PHYS 1443 – Section 002
Lecture #12

Monday, Oct. 13, 2008
Dr. Jaehoon Yu

- Work done by a Constant Force
- Work done by a Varying Force
- Work and Kinetic Energy Theorem

Today's homework is homework #7, due 9pm, Monday, Oct. 22!!

Announcements

- Reading assignments
  - CH. 8 – 5, 8 – 6 and 8 – 7
- No colloquium this week
- 2nd term exam on Wednesday, Oct. 22
  - Covers from Ch. 1 to what we cover up to this Wednesday, Oct. 15
  - Time: 1 – 2:20pm in class
  - Location: SH103
  - Jason will conduct a summary session Monday, Oct. 20
- Please do NOT miss the exam
- Will have a mid-term grade discussion
  - as soon as I am done with grading the exam and

Reminder: Special Project

- Derive the formula for the gravitational acceleration (g_{in}) at the radius $R_{in}\left(< R_E\right)$ from the center, inside of the Earth. (10 points)
- Compute the fractional magnitude of the gravitational acceleration 1km and 500km inside the surface of the Earth with respect to that on the surface. (6 points, 3 points each)
- Due at the beginning of the class Wednesday, Oct. 15

Work Done by a Constant Force

A meaningful work in physics is done only when a sum of forces exerted on an object made a motion to the object.

Which force did the work? Force $\vec{F}$ Why?

How much work did it do? $W = \left(\sum \vec{F}\right) \cdot \vec{d} = Fd\cos\theta$ Unit? $N \cdot m$ $= J$ (for Joule)

What does this mean?

Physical work is done only by the component of the force along the movement of the object.

Work is an energy transfer!!

Scalar Product of Two Vectors

- Product of magnitude of the two vectors and the cosine of the angle between them $\vec{A}\cdot\vec{B} \equiv \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta$
- Operation is commutative $\vec{A}\cdot\vec{B} = \left|\vec{A}\right|\left|\vec{B}\right|\cos\theta = \left|\vec{B}\right|\left|\vec{A}\right|\cos\theta = \vec{B}\cdot\vec{A}$
- Operation follows the distribution law of multiplication $\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- Scalar products of Unit Vectors $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$
- How does scalar product look in terms of components?

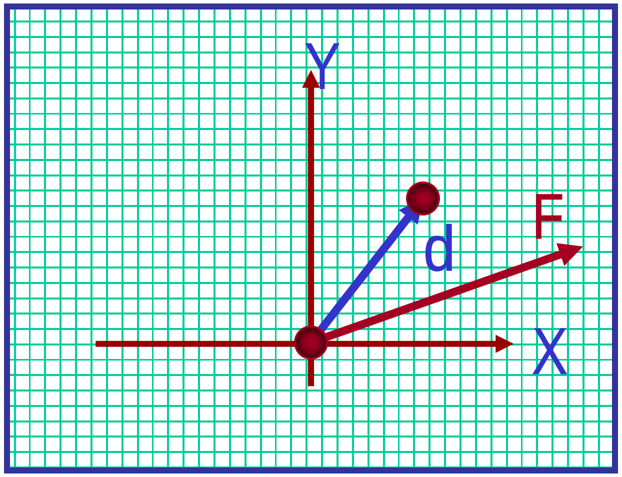
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \qquad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A}\cdot\vec{B} = \left(A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\cdot\left(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\right) = \left(A_xB_x\,\hat{i}\cdot\hat{i} + A_yB_y\,\hat{j}\cdot\hat{j} + A_zB_z\,\hat{k}\cdot\hat{k}\right) + \underbrace{cross\ terms}_{=0}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

Example of Work by Scalar Product

A particle moving on the xy plane undergoes a displacement **d**=(2.0**i**+3.0**j**)m as a constant force **F**=(5.0**i**+2.0**j**) N acts on the particle.



a) Calculate the magnitude of the displacement and that of the force.

$$\left|\vec{d}\right| = \sqrt{d_x^2 + d_y^2} = \sqrt{(2.0)^2 + (3.0)^2} = 3.6m$$

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{(5.0)^2 + (2.0)^2} = 5.4N$$

b) Calculate the work done by the force F.

$$W = \vec{F} \cdot \vec{d} = \left(2.0\hat{i} + 3.0\hat{j}\right) \cdot \left(5.0\hat{i} + 2.0\hat{j}\right) = 2.0 \times 5.0\,\hat{i} \cdot \hat{i} + 3.0 \times 2.0\,\hat{j} \cdot \hat{j} = 10 + 6 = 16(J)$$

Can you do this using the magnitudes and the angle between d and F?

