PHYS 1443 – Section 002
Lecture #13

Wednesday, Oct. 15, 2008

Dr. Jaehoon Yu

- Potential Energy and the Conservative Force
  - Gravitational Potential Energy
  - Elastic Potential Energy
- Conservation of Energy
- Power

Announcements

- 2^{nd} term exam on Wednesday, Oct. 22
  - Covers from Ch. 1 – CH8 – 7 + appendices
  - Time: 1 – 2:20pm in class
  - Location: SH103
  - Jason will conduct a summary session Monday, Oct. 20
  - Please do NOT miss the exam
- Quiz results
  - Class Average: 4.5/8
    - Equivalent to: 56/100
  - Top score: 8/8
- There is a colloquium today, after all...

Physics Department
The University of Texas at Arlington

COLLOQUIUM

Novel Methodologies for Optical Microscopy and Tissue Imaging Applications

Dr. Georgios Alexandrakis

**University of Texas Southwestern Medical School
Radiation Oncology**

Wednesday, October 15, 2008 at 4:00 pm in Room 101 SH

Abstract

In the first half of the talk I will be presenting results from collaborative work with the University of Texas Southwestern Medical School (UTSW, Radiation Oncology) on the application of quantitative microscopy methods to the study of protein-DNA interaction kinetics. More specifically, we have established a confocal Fluorescence Correlation Spectroscopy setup to study the mechanism by which DNA double-strand breaks are recognized by the DNA dependent protein kinase complex and other key components of the non-homologous end-joining pathway *in vitro*. Future plans on extending these methods to monitor DNA damage recognition and repair *in vivo* will be outlined.

In the second half of the talk I will be presenting some recent findings from collaborative projects with Dr. Hanli Liu (UTA, Bioengineering) and clinicians at UTSW on the use of diffuse optical imaging methods to improve surgical outcomes. Examples will include imaging-assisted cholecystectomy and renal and prostate tumor margin delineation. I will also discuss some of our recent collaborative work on optical tomography-based brain activation studies in children that are handicapped by Cerebral Palsy.

Refreshments will be served in the Physics Lounge at 3:30 pm

Work and Kinetic Energy

A meaningful work in physics is done only when the sum of the forces exerted on an object made a motion to the object.

What does this mean?

However much tired your arms feel, if you were just holding an object without moving it you have not done any physical work to the object.

Mathematically, the work is written as the product of magnitudes of the net force vector, the magnitude of the displacement vector and the angle between them.

$$W = \sum\left(\vec{F}_i\right)\bullet\vec{d} = \left|\sum\left(\vec{F}_i\right)\right|\left|\vec{d}\right|\cos\theta$$

Kinetic Energy is the energy associated with the motion and capacity to perform work. Work causes change of energy after the completion ← <u>Work-Kinetic energy theorem</u>

$$K = \frac{1}{2}mv^2$$

$$\sum W = K_f - K_i = \Delta K$$

Nm=Joule

Potential Energy

Energy associated with a system of objects → Stored energy which has the potential or the possibility to work or to convert to kinetic energy

What does this mean?

In order to describe potential energy, U, a system must be defined.

The concept of potential energy can only be used under the special class of forces called the conservative force which results in the principle of conservation of mechanical energy.

$$E_M \equiv KE_i + PE_i = KE_f + PE_f$$

What are other forms of energies in the universe?

Mechanical Energy *Chemical Energy* *Biological Energy*

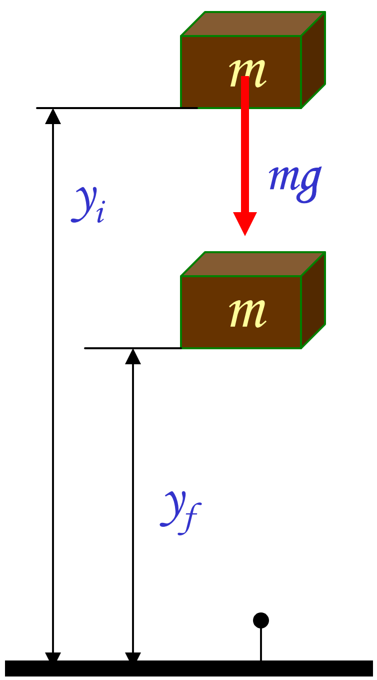
Electromagnetic Energy *Nuclear Energy*

These different types of energies are stored in the universe in many different forms!!!

