PHYS 1443 – Section 002
Lecture #8

Monday, Sept. 29, 2008
Dr. Jaehoon Yu

- Newton's Laws of Motion
  - Mass
  - Newton's second law of motion
  - Gravitational Force and Weight
  - Newton's third law of motion
- Free Body Diagram
- Application of Newton's Laws
  - Motion without friction

Today's homework is homework #5, due 9pm, Monday, Oct. 6!!

Special Project for Extra Credit

- Show that the trajectory of a projectile motion is a parabola!!
  - 20 points
  - Due: Monday, Oct. 6
  - You MUST show full details of computations to obtain any credit
    - Beyond what was covered in the lecture note!!

Mass

Mass: *A measure of the inertia of a body* or *the quantity of matter*

- A fundamental property of matter!!
- Independent of the object's surroundings: The same no matter where you go.
- Independent of the method of measurement: The same no matter how you measure it.

The heavier the object, the bigger the inertia !!

It is harder to make changes of motion of a heavier object than a lighter one.

The same forces applied to two different masses result in different acceleration depending on the mass.

$$\boxed{\frac{m_1}{m_2} \equiv \frac{a_2}{a_1}}$$

Note that the mass and the weight of an object are two different quantities!!

Weight of an object is the magnitude of the gravitational force exerted on the object.

Not an inherent property of an object!!!

Weight will change if you measure on the Earth or on the moon but the mass won't!!

Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force exerted on it and is inversely proportional to the object's mass.

How do we write the above statement in a mathematical expression?

$$\sum_i \vec{F}_i = m\vec{a}$$

Newton's 2nd Law of Motion

Since it's a vector expression, each component must also satisfy:

$$\sum_i F_{ix} = ma_x \qquad \sum_i F_{iy} = ma_y \qquad \sum_i F_{iz} = ma_z$$

From the above vector expression, what do you conclude the dimension and the unit of the force are?

The dimension of force is $[m][a] = [M][LT^{-2}]$

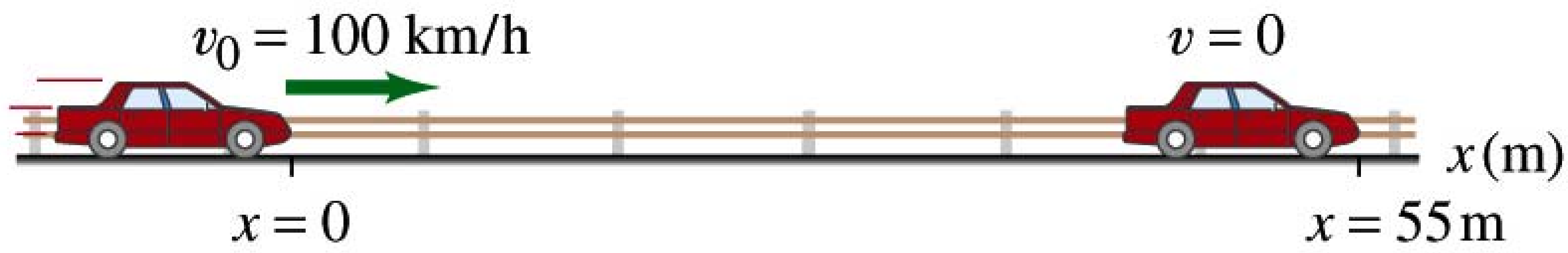
The unit of force in SI is $[Force] = [m][a] = [M][LT^{-2}] = kg \cdot m/s^2$

For ease of use, we define a new derived unit called, Newton (N)

$$1N \equiv 1kg \cdot m/s^2 \approx 1/4\ lbs$$

Example

What constant net force is required to bring a 1500kg car to rest from a speed of 100km/h within a distance of 55m?



What do we need to know to figure out the force?

Acceleration!!

What are given? Initial speed: $v_{xi} = 100\ km/h = 28\ m/s$ Final speed: $v_{xf} = 0\ m/s$

Displacement: $\Delta x = x_f - x_i = 55\ m$

This is a one dimensional motion. Which kinetic formula do we use to find acceleration?

