

PHYS 1443 – Section 002

Lecture #14

Wednesday, Oct. 29, 2008

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(Disguised as Dr. Yu)

- Energy Diagram
- General Energy Conservation & Mass Equivalence
- More on gravitational potential energy
 - Escape speed
- Power
- Linear Momentum
- Conservation of Momentum



Energy Diagram and the Equilibrium of a System

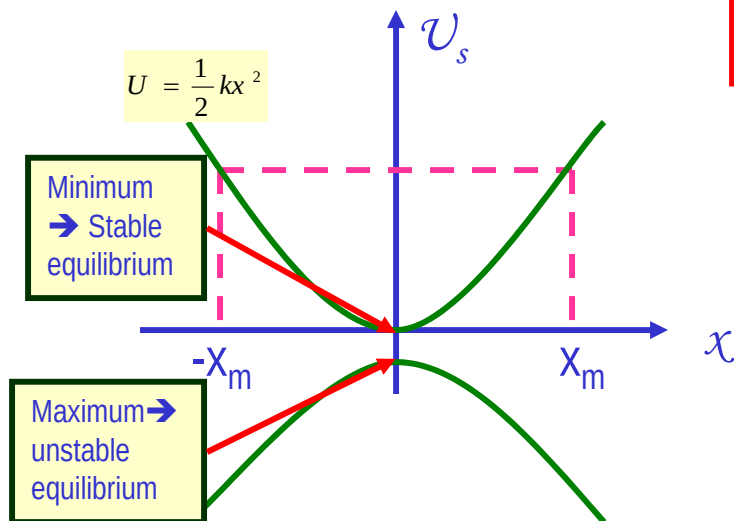
One can draw potential energy as a function of position → *Energy Diagram*

Let's consider potential energy of a spring-ball system

$$U_s = \frac{1}{2} kx^2$$

What shape is this diagram?

A Parabola



What does this energy diagram tell you?

1. *Potential energy for this system is the same independent of the sign of the position.*
2. *The force is 0 when the slope of the potential energy curve is 0 at the position.*
3. *$x=0$ is the stable equilibrium position of this system where the potential energy is minimum.*

Position of a stable equilibrium corresponds to points where potential energy is at a minimum.

Position of an unstable equilibrium corresponds to points where potential energy is a maximum.

General Energy Conservation and Mass-Energy Equivalence

General Principle of Energy Conservation

The total energy of an isolated system is conserved as long as all forms of energy are taken into account.

What about friction?

Friction is a non-conservative force and causes mechanical energy to change to other forms of energy.

However, if you add the new forms of energy altogether, the system as a whole did not lose any energy, as long as it is self-contained or isolated.

In the grand scale of the universe, no energy can be destroyed or created but just transformed or transferred from one to another. The total energy of universe is constant as a function of time!! The total energy of the universe is conserved!

Principle of Conservation of Mass

In any physical or chemical process, mass is neither created nor destroyed. Mass before a process is identical to the mass after the process.

Einstein's Mass-Energy equality.

$$E_R = mc^2$$

How many joules does your body correspond to?



The Gravitational Field

The gravitational force is a field force. The force exists everywhere in the universe.

If one were to place a test object of mass m at any point in the space in the existence of another object of mass M , the test object will feel the gravitational force exerted by M , $\vec{F}_g = m\vec{g}$.

Therefore the gravitational field $\vec{g}$ is defined as $\vec{g} \equiv \frac{\vec{F}_g}{m}$

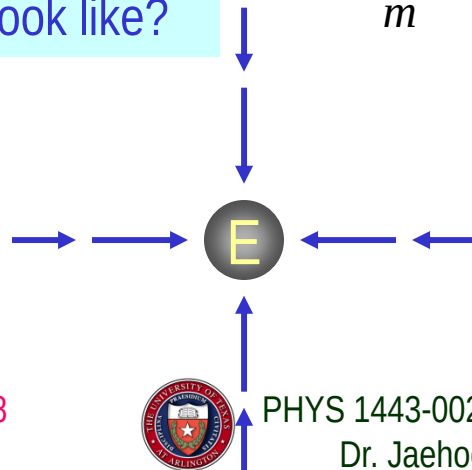
In other words, the gravitational field at a point in the space is the gravitational force experienced by a test particle placed at the point divided by the mass of the test particle.