If one takes into account ALL forms of energy, the total energy in the entire universe is conserved. It just transforms from one form to another.

Gravitational Potential Energy

The potential energy given to an object by the gravitational field in the system of Earth due to the object's height from the surface



When an object is falling, the gravitational force, Mg, performs the work on the object, increasing the object's kinetic energy. So the potential energy of an object at a height y, which is the potential to do work is expressed as

$$U_g = \vec{F}_g \cdot \vec{y} = \left|\vec{F}_g\right|\left|\vec{y}\right|\cos\theta = \left|\vec{F}_g\right|\left|\vec{y}\right| = mgy \qquad U_g \equiv mgy$$

The work done on the object by the gravitational force as the brick drops from y_i to y_f is:

$$W_g = U_i - U_f$$
$$= mgy_i - mgy_f = -\Delta U_g$$

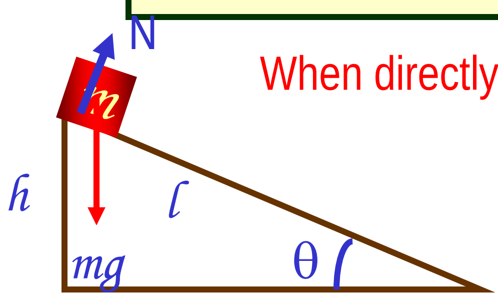
What does this mean?

Work by the gravitational force as the brick drops from y_i to y_f is the negative change of the system's potential energy

➔ Potential energy was lost in order for the gravitational force to increase the brick's kinetic energy.

Conservative and Non-conservative Forces

The work done on an object by the gravitational force does not depend on the object's path in the absence of a retardation force.



When directly falls, the work done on the object by the gravitation force is $W_g = mgh$

When sliding down the hill of length l, the work is

$$W_g = F_{g-incline} \times l = mg \sin\theta \times l$$
$$= mg(l \sin\theta) = mgh$$

How about if we lengthen the incline by a factor of 2, keeping the height the same??

Still the same amount of work☺

$$\boxed{W_g = mgh}$$

So the work done by the gravitational force on an object is independent of the path of the object's movements. It only depends on the difference of the object's initial and final position in the direction of the force.

Forces like gravitational and elastic forces are called the conservative force

1. If the work performed by the force does not depend on the path.
2. If the work performed on a closed path is 0.

Total mechanical energy is conserved!!

$$\boxed{E_M \equiv KE_i + PE_i = KE_f + PE_f}$$

Conservative Forces and Potential Energy

The work done on an object by a conservative force is equal to the decrease in the potential energy of the system

$$W_c = \int_{x_i}^{x_f} F_x dx = -\Delta U$$

What does this statement tell you?

The work done by a conservative force is equal to the negative change of the potential energy associated with that force.

Only the changes in potential energy of a system is physically meaningful!!

We can rewrite the above equation in terms of the potential energy U

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

So the potential energy associated with a conservative force at any given position becomes

$$U_f(x) = -\int_{x_i}^{x_f} F_x dx + U_i$$

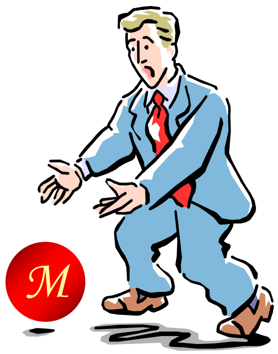
Potential energy function

What can you tell from the potential energy function above?

Since U_i is a constant, it only shifts the resulting $U_f(x)$ by a constant amount. One can always change the initial potential so that U_i can be 0.

Example for Potential Energy

A bowler drops bowling ball of mass 7kg on his toe. Choosing the floor level as y=0, estimate the total work done on the ball by the gravitational force as the ball falls on the toe.



Let's assume the top of the toe is 0.03m from the floor and the hand was 0.5m above the floor.

$$U_i = mgy_i = 7\times 9.8\times 0.5 = 34.3J \quad U_f = mgy_f = 7\times 9.8\times 0.03 = 2.06J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.24J \cong 30J$$

b) Perform the same calculation using the top of the bowler's head as the origin.