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Acceleration

$$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \frac{-\left(28\ m/s\right)^2}{2\left(55\ m\right)} = -7.1\ m/s^2$$

Thus, the force needed to stop the car is

$$F_x = ma_x = 1500kg \times \left(-7.1m/s^2\right) = -1.1 \times 10^4\ N$$

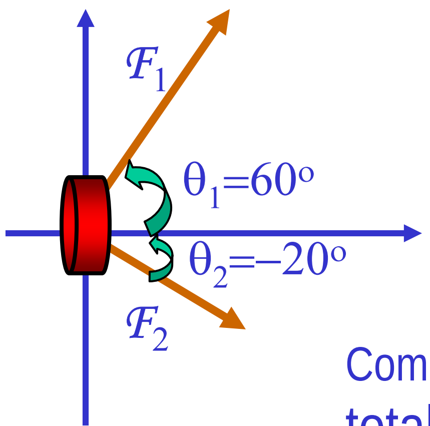
Given the force how far does the car move till it stops?

$$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{m\left(v_{xf}^2 - v_{xi}^2\right)}{2ma_x} = \frac{m\left(v_{xf}^2 - v_{xi}^2\right)}{2F_x}$$

- Linearly proportional to the mass of the car
- Squarely proportional to the speed of the car
- Inversely proportional to the force by the brake

Example for Newton's 2nd Law of Motion

Determine the magnitude and direction of the acceleration of the puck whose mass is 0.30kg and is being pulled by two forces, F1 and F2, as shown in the picture, whose magnitudes of the forces are 8.0 N and 5.0 N, respectively.



Components of $\mathcal{F}_1$

$$F_{1x} = \left|\vec{F}_1\right| \cos\theta_1 = 8.0\times\cos\left(60^o\right) = 4.0N$$

$$F_{1y} = \left|\vec{F}_1\right| \sin\theta_1 = 8.0\times\sin\left(60^o\right) = 6.9N$$

Components of $\mathcal{F}_2$

$$F_{2x} = \left|\vec{F}_2\right| \cos\theta_2 = 5.0\times\cos\left(-20^o\right) = 4.7N$$

$$F_{2y} = \left|\vec{F}_2\right| \sin\theta_2 = 5.0\times\sin\left(-20^o\right) = -1.7N$$

Components of total force $\mathcal{F}$

$$F_x = F_{1x} + F_{2x} = 4.0 + 4.7 = 8.7N = ma_x$$

$$F_y = F_{1y} + F_{2y} = 6.9 - 1.7 = 5.2N = ma_y$$

Magnitude and direction of acceleration a

$$a_x = \frac{F_x}{m} = \frac{8.7}{0.3} = 29m/s^2 \quad a_y = \frac{F_y}{m} = \frac{5.2}{0.3} = 17m/s^2 \quad \left|\vec{a}\right| = \sqrt{\left(a_x\right)^2 + \left(a_y\right)^2} = \sqrt{\left(29\right)^2 + \left(17\right)^2}$$

$$= 34m/s^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^\circ$$

Acceleration Vector a

$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \left(29\hat{i} + 17\hat{j}\right)m/s^2$$

Gravitational Force and Weight

Gravitational Force, $\mathcal{F}_g$

The attractive force exerted on an object by the Earth

$$\vec{F}_G = m\vec{a} = m\vec{g}$$

Weight of an object with mass M is

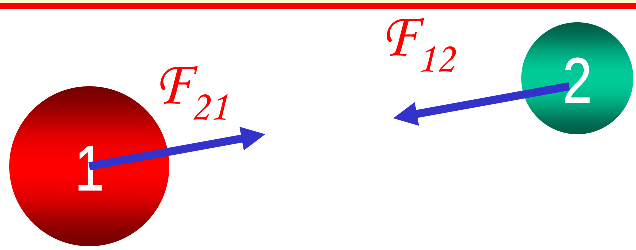
$$W = \left|\vec{F}_G\right| = M\left|\vec{g}\right| = Mg$$

Since weight depends on the magnitude of gravitational acceleration, g, it varies depending on geographical location.