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta$$

Work Done by Varying Force

- If the force depends on the position of the object in motion,

$\longrightarrow$ one must consider the work in small segments of the displacement where the force can be considered constant

$$\Delta W = F_x \cdot \Delta x$$

– Then add all the work-segments throughout the entire motion ($x_i \rightarrow x_f$)

$$W \approx \sum_{x_i}^{x_f} F_x \cdot \Delta x$$

In the limit where $\Delta x \rightarrow 0$

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \cdot \Delta x = \int_{x_i}^{x_f} F_x dx = W$$

– If more than one force is acting, the net work done by the net force is

$$W(net) = \int_{x_i}^{x_f} \left(\sum F_{ix} \right) dx$$

One of the position dependent forces is the force by the spring $\boxed{F_s = -kx}$

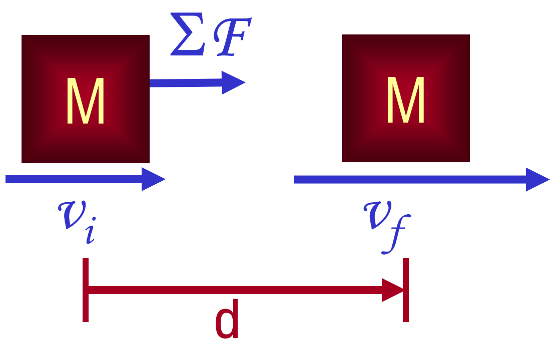
Hooke's Law

The work done by the spring force is

$$W = \int_{-x_{max}}^{0} F_s dx = \int_{-x_{max}}^{0} (-kx) dx = \frac{1}{2} k x_{max}^2$$

Kinetic Energy and Work-Kinetic Energy Theorem

- Some problems are hard to solve using Newton's second law
  - If forces exerting on an object during the motion are complicated
  - Relate the work done on the object by the net force to the change of the speed of the object



Suppose net force ΣF was exerted on an object for displacement d to increase its speed from v_i to v_f

The work on the object by the net force ΣF is

$$W = \left(\sum \vec{F}\right)\cdot \vec{d} = (ma)d\cos 0 = (ma)d$$

Displacement $d = \frac{1}{2}\left(v_f + v_i\right)t$ Acceleration $a = \frac{v_f - v_i}{t}$

Work $W = (ma)d = \left[m\left(\frac{v_f - v_i}{\cancel{t}}\right)\right]\frac{1}{2}\left(v_f + v_i\right)\cancel{t} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Kinetic Energy $\boxed{KE \equiv \frac{1}{2}mv^2}$

Haven't we seen this equation somewhere?

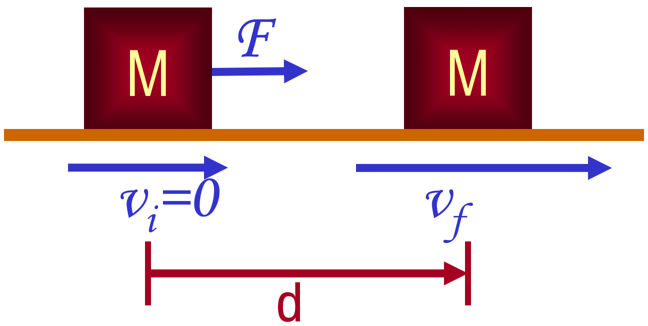
Work $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = KE_f - KE_i = \Delta KE$

Work done by the net force causes change of the object's kinetic energy.