What has to change?

First we must re-compute the positions of the ball in his hand and on his toe.

Assuming the bowler's height is 1.8m, the ball's original position is –1.3m, and the toe is at –1.77m.

$$U_i = mgy_i = 7\times 9.8\times\left(-1.3\right) = -89.2J \quad U_f = mgy_f = 7\times 9.8\times\left(-1.77\right) = -121.4J$$

$$W_g = -\Delta U = -\left(U_f - U_i\right) = 32.2J \cong 30J$$

Elastic Potential Energy

Potential energy given to an object by a spring or an object with elasticity in the system that consists of an object and the spring.

The force spring exerts on an object when it is distorted from its equilibrium by a distance x is

$$F_s = -kx$$

Hooke's Law

The work performed on the object by the spring is

$$W_s = \int_{x_i}^{x_f} (-kx)dx = \left[-\frac{1}{2}kx^2\right]_{x_i}^{x_f} = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

The potential energy of this system is

$$U_s \equiv \frac{1}{2}kx^2$$

What do you see from the above equations?

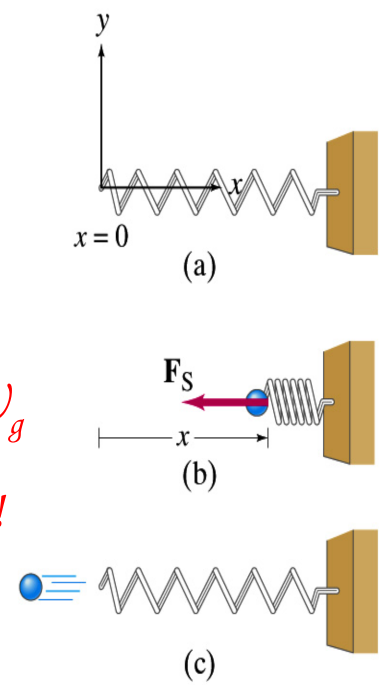
The work done on the object by the spring depends only on the initial and final position of the distorted spring.

Where else did you see this trend?

The gravitational potential energy, U_g

So what does this tell you about the elastic force?

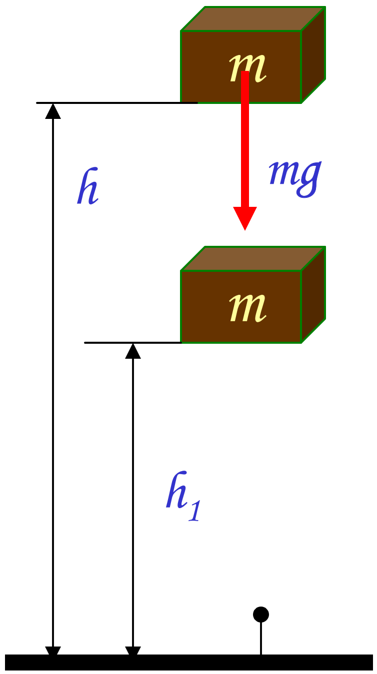
A conservative force!!!



Conservation of Mechanical Energy

Total mechanical energy is the sum of kinetic and potential energies

$$E \equiv K + U$$



Let's consider a brick of mass m at the height h from the ground

What is the brick's potential energy?

$$U_g = mgh$$

What happens to the energy as the brick falls to the ground?

$$\Delta U = U_f - U_i = -\int_{x_i}^{x_f} F_x dx$$

The brick gains speed *By how much?*

$$v = gt$$

So what? *The brick's kinetic energy increased*

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mg^2t^2$$

And? *The lost potential energy is converted to kinetic energy!!*

The total mechanical energy of a system remains constant in any isolated system of objects that interacts only through conservative forces: ***Principle of mechanical energy conservation***

