PHYS 1443 – Section 002 Lecture #5

Wednesday, September 10, 2008
Dr. Mark Sosebee

- Free Fall
- Coordinate System
- Vectors and Scalars
- Motion in Two Dimensions
  - Motion under constant acceleration
  - Projectile Motion
  - Maximum ranges and heights

Announcements

- The first term exam is to be next Wednesday, Sept. 17
  – Will cover CH1 – what we finish today (likely to be CH3) + Appendices A and B
  – Mixture of multiple choices and essay problems
  – In class, from 1 – 2:20pm, SH103
  – Jason will conduct a review in the class Monday, Sept. 15
- There will be a department colloquium this afternoon at 4pm in SH101
  – Extra credit
  – Be sure to sign in
  – Refreshment at 3:30pm in SH108, the Physics Lounge!

Physics Department
The University of Texas at Arlington
COLLOQUIUM

An Overview of Modeling at the Community Coordinated Modeling Center (CCMC) at Goddard Space Flight Center

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Wednesday, September 10, 2008 at 4:00 pm in Room 101 SH

Abstract

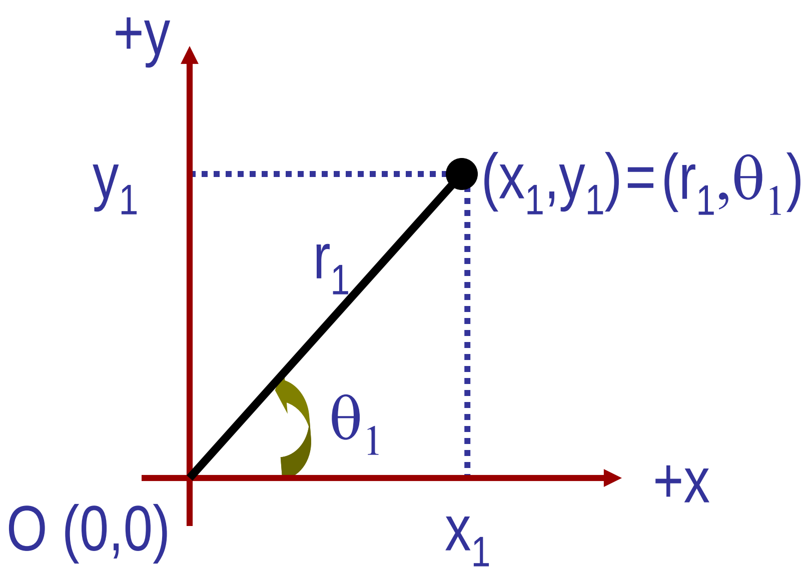
Part 1 - Magnetospheric Modeling During prolonged intervals of negative IMF Bz the magnetosphere often enters a state in which quasi-periodic, large-amplitude oscillations of energetic particle fluxes are observed at the geosynchronous orbit. We use the global magnetosphere MHD code BATS-R-US output during a long period of steady southward IMF Bz to drive the Fok Ring Current Model. We use a global magnetosphere MHD code that reproduces fast magnetotail reconnection rates observed in kinetic simulations. This results in periodical loading-unloading cycles in the magnetotail even for steady southward Bz and can explain quasi-periodic oscillations of geosynchronous energetic particle fluxes. The total proton energy within geosynchronous orbit exhibits overall growth in time due to quasi-steady convection and oscillates due to injection through inductive electric fields caused by multiple dipolarizations.

Part 2 - Heliospheric Modeling A cone model-based halo CME representation is inserted into the combined WSA (corona) and ENLIL (heliosphere) models. We studied the performance of the combined models by analyzing different halo CME propagation and evolution to the L1 point and comparing the result to ACE observations. We simulated CMEs related to a number of geomagnetic storms and events, including the series of the October 2003 Halloween Storm CMEs and the fall AGU storm CME on December, 2006. We introduced 4 parameters characterizing cone model performance: CME arrival time, magnitude of impact, magnetopause standoff distance, duration of the event. We also describe real-time setup for the ENLIL cone model to trigger CME alert capability at the Community Coordinated Modeling Center.