By measuring the forces one can determine masses. This is why you can measure mass using the spring scale.

Newton's Third Law (Law of Action and Reaction)

If two objects interact, the force F_{21} exerted on object 1 by object 2 is equal in magnitude and opposite in direction to the force F_{12} exerted on object 2 by object 1.



$$\vec{F}_{12} = -\vec{F}_{21}$$

The action force is equal in magnitude to the reaction force but in opposite direction. These two forces always act ***on different objects****.*

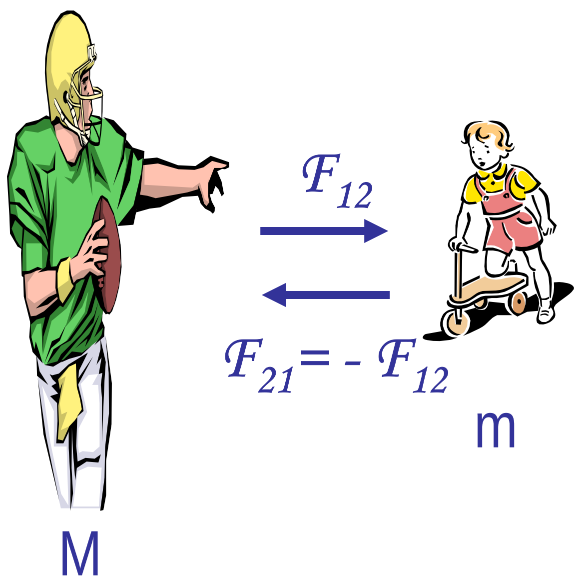
What is the reaction force to the force of a free falling object?

The gravitational force exerted by the object to the Earth!

Stationary objects on top of a table has a reaction force (called the normal force) from table to balance the action force, the gravitational force.

Example of Newton's 3rd Law

A large man and a small boy stand facing each other on frictionless ice. They put their hands together and push against each other so that they move apart. a) Who moves away with the higher speed and by how much?



$$\vec{F}_{12} = -\vec{F}_{21} \Rightarrow \left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

$$\vec{F}_{12} = m\vec{a}_b \Rightarrow F_{12x} = ma_{bx}, \quad F_{12y} = ma_{by} = 0$$

$$\vec{F}_{21} = M\vec{a}_M \Rightarrow F_{21x} = Ma_{Mx}, \quad F_{21y} = Ma_{My} = 0$$

Since $\vec{F}_{12} = -\vec{F}_{21}$ and $\left|\vec{F}_{12}\right| = \left|-\vec{F}_{21}\right| = F$

Establish the equation $\Rightarrow$ $ma_{bx} = F = Ma_{Mx}$ Divide by m $\Rightarrow$ $a_{bx} = \frac{F}{m} = \frac{M}{m} a_{Mx}$

Example of Newton's 3rd Law, cnt'd

Man's velocity

$$v_{Mxf} = v_{Mxi} + a_{Mx}t = a_{Mx}t$$

Boy's velocity

$$v_{bxf} = v_{bxi} + a_{bx}t = a_{bx}t = \frac{M}{m} a_{Mx} t = \frac{M}{m} v_{Mxf}$$

So boy's velocity is higher than man's, if M>m, by the ratio of the masses.

b) Who moves farther while their hands are in contact?

Boy's displacement

$$x_b = v_{bxi}t + \frac{1}{2}a_{bx}t^2 = \frac{M}{2m} a_{Mx}t^2$$

$$x_b = \frac{M}{m}\left(\frac{1}{2}a_{Mx}t^2\right) = \frac{M}{m} x_M$$

Man's displacement

Given in the same time interval, since the boy has higher acceleration and thereby higher speed, he moves farther than the man.