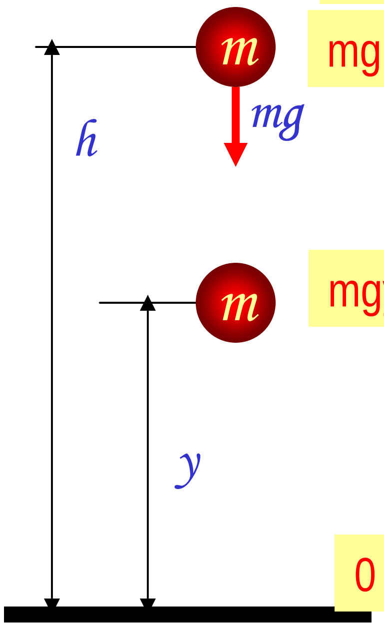
$$E_i = E_f$$

$$K_i + \sum U_i = K_f + \sum U_f$$

What does this mean?

Example

A ball of mass m at rest is dropped from the height h above the ground. a) Neglecting air resistance determine the speed of the ball when it is at the height y above the ground.



PE	KE	
mgh	0	$mv_i^2/2$
mgy	$mv^2/2$	$mv_f^2/2$
0		

Using the principle of mechanical energy conservation

$$K_i + U_i = K_f + U_f \quad 0 + mgh = \frac{1}{2}mv^2 + mgy$$

$$\frac{1}{2}mv^2 = mg(h-y)$$

$$\therefore v = \sqrt{2g(h-y)}$$

b) Determine the speed of the ball at y if it had initial speed v_i at the time of the release at the original height h.

Again using the principle of mechanical energy conservation but with non-zero initial kinetic energy!!!

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mgy$$

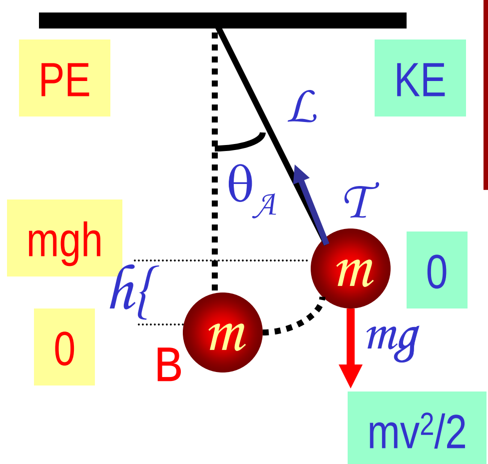
$$\frac{1}{2}m\left(v_f^2 - v_i^2\right) = mg(h-y)$$

$$\therefore v_f = \sqrt{v_i^2 + 2g(h-y)}$$

This result look very similar to a kinematic expression, doesn't it? Which one is it?

Example

A ball of mass m is attached to a light cord of length L, making up a pendulum. The ball is released from rest when the cord makes an angle θ_A with the vertical, and the pivoting point P is frictionless. Find the speed of the ball when it is at the lowest point, B.



Compute the potential energy at the maximum height, h. Remember where the 0 is.

$$h = L - L\cos\theta_A = L\left(1-\cos\theta_A\right)$$

$$U_i = mgh = mgL\left(1-\cos\theta_A\right)$$

Using the principle of mechanical energy conservation

$$\boxed{K_i + U_i = K_f + U_f}$$

$$0 + mgh = mgL\left(1-\cos\theta_A\right) = \frac{1}{2}mv^2$$

$$v^2 = 2gL\left(1-\cos\theta_A\right) \qquad \boxed{\therefore v = \sqrt{2gL\left(1-\cos\theta_A\right)}}$$

b) Determine tension T at the point B.

Using Newton's 2nd law of motion and recalling the centripetal acceleration of a circular motion

$$\sum F_r = T - mg = ma_r = m\frac{v^2}{r} = m\frac{v^2}{L}$$

$$T = mg + m\frac{v^2}{L} = m\left(g + \frac{v^2}{L}\right) = m\left(g + \frac{2gL\left(1-\cos\theta_A\right)}{L}\right)$$

$$= m\frac{gL + 2gL\left(1-\cos\theta_A\right)}{L} \qquad \boxed{\therefore T = mg\left(3-2\cos\theta_A\right)}$$

Cross check the result in a simple situation. What happens when the initial angle θ_A is 0? $T = mg$

Work Done by Non-conservative Forces

Mechanical energy of a system is not conserved when any one of the forces in the system is a non-conservative (dissipative) force.