Refreshments will be served in the Physics Lounge at 3:30 pm

Coordinate Systems

- They make it easy and consistent to express locations or positions
- Two commonly used systems, depending on convenience, are
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in distance from the origin ® and the angle measured from the x-axis, θ (r,θ)
- Vectors become a lot easier to express and compute



How are Cartesian and Polar coordinates related?

$$x_1 = r_1 \cos\theta_1$$

$$y_1 = r_1 \sin\theta_1$$

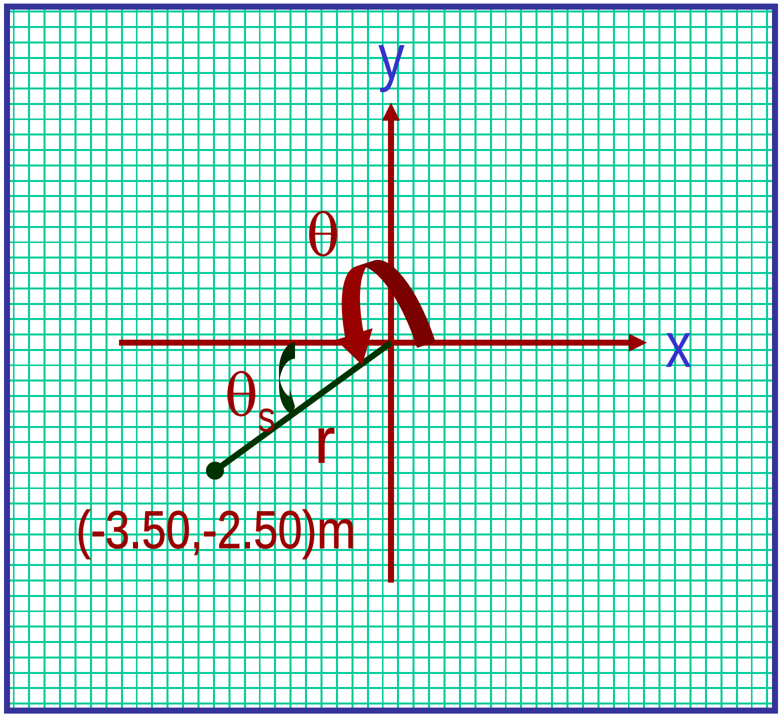
$$r_1 = \sqrt{\left(x_1^2 + y_1^2\right)}$$

$$\tan\theta_1 = \frac{y_1}{x_1}$$

$$\theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right)$$

Example

Cartesian Coordinate of a point in the xy plane are (x,y)= (-3.50,-2.50)m. Find the equivalent polar coordinates of this point.



$$r = \sqrt{\left(x^2 + y^2\right)}$$

$$= \sqrt{\left((-3.50)^2 + (-2.50)^2\right)}$$

$$= \sqrt{18.5} = 4.30(m)$$

$$\theta = 180 + \theta_s$$

$$\tan\theta_s = \frac{-2.50}{-3.50} = \frac{5}{7}$$

$$\theta_s = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\therefore \theta = 180 + \theta_s = 180^\circ + 35.5^\circ = 216^\circ$$

Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

Force, gravitational acceleration, momentum

Normally denoted in **BOLD** letters, $\mathbf{F}$, or a letter with arrow on top $\vec{F}$

Their sizes or magnitudes are denoted with normal letters, F, or absolute values: $\left|\vec{F}\right|$ or $\left|F\right|$

