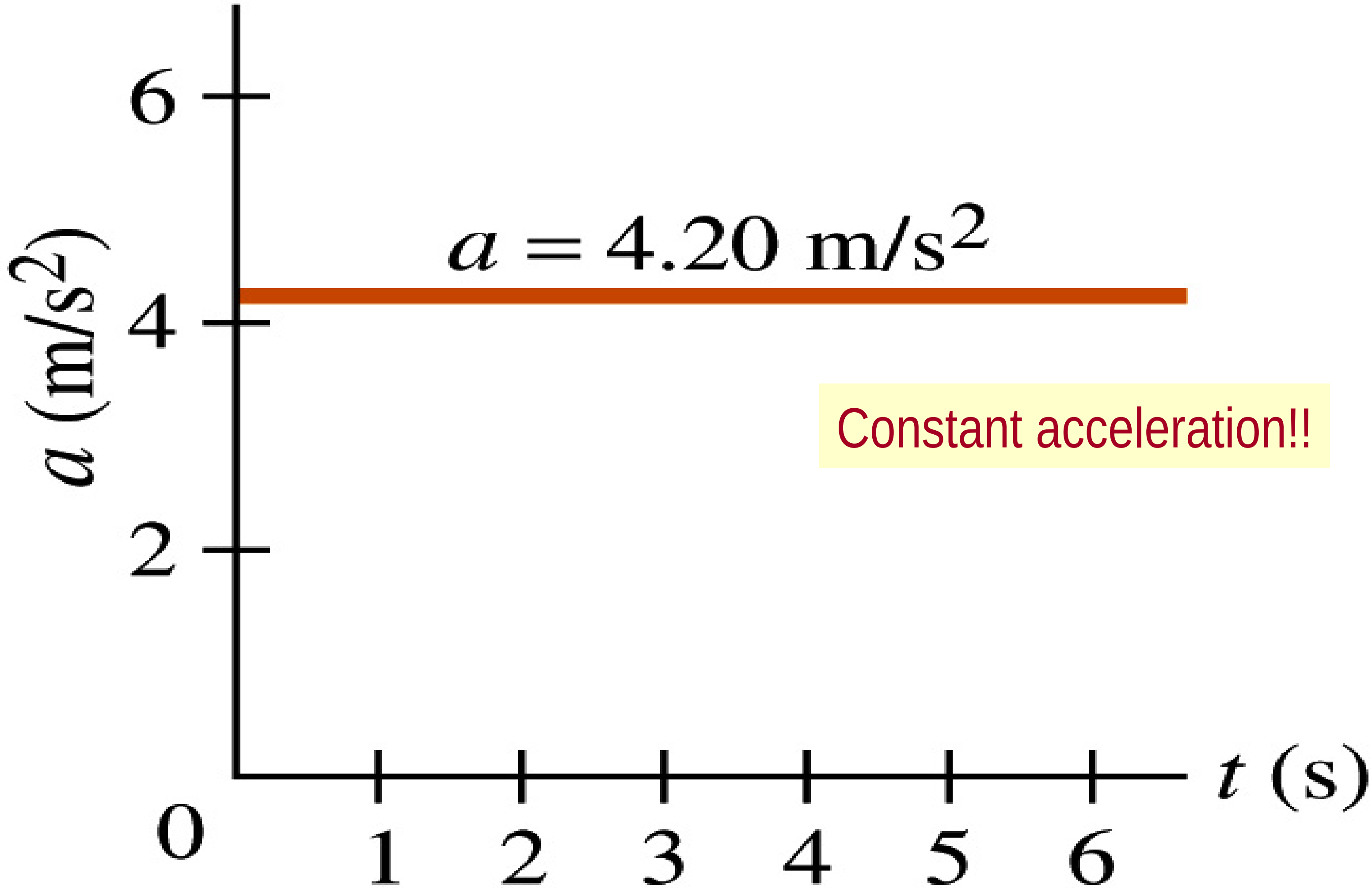
$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogous to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Definition of the instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \quad \text{analogous to} \quad \boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

- In calculus terms: The slope (derivative) of the velocity vector with respect to time or the change of slopes of position as a function of time

Acceleration vs Time Plot



Constant acceleration!!

Meanings of Acceleration

- What is the acceleration when an object is moving at a constant velocity ($v=v_0$)?
  - Acceleration is zero.
  - Is there any net acceleration when an object is not moving? Nope!!
- When an object speeds up as time goes on....
  - The acceleration has the same sign as v.
- When an object slows down as time goes on
  - The acceleration has the opposite sign as v.
- Is there acceleration if an object moves in a constant speed but changes direction? YES!!

One Dimensional Motion

- Let's start with the simplest case: the acceleration is constant ($a=a_0$)
- Using definitions of average acceleration and velocity, we can derive equation of motion (description of motion or position *wrt* time)

$$a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

If $t_f=t$ and $t_i=0$

$$a_x = \frac{v_{xf} - v_{xi}}{t}$$

Solve for v_{xf}

$$\boxed{v_{xf} = v_{xi} + a_x t}$$

For constant acceleration, simple numeric average

$$\overline{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2}a_x t$$

$$\overline{v}_x = \frac{x_f - x_i}{t_f - t_i}$$

If $t_f=t$ and $t_i=0$

$$\overline{v}_x = \frac{x_f - x_i}{t}$$

Solve for x_f

$$x_f = x_i + \overline{v}_x t$$

Resulting Equation of Motion becomes

$$\boxed{x_f = x_i + \overline{v}_x t = x_i + v_{xi} t + \frac{1}{2}a_x t^2}$$

Kinematic Equations of Motion on a Straight Line Under Constant Acceleration

$$\boxed{v_{xf}(t) = v_{xi} + a_x t}$$

Velocity as a function of time

$$\boxed{x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t}$$

Displacement as a function of velocities and time

$$\boxed{x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2}$$

Displacement as a function of time, velocity, and acceleration

$$\boxed{v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)}$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you in specific physical problems!!

How do we solve a problem using a kinematic formula under constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance, initial position or final position?
  - Time?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem wants
- Identify which kinematic formula is **appropriate and easiest** to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equations for the quantity or quantities wanted.

Example 2.11

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇒ As long as it takes for it to crumple.

The initial speed of the car is $v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$

The acceleration is $a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \frac{0-\left(28m/s\right)^2}{2\times 1m} = -390m/s^2$

Thus the time for air-bag to deploy is $t = \frac{v_{xf} - v_{xi}}{a} = \frac{0 - 28m/s}{-390m/s^2} = 0.07s$

Free Fall

- Free fall is a motion under the influence of gravitational pull (gravity) only; Which direction is a freely falling object moving?
  - A motion under constant acceleration
  - All kinematic formulae we learned can be used to solve for falling motions.

Down to the center of the Earth!

- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $|g|=9.80m/s^2$ on the surface of the Earth, most of the time.
- The direction of gravitational acceleration is <u>ALWAYS</u> toward the center of the earth, which we normally call (-y); when the vertical directions are indicated with the variable "y"
- Thus the correct denotation of the gravitational acceleration on the surface of the earth is $g=-9.80m/s^2$

Note the negative sign!!

Example for Using 1D Kinematic Equations on a Falling object (sim to Ex. 2.16)

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m high building,

What is the acceleration in this motion? $g=-9.80m/s^2$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00 m/s$$

Solve for t

$$t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

Position

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$