Two kinds of non-conservative forces:

*Applied forces: Forces that are external to the system. These forces can take away or add energy to the system. So the **mechanical energy of the system is no longer conserved.***

If you were to hit a free falling ball, the force you apply to the ball is external to the system of ball and the Earth. Therefore, you add kinetic energy to the ball-Earth system.

$$W_{you} + W_g = \Delta K; \quad W_g = -\Delta U$$

$$W_{you} = W_{applied} = \Delta K + \Delta U$$

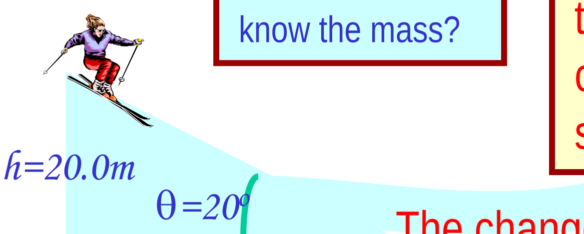
Kinetic Friction: Internal non-conservative force that causes irreversible transformation of energy. The friction force causes the kinetic and potential energy to transfer to internal energy

$$W_{friction} = \Delta K_{friction} = -f_k d$$

$$\Delta E = E_f - E_i = \Delta K + \Delta U = -f_k d$$

Example of Non-Conservative Force

A skier starts from rest at the top of frictionless hill whose vertical height is 20.0m and the inclination angle is 20°. Determine how far the skier can get on the snow at the bottom of the hill with a coefficient of kinetic friction between the ski and the snow is 0.210.



Don't we need to know the mass?

Compute the speed at the bottom of the hill, using the mechanical energy conservation on the hill before friction starts working at the bottom

$$ME = mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{2\times 9.8\times 20.0} = 19.8m/s$$

The change of kinetic energy is the same as the work done by the kinetic friction.

What does this mean in this problem?

$$\Delta K = K_f - K_i = -f_k d$$

Since we are interested in the distance the skier can get to before stopping, the friction must do as much work as the available kinetic energy to take it all away.

Since $K_f = 0$

$$-K_i = -f_k d; \quad f_k d = K_i$$

$$f_k = \mu_k n = \mu_k mg$$

$$d = \frac{K_i}{\mu_k mg} = \frac{\frac{1}{2}mv^2}{\mu_k mg} = \frac{v^2}{2\mu_k g} = \frac{(19.8)^2}{2\times 0.210\times 9.80} = 95.2m$$

Well, it turns out we don't need to know the mass.

What does this mean?

No matter how heavy the skier is he will get as far as anyone else has gotten starting from the same height.

How is the conservative force related to the potential energy?

Work done by a force component on an object through the displacement Δx is

$$W = F_x \Delta x = -\Delta U$$

For an infinitesimal displacement Δx

$$\lim_{\Delta x \to 0} \Delta U = -\lim_{\Delta x \to 0} F_x \Delta x$$

$$dU = -F_x dx$$

Results in the conservative force-potential relationship

$$F_x = -\frac{dU}{dx}$$

This relationship says that any conservative force acting on an object within a given system is the same as the negative derivative of the potential energy of the system with respect to the position.

Does this statement make sense?

1. spring-ball system:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

2. Earth-ball system:

$$F_g = -\frac{dU_g}{dy} = -\frac{d}{dy}(mgy) = -mg$$

The relationship works in both the conservative force cases we have learned!!!

Energy Diagram and the Equilibrium of a System

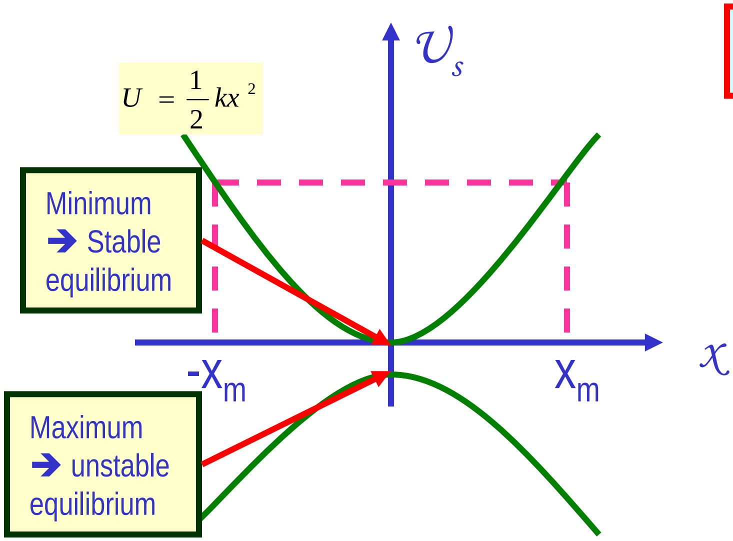
One can draw potential energy as a function of position → *Energy Diagram*