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

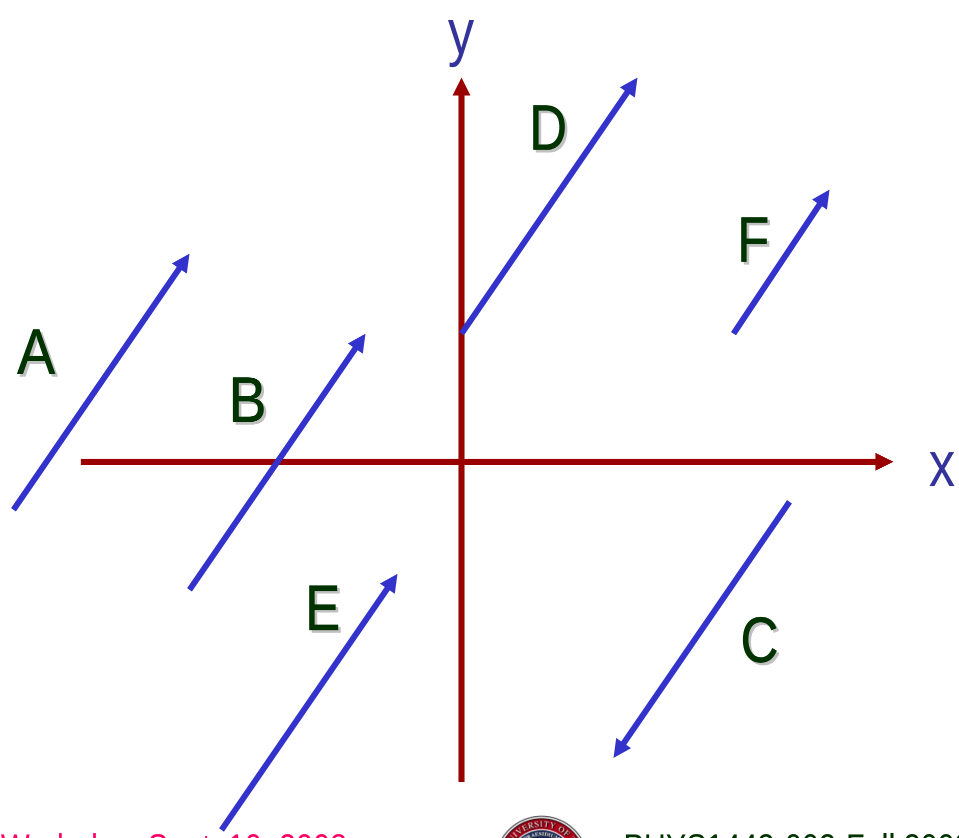
Energy, heat, mass, time

Normally denoted in normal letters, E

Both have units!!!

Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!!



Which ones are the same vectors?

A=B=E=D

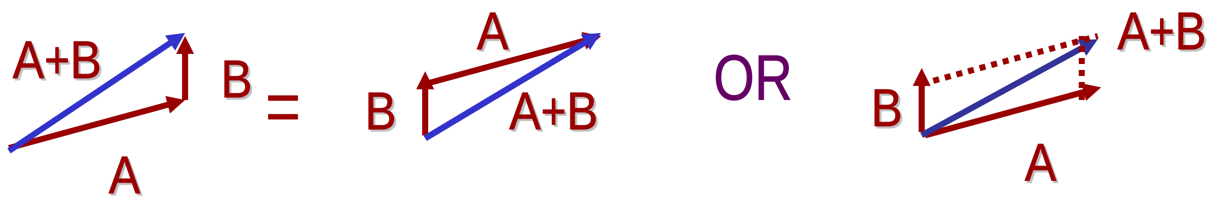
Why aren't the others?

C: The same magnitude but opposite direction: **C=-A:** A negative vector

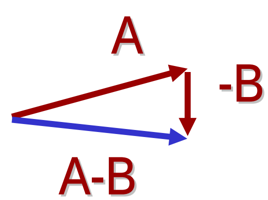
F: The same direction but different magnitude

Vector Operations

- Addition:
  - Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  - Parallelogram method: Connect the tails of the two vectors and extend
  - Addition is commutative: Changing order of operation does not affect the results A+B=B+A, A+B+C+D+E=E+C+A+B+D



- Subtraction:
  - The same as adding a negative vector: A - B = A + (-B)