So how does the Earth's gravitational field look like?

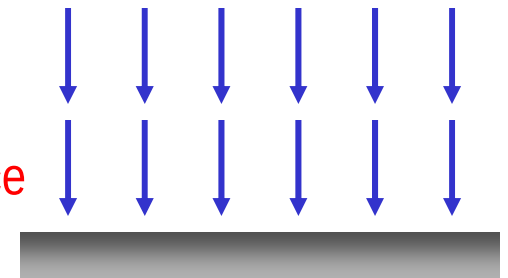
$$\vec{g} = \frac{\vec{F}_g}{m} = -\frac{GM_E}{R_E^2} \hat{r}$$

Where $\hat{r}$ is the unit vector pointing outward from the center of the Earth

Far away from the Earth's surface



Close to the Earth's surface



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The Gravitational Potential Energy

What is the potential energy of an object at the height y from the surface of the Earth?

$$U = mgy$$

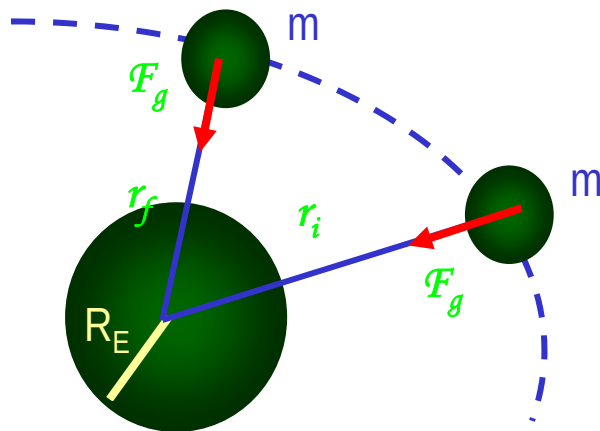
Do you think this would work in general cases?

No, it would not.

Why not?

Because this formula is only valid for the case where the gravitational force is constant, near the surface of the Earth, and the generalized gravitational force is inversely proportional to the square of the distance.

OK. Then how would we generalize the potential energy in the gravitational field?



Since the gravitational force is a central force, and a central force is a conservative force, the work done by the gravitational force is independent of the path.

The path can be considered as consisting of many tangential and radial motions.

Tangential motions do not contribute to work!!!

More on The Gravitational Potential Energy

Since the gravitational force is a radial force, it performs work only when the path has component in radial direction. Therefore, the work performed by the gravitational force that depends on the position becomes:

$$dW = \vec{F} \cdot d\vec{r} = F(r)dr \quad \xrightarrow{\text{For the whole path}} \quad W = \int_{r_i}^{r_f} F(r)dr$$

Potential energy is the negative change of the work done through the path

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r)dr$$

Since the Earth's gravitational force is

$$F(r) = -\frac{GM_E m}{r^2}$$

Thus the potential energy function becomes

$$U_f - U_i = \int_{r_i}^{r_f} \frac{GM_E m}{r^2} dr = -GM_E m \left[\frac{1}{r_f} - \frac{1}{r_i} \right]$$

Since only the difference of potential energy matters, by taking the infinite distance as the initial point of the potential energy, we obtain

$$U = -\frac{GM_E m}{r}$$

For any two particles?

$$U = -\frac{Gm_1 m_2}{r}$$

The energy needed to take the particles infinitely apart.

For many particles?

$$U = \sum_{i,j} U_{i,j}$$

Example of Gravitational Potential Energy

A particle of mass m is displaced through a small vertical distance Δy near the Earth's surface. Show that in this situation the general expression for the change in gravitational potential energy is reduced to the $\Delta U = -mg\Delta y$.