Work-Kinetic Energy Theorem

Example for Work-KE Theorem

A 6.0kg block initially at rest is pulled to East along a horizontal, frictionless surface by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W = \vec{F} \cdot \vec{d} = \left|\vec{F}\right|\left|\vec{d}\right| \cos\theta = 12 \times 3.0 \cos 0 = 36(J)$$

From the work-kinetic energy theorem, we know $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Since initial speed is 0, the above equation becomes $W = \frac{1}{2}mv_f^2$

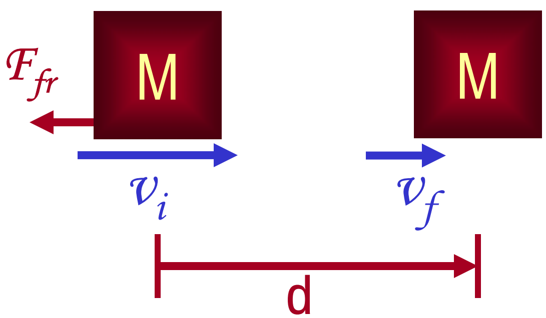
Solving the equation for v_f, we obtain $v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 36}{6.0}} = 3.5m/s$

Work and Energy Involving Kinetic Friction

- What do you think the work looks like if there is friction?
  - Static friction does not matter! Why?

    It isn't there when the object is moving.
  - Then which friction matters?

    Kinetic Friction



Friction force F_{fr} works on the object to slow down

The work on the object by the friction F_{fr} is

$$W_{fr} = F_{fr} d \cos(180) = -F_{fr} d \quad \Delta KE = -F_{fr} d$$

The negative sign means that the work is done on the friction!!

The final kinetic energy of an object, taking into account its initial kinetic energy, friction force and other source of work, is

$$KE_f = KE_i + \sum W - F_{fr} d$$

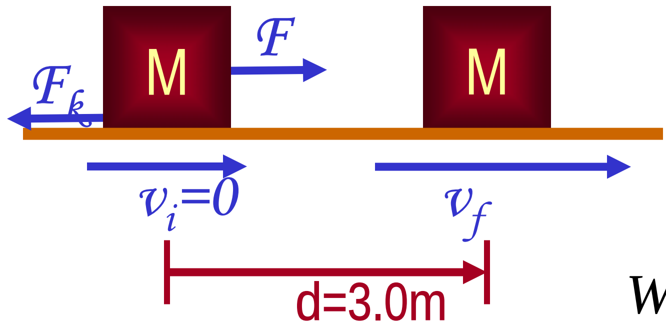
t=0, KE_i

Friction, Engine work

t=T, KE_f

Example of Work Under Friction

A 6.0kg block initially at rest is pulled to East along a horizontal surface with coefficient of kinetic friction μ_k=0.15 by a constant horizontal force of 12N. Find the speed of the block after it has moved 3.0m.



Work done by the force $\mathcal{F}$ is

$$W_F = \left|\vec{F}\right|\left|\vec{d}\right|\cos\theta = 12\times 3.0\cos 0 = 36\,(J)$$

Work done by friction $\mathcal{F}_k$ is

$$W_k = \vec{F}_k \bullet \vec{d} = \left|\vec{F}_k\right|\left|\vec{d}\right|\cos\theta = \left|\mu_k mg\right|\left|\vec{d}\right|\cos\theta$$

$$= 0.15\times 6.0\times 9.8\times 3.0\cos 180 = -26\,(J)$$

Thus the total work is

$$W = W_F + W_k = 36 - 26 = 10(J)$$

Using work-kinetic energy theorem and the fact that initial speed is 0, we obtain

$$W = W_F + W_k = \frac{1}{2}mv_f^2$$

Solving the equation for v_f, we obtain

$$v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\times 10}{6.0}} = 1.8m/s$$

Work and Kinetic Energy

A meaningful work in physics is done only when the sum of the forces exerted on an object made a motion to the object.

What does this mean?

However much tired your arms feel, if you were just holding an object without moving it you have not done any physical work to the object.

Mathematically, the work is written as the product of magnitudes of the net force vector, the magnitude of the displacement vector and the angle between them.

$$W = \sum\left(\vec{F}_i\right)\bullet\vec{d} = \left|\sum\left(\vec{F}_i\right)\right|\left|\vec{d}\right|\cos\theta$$

Kinetic Energy is the energy associated with the motion and capacity to perform work. Work causes change of energy after the completion ← Work-Kinetic energy theorem

$$K = \frac{1}{2}mv^2$$

$$\sum W = K_f - K_i = \Delta K$$

Nm=Joule