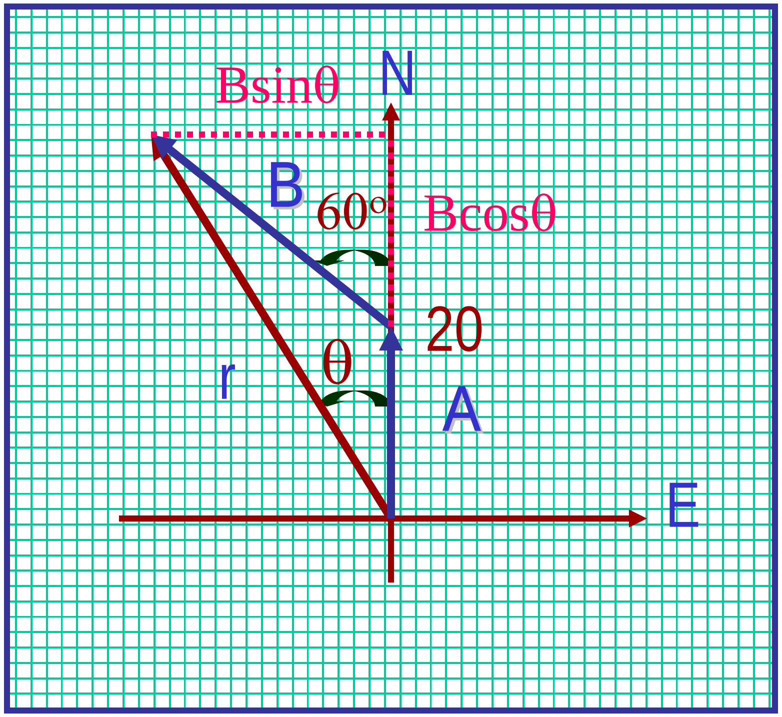
Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

- Multiplication by a scalar is increasing the magnitude A, B=2A

$$\left|\mathcal{B}\right| = 2\left|\mathcal{A}\right|$$

Example for Vector Addition

A car travels 20.0km due north followed by 35.0km in a direction 60.0º west of north. Find the magnitude and direction of resultant displacement.



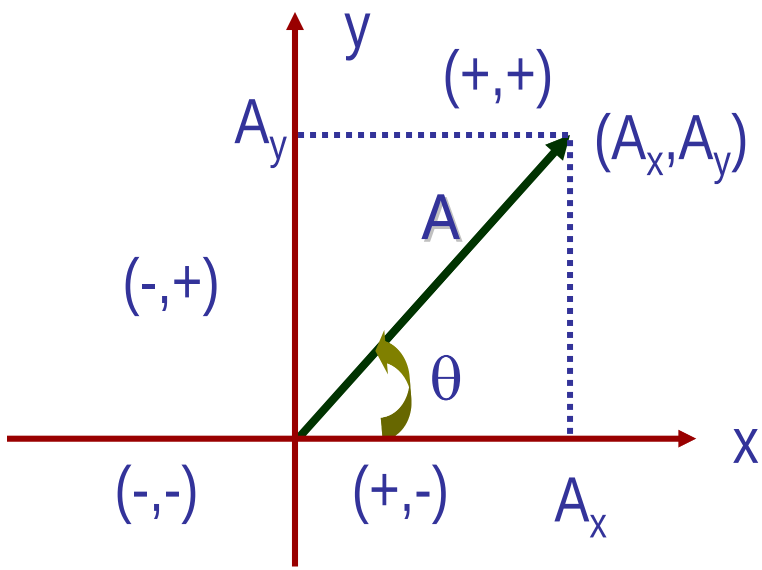
$$r = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$$
$$= \sqrt{A^2 + B^2\left(\cos^2\theta + \sin^2\theta\right) + 2AB\cos\theta}$$
$$= \sqrt{A^2 + B^2 + 2AB\cos\theta}$$
$$= \sqrt{(20.0)^2 + (35.0)^2 + 2\times 20.0\times 35.0\cos 60}$$
$$= \sqrt{2325} = 48.2(km)$$

$$\theta = \tan^{-1}\frac{\left|\vec{B}\right|\sin 60}{\left|\vec{A}\right| + \left|\vec{B}\right|\cos 60}$$
$$= \tan^{-1}\frac{35.0\sin 60}{20.0 + 35.0\cos 60}$$
$$= \tan^{-1}\frac{30.3}{37.5} = 38.9^{\circ} \text{ to W wrt N}$$