Taking the general expression of gravitational potential energy

$$\Delta U = -GM_E m \left(\frac{1}{r_f} - \frac{1}{r_i} \right)$$

Reorganizing the terms w/
the common denominator

$$= -GM_E m \frac{(r_f - r_i)}{r_f r_i} = -GM_E m \frac{\Delta y}{r_f r_i}$$

Since the situation is close to the surface of the Earth

$$r_i \approx R_E \quad \text{and} \quad r_f \approx R_E$$

Therefore, ΔU becomes

$$\Delta U = -GM_E m \frac{\Delta y}{R_E^2}$$

Since on the surface of the Earth the gravitational field is

$$g = \frac{GM_E}{R_E^2}$$

The potential energy becomes

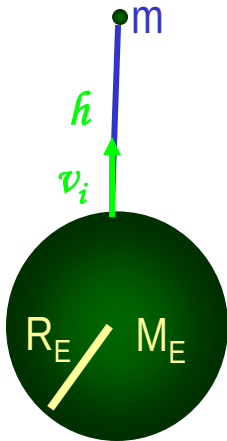
$$\Delta U = -mg\Delta y$$



$v_f=0$ at $h=r_{\max}$

Escape Speed

Consider an object of mass m is projected vertically from the surface of the Earth with an initial speed v_i and eventually comes to stop $v_f=0$ at the distance $r_{\max}$.



Since the total mechanical energy is conserved

$$ME = K + U = \frac{1}{2}mv_i^2 - \frac{GM_E m}{R_E} = - \frac{GM_E m}{r_{\max}}$$

Solving the above equation for v_i one obtains

$$v_i = \sqrt{2GM_E \left(\frac{1}{R_E} - \frac{1}{r_{\max}} \right)}$$

Therefore if the initial speed v_i is known, one can use this formula to compute the final height h of the object.

$$h = r_{\max} - R_E = \frac{v_i^2 R_E^2}{2GM_E - v_i^2 R_E}$$

In order for an object to escape Earth's gravitational field completely, the initial speed needs to be

$$v_{\text{esc}} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.37 \times 10^6}} \\ = 1.12 \times 10^4 \text{ m/s} = 11.2 \text{ km/s}$$

This is called the escape speed. This formula is valid for any planet or large mass objects.

How does this depend on the mass of the escaping object?

Independent of the mass of the escaping object

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Power

- Rate at which the work is done or the energy is transferred
 - What is the difference for the same car with two different engines (4 cylinder and 8 cylinder) climbing the same hill?
 - ➔ The time... 8 cylinder car climbs up the hill faster!

Is the total amount of work done by the engines different? **NO**

Then what is different? **The rate at which the same amount of work performed is higher for 8 cylinders than 4.**

Average power

$$\bar{P} \equiv \frac{\Delta W}{\Delta t}$$

Instantaneous power $P \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \lim_{\Delta t \rightarrow 0} \left(\sum \vec{F} \right) \cdot \frac{\Delta \vec{s}}{\Delta t} = \left(\sum \vec{F} \right) \cdot \vec{v} = \left| \sum \vec{F} \right| |\vec{v}| \cos \theta$

Unit?

$$J / s = \text{Watts}$$

$$1HP \equiv 746Watts$$

What do power companies sell? $1kWH = 1000Watts \times 3600s = 3.6 \times 10^6 J$

Energy



Energy Loss in Automobile

Automobile uses only 13% of its fuel to propel the vehicle.

Why?

67% in the engine:

- Incomplete burning
- Heat
- Sound

16% in friction in mechanical parts

4% in operating other crucial parts such as oil and fuel pumps, etc

13% used for balancing energy loss related to moving vehicle, like air resistance and road friction to tire, etc

Two frictional forces involved in moving vehicles

Coefficient of Rolling Friction; $\mu=0.016$

Air Drag

$$f_a = \frac{1}{2} D \rho A v^2 = \frac{1}{2} \times 0.5 \times 1.293 \times 2v^2 = 0.647v^2$$