Let's consider potential energy of a spring-ball system

$$U_s = \frac{1}{2}kx^2$$

What shape is this diagram?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 at the position.*
3. *x=0 is the stable equilibrium position of this system where the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new forms of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one to another. The total energy of universe is constant as a function of time!! The total energy of the universe is conserved!

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$\boxed{E_R = mc^2}$$

How many joules does your body correspond to?

The Gravitational Field

The gravitational force is a field force. The force exists everywhere in the universe.

If one were to place a test object of mass m at any point in the space in the existence of another object of mass M, the test object will feel the gravitational force exerted by M, $\vec{F}_g = m\vec{g}$.

Therefore the gravitational field g is defined as $\vec{g} \equiv \frac{\vec{F}_g}{m}$

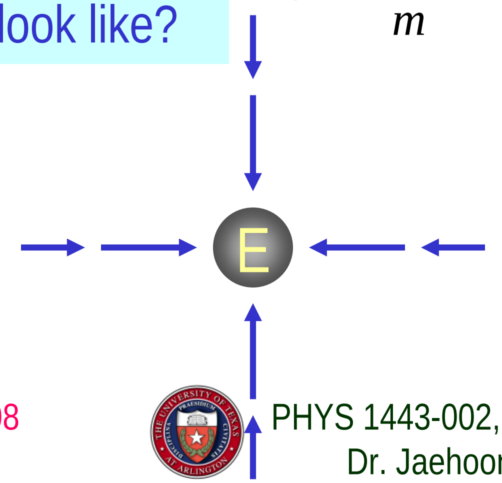
In other words, the gravitational field at a point in the space is the gravitational force experienced by a test particle placed at the point divided by the mass of the test particle.

So how does the Earth's gravitational field look like?

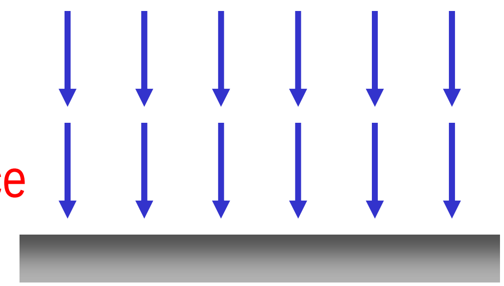
$$\vec{g} = \frac{\vec{F}_g}{m} = -\frac{GM_E}{R_E^2}\hat{r}$$

Where $\hat{r}$ is the unit vector pointing outward from the center of the Earth

Far away from the Earth's surface



Close to the Earth's surface



The Gravitational Potential Energy

What is the potential energy of an object at the height y from the surface of the Earth?

$$U = mgy$$

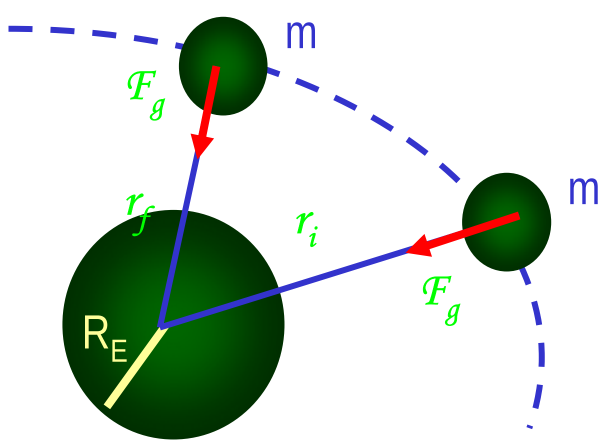
Do you think this would work in general cases?