Find other ways to solve this problem…

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$\left.\begin{array}{l} \boxed{A_x = \left|\vec{A}\right| \cos\theta} \\ \boxed{A_y = \left|\vec{A}\right| \sin\theta} \end{array}\right\} \text{Components}$$

$$\boxed{\left|\vec{A}\right| = \sqrt{A_x^{\;2} + A_y^{\;2}}} \;\; \} \text{ Magnitude}$$

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right| \cos\theta\right)^2 + \left(\left|\vec{A}\right| \sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2 \left(\cos^2\theta + \sin^2\theta\right)} \quad = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- <u>Dimensionless</u>
- <u>Magnitudes are exactly 1</u>
- Unit vectors are usually expressed in **i**, **j**, **k** or

$$\vec{i},\ \vec{j},\ \vec{k}$$

So the vector **A** can be re-written as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} = \left|\vec{A}\right| \cos\theta \vec{i} + \left|\vec{A}\right| \sin\theta \vec{j}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$

$$= (2.0+2.0)\vec{i} \quad + (2.0-4.0)\vec{j} \qquad = 4.0\vec{i} - 2.0\vec{j}\,(m)$$

$$\left|\vec{C}\right| = \sqrt{(4.0)^2 + (-2.0)^2}$$

$$= \sqrt{16+4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements: $\mathbf{d}_1$=(15**i**+30**j** +12**k**)cm, $\mathbf{d}_2$=(23**i**+14**j** -5.0**k**)cm, and $\mathbf{d}_3$=(-13**i**+15**j**)cm

$$\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

$$= (15+23-13)\vec{i} + (30+14+15)\vec{j} + (12-5.0)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}\,(cm)$$

Magnitude $$\left|\vec{D}\right| = \sqrt{(25)^2 + (59)^2 + (7.0)^2} = 65(cm)$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2}$$

How is each of these quantities defined in 1-D?

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$

What is the difference between 1D and 2D quantities?

2-dim Motion Under Constant Acceleration

- Position vectors in x-y plane:

$$\vec{r}_i = x_i\vec{i} + y_i\vec{j} \qquad \vec{r}_f = x_f\vec{i} + y_f\vec{j}$$

- Velocity vectors in x-y plane:

$$\vec{v}_i = v_{xi}\vec{i} + v_{yi}\vec{j} \qquad \vec{v}_f = v_{xf}\vec{i} + v_{yf}\vec{j}$$

Velocity vectors in terms of the acceleration vector

X-comp $v_{xf} = v_{xi} + a_x t$ Y-comp $v_{yf} = v_{yi} + a_y t$

$$\vec{v}_f = (v_{xi} + a_x t)\vec{i} + (v_{yi} + a_y t)\vec{j} = (v_{xi}\vec{i} + v_{yi}\vec{j}) + (a_x\vec{i} + a_y\vec{j})t =$$

$$= \vec{v}_i + \vec{a}t$$

2-dim Motion Under Constant Acceleration

- How are the 2D position vectors written in acceleration vectors?

Position vector components

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Putting them together in a vector form

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} =$$

$$= \left(x_i + v_{xi}t + \frac{1}{2}a_x t^2\right)\vec{i} + \left(y_i + v_{yi}t + \frac{1}{2}a_y t^2\right)\vec{j}$$

Regrouping the above

$$= \left(x_i\vec{i} + y_i\vec{j}\right) + \left(v_{xi}\vec{i} + v_{yi}\vec{j}\right)t + \frac{1}{2}\left(a_x\vec{i} + a_y\vec{j}\right)t^2$$

$$= \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

2D problems can be interpreted as two 1D problems in x and y

Example for 2-D Kinematic Equations

A particle starts at origin when t=0 with an initial velocity **v**=(20**i**-15**j**)m/s. The particle moves in the xy plane with $a_x=4.0m/s^2$. Determine the components of the velocity vector at any time t.

$$v_{xf} = v_{xi} + a_x t = 20 + 4.0t\,(m/s) \qquad v_{yf} = v_{yi} + a_y t = -15 + 0t = -15\,(m/s)$$