Total Resistance

$$f_t = f_r + f_a$$

Total power to keep speed $v=26.8\text{m/s}=60\text{mi/h}$

Power to overcome each component of resistance

$$m_{\text{car}} = 1450\text{kg} \quad \text{Weight} = mg = 14200\text{N}$$

$$\mu n = \mu mg = 227\text{N}$$

$$P = f_t v = (691\text{N}) \cdot 26.8 = 18.5\text{kW}$$

$$P_r = f_r v = (227) \cdot 26.8 = 6.08\text{kW}$$

$$P_a = f_a v = (464.7) \cdot 26.8 = 12.5\text{kW}$$

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Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at a velocity of $\vec{v}$ is defined as

$$\vec{p} \equiv m\vec{v}$$

What can you tell from this definition about momentum?

1. Momentum is a vector quantity.
2. The heavier the object the higher the momentum
3. The higher the velocity the higher the momentum
4. Its unit is kg.m/s

What else can we see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m(\vec{v} - \vec{v}_0)}{\Delta t} = m \frac{\Delta \vec{v}}{\Delta t} = m\vec{a} = \sum \vec{F}$$



Linear Momentum and Forces

$$\sum \vec{F} = \frac{d\vec{p}}{dt}$$

What can we learn from this Force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When net force is 0, the particle's linear momentum is constant as a function of time.
- If a particle is isolated, the particle experiences no net force. Therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

The relationship can be used to study the case where the mass changes as a function of time.

Can you think of a few cases like this?

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{dm}{dt}\vec{v} + m\frac{d\vec{v}}{dt}$$

Motion of a meteorite

Motion of a rocket



Conservation of Linear Momentum in a Two Particle System

Consider an isolated system with two particles that do not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

If particle #1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces, and the net force in the entire SYSTEM is still 0.

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum p_1 and #2 has p_2 at some point of time.

Using momentum-force relationship

$$\vec{F}_{21} = \frac{d\vec{p}_1}{dt} \quad \text{and} \quad \vec{F}_{12} = \frac{d\vec{p}_2}{dt}$$