No, it would not.

Why not?

Because this formula is only valid for the case where the gravitational force is constant, near the surface of the Earth, and the generalized gravitational force is inversely proportional to the square of the distance.

OK. Then how would we generalize the potential energy in the gravitational field?



Since the gravitational force is a central force, and a central force is a conservative force, the work done by the gravitational force is independent of the path.

The path can be considered as consisting of many tangential and radial motions.
Tangential motions do not contribute to work!!!

More on The Gravitational Potential Energy

Since the gravitational force is a radial force, it performs work only when the path has component in radial direction. Therefore, the work performed by the gravitational force that depends on the position becomes:

$$dW = \vec{F} \cdot d\vec{r} = F(r)dr \xrightarrow{\text{For the whole path}} W = \int_{r_i}^{r_f} F(r)dr$$

Potential energy is the negative change of the work done through the path

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$$

Since the Earth's gravitational force is $F(r) = -\frac{GM_E m}{r^2}$

Thus the potential energy function becomes

$$U_f - U_i = \int_{r_i}^{r_f} \frac{GM_E m}{r^2} dr = -GM_E m\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

Since only the difference of potential energy matters, by taking the infinite distance as the initial point of the potential energy, we obtain

$$\boxed{U = -\frac{GM_E m}{r}}$$

For any two particles?

$$\boxed{U = -\frac{Gm_1m_2}{r}}$$

The energy needed to take the particles infinitely apart.

For many particles?

$$\boxed{U = \sum_{i,j} U_{i,j}}$$

Example of Gravitational Potential Energy

A particle of mass m is displaced through a small vertical distance Δy near the Earth's surface. Show that in this situation the general expression for the change in gravitational potential energy is reduced to the $\Delta U = -mg\Delta y$.

Taking the general expression of gravitational potential energy

$$\Delta U = -GM_E m\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

Reorganizing the terms w/ the common denominator

$$= -GM_E m\frac{(r_f - r_i)}{r_f r_i} = -GM_E m\frac{\Delta y}{r_f r_i}$$

Since the situation is close to the surface of the Earth

$$r_i \approx R_E \quad \text{and} \quad r_f \approx R_E$$

Therefore, ΔU becomes

$$\Delta U = -GM_E m\frac{\Delta y}{R_E^2}$$

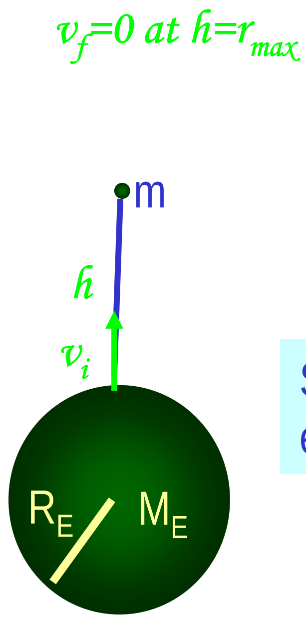
Since on the surface of the Earth the gravitational field is

$$g = \frac{GM_E}{R_E^2}$$

The potential energy becomes

$$\Delta U = -mg\Delta y$$

Escape Speed



Consider an object of mass m is projected vertically from the surface of the Earth with an initial speed v_i and eventually comes to stop v_f=0 at the distance r_{max}.

Since the total mechanical energy is conserved

$$ME = K + U = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = -\frac{GM_E m}{r_{\max}}$$

Solving the above equation for v_i, one obtains

$$v_i = \sqrt{2GM_E\left(\frac{1}{R_E} - \frac{1}{r_{\max}}\right)}$$

Therefore if the initial speed v_i is known, one can use this formula to compute the final height h of the object.

$$h = r_{\max} - R_E = \frac{v_i^2 R_E^2}{2GM_E - v_i^2 R_E}$$

In order for an object to escape Earth's gravitational field completely, the initial speed needs to be

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.37 \times 10^6}}$$

$$= 1.12 \times 10^4 m/s = 11.2 km/s$$

This is called the escape speed. This formula is valid for any planet or large mass objects.

How does this depend on the mass of the escaping object?

Independent of the mass of the escaping object