Velocity vector

$$\vec{v}(t) = v_x(t)\vec{i} + v_y(t)\vec{j} = (20 + 4.0t)\vec{i} - 15\vec{j}\,(m/s)$$

Compute the velocity and the speed of the particle at t=5.0 s.

$$\vec{v}_{t=5} = v_{x,t=5}\vec{i} + v_{y,t=5}\vec{j} = (20 + 4.0\times 5.0)\vec{i} - 15\vec{j} = \left(40\vec{i} - 15\vec{j}\right)\,m/s$$

$$speed = \left|\vec{v}\right| = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{(40)^2 + (-15)^2} = 43m/s$$

Example for 2-D Kinematic Eq. Cnt'd

Angle of the Velocity vector

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = \tan^{-1}\left(\frac{-3}{8}\right) = -21^\circ$$

Determine the x and y components of the particle at t=5.0 s.

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = 20\times 5 + \frac{1}{2}\times 4\times 5^2 = 150(m)$$

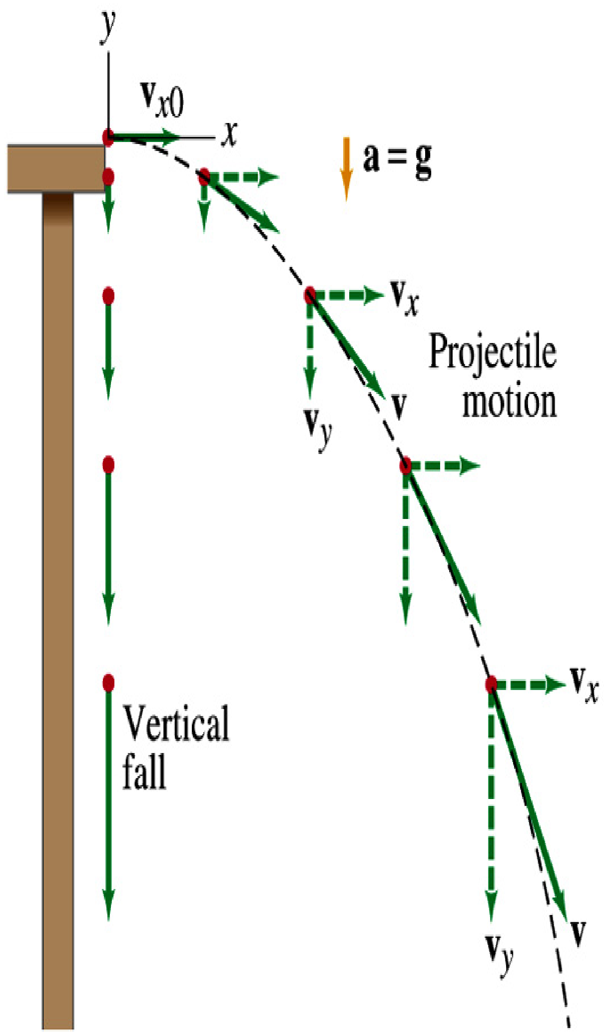
$$y_f = v_{yi}t = -15\times 5 = -75\,(m)$$

Can you write down the position vector at t=5.0s?

$$\vec{r}_f = x_f\vec{i} + y_f\vec{j} = 150\vec{i} - 75\vec{j}\,(m)$$

Projectile Motion

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, g, is constant over the range of the motion
    - $\vec{g} = -9.8\vec{j}\left(m/s^2\right)$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity (<u>no acceleration</u>)
  - Vertical motion under constant acceleration (g)



Show that a projectile motion is a parabola!!!