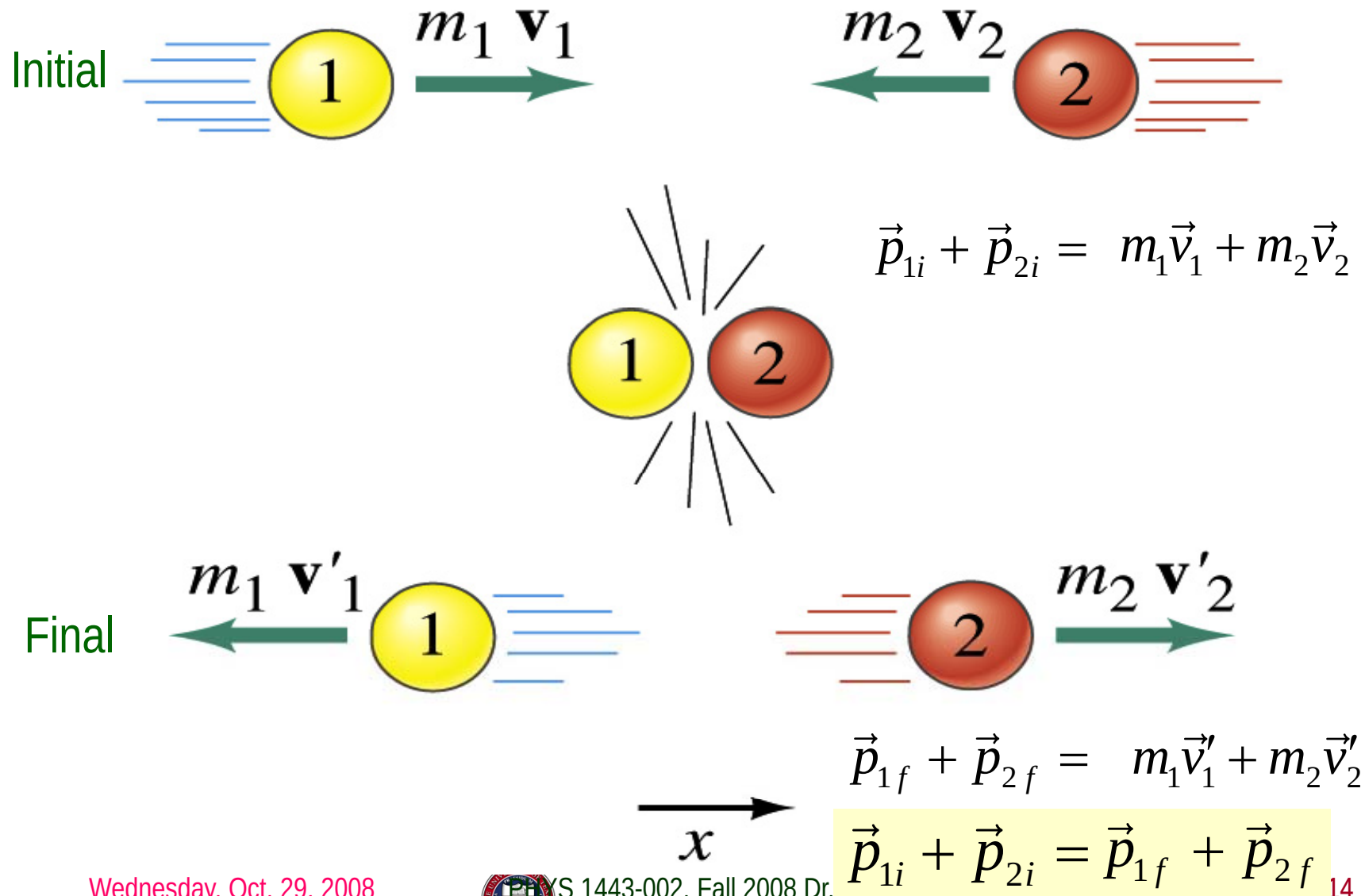
And since net force of this system is 0

$$\sum \vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{d\vec{p}_2}{dt} + \frac{d\vec{p}_1}{dt} = \frac{d}{dt}(\vec{p}_2 + \vec{p}_1) = 0$$

Therefore $\vec{p}_2 + \vec{p}_1 = \text{const}$

The total linear momentum of the system is conserved!!!

Linear Momentum Conservation



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More on Conservation of Linear Momentum in a Two Body System

From the previous slide we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = \text{const}$$

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interactions

Mathematically this statement can be written as

$$\vec{p}_{2i} + \vec{p}_{1i} = \vec{p}_{2f} + \vec{p}_{1f}$$

$$\sum_{\text{system}} P_{xi} = \sum_{\text{system}} P_{xf}$$

$$\sum_{\text{system}} P_{yi} = \sum_{\text{system}} P_{yf}$$

$$\sum_{\text{system}} P_{zi} = \sum_{\text{system}} P_{zf}$$

This can be generalized into conservation of linear momentum in many particle systems.

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

Example for Linear Momentum Conservation

Estimate an astronaut's ($M=70\text{kg}$) resulting velocity after he throws his book ($m=1\text{kg}$) to a direction in the space to move to another direction.



From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = m_A \vec{v}_A + m_B \vec{v}_B$$

Assuming the astronaut's mass is 70kg, and the book's mass is 1kg and using linear momentum conservation

$$\vec{v}_A = - \frac{m_B \vec{v}_B}{m_A} = - \frac{1}{70} \vec{v}_B$$

Now if the book gained a velocity of 20 m/s in +x-direction, the Astronaut's velocity is

$$\vec{v}_A = - \frac{1}{70} (20\vec{i}) = -0.3\vec{i} \text{ (m/s)}$$

Impulse and Linear Momentum

*Net force causes change of momentum →
Newton's second law*

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \Rightarrow \quad d\vec{p} = \vec{F} dt$$

By integrating the above equation in a time interval t_i to t_f one can obtain impulse I .

$$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F} dt = \vec{I}$$

So what do you think an impulse is?

Effect of the force F acting on an object over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the object caused by that force. Impulse is the degree of which an external force changes an object's momentum.

The above statement is called the impulse-momentum theorem and is equivalent to Newton's second law.