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

a_x=0

$$x_f = v_{xi}t = v_i \cos\theta_i t$$

$$\boxed{t = \frac{x_f}{v_i \cos\theta_i}}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)\,t^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

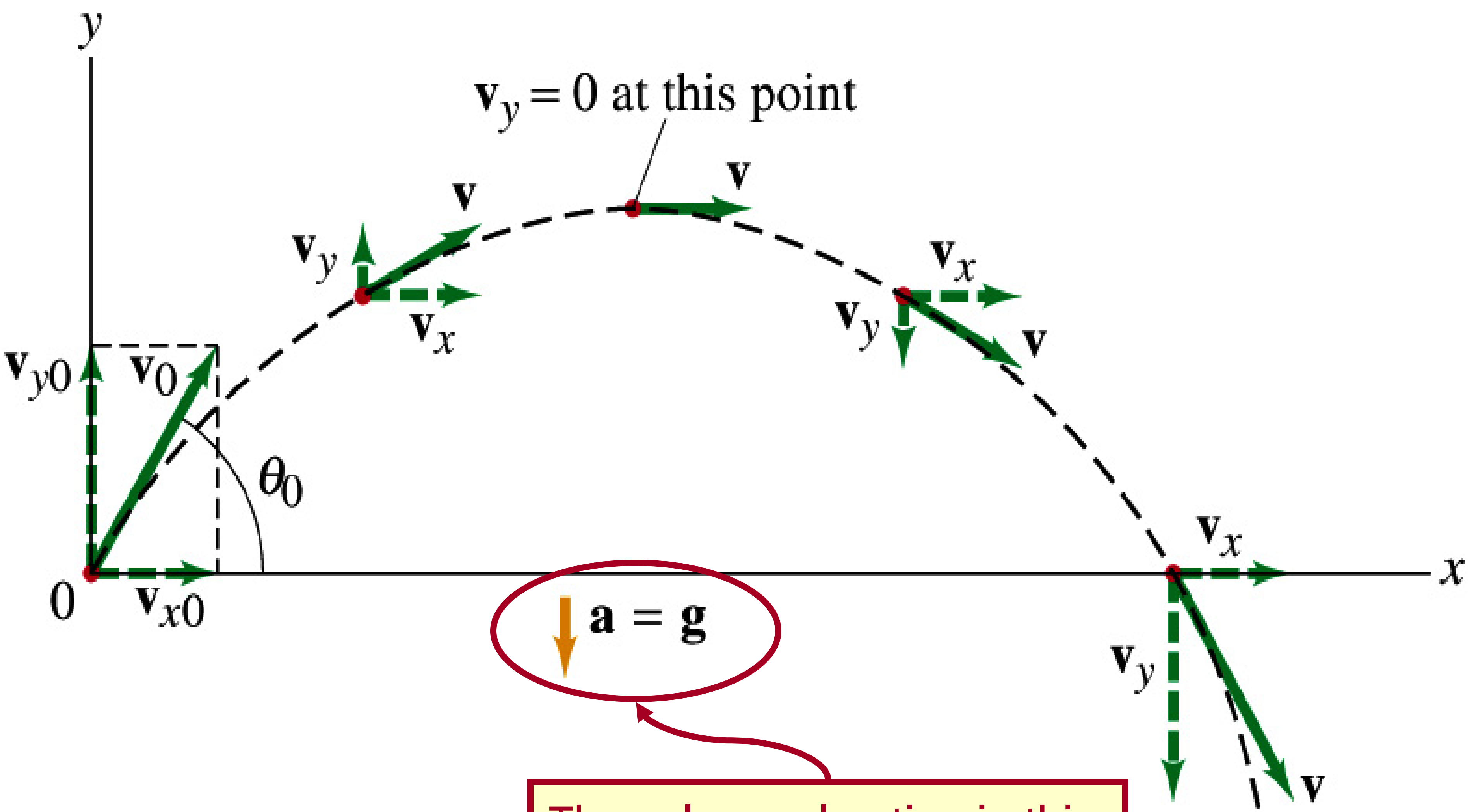
Plug t into the above

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$\boxed{y_f = x_f \tan\theta_i - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right)x_f^2}$$

What kind of parabola is this?

Projectile Motion



The only acceleration in this motion. It is a constant!!

Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by the y component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore \ t \approx 8\sec$$

Why isn't 0 the solution?

Distance is determined by the x component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

$$x_f = v_{xi}t = 20 \times 8 = 160(m)$$