*What are the dimension and unit of Impulse?
What is the direction of an impulse vector?*

Defining a time-averaged force

$$\vec{F} \equiv \frac{1}{\Delta t} \sum_i \vec{F}_i \Delta t$$

Impulse can be rewritten

$$\vec{I} \equiv \vec{F} \Delta t$$

If force is constant

$$\vec{I} \equiv \vec{F} \Delta t$$

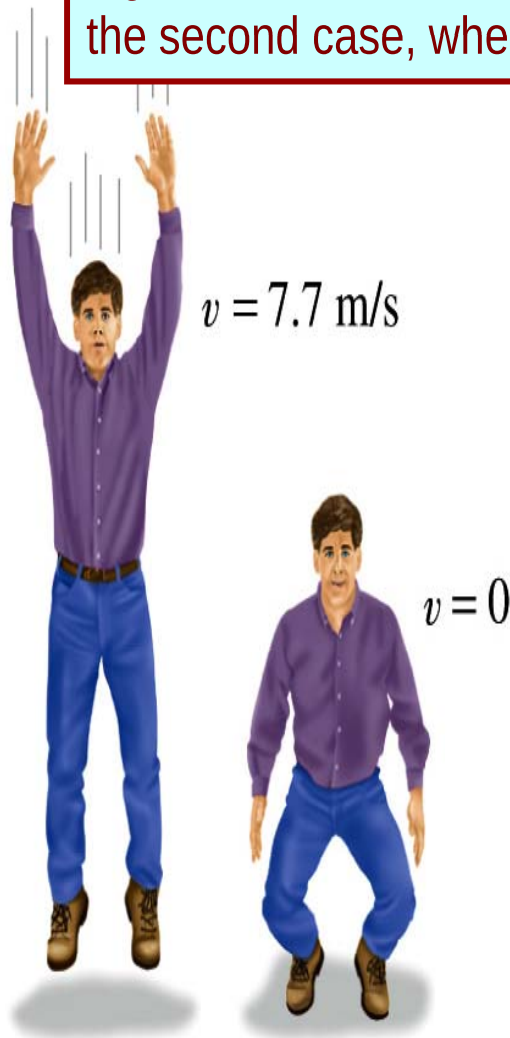
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It is generally assumed that the impulse force acts on a short time but much greater than any other forces present.

Example for Impulse

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \quad \frac{1}{2}mv^2 = -mg(y - y_i) = mgy_i$$

Solving the above for velocity v , we obtain

$$v = \sqrt{2gy_i} = \sqrt{2 \cdot 9.8 \cdot 3} = 7.7 \text{ m/s}$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$\begin{aligned} \vec{I} &= \vec{F}\Delta t = \Delta \vec{p} = \vec{p}_f - \vec{p}_i = 0 - m\vec{v} = \\ &= -70\text{kg} \cdot 7.7\text{m/s} \vec{j} = -540 \vec{j} \text{ N} \cdot \text{s} \end{aligned}$$

Example cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance $d=1.0\text{cm}=0.01\text{m}$.

The average speed during this period is $\bar{v} = \frac{0 + v_i}{2} = \frac{7.7}{2} = 3.8\text{m / s}$

The time period the collision lasts is $\Delta t = \frac{d}{\bar{v}} = \frac{0.01\text{m}}{3.8\text{m / s}} = 2.6 \times 10^{-3}\text{s}$

Since the magnitude of impulse is $|\vec{I}| = |\vec{F}\Delta t| = 540\text{N} \cdot \text{s}$

The average force on the feet during this landing is $\bar{F} = \frac{I}{\Delta t} = \frac{540}{2.6 \times 10^{-3}} = 2.1 \times 10^5\text{N}$

How large is this average force? $\text{Weight} = 70\text{kg} \cdot 9.8\text{m / s}^2 = 6.9 \times 10^2\text{N}$

$$\bar{F} = 2.1 \times 10^5\text{N} = 304 \times 6.9 \times 10^2\text{N} = 304 \times \text{Weight}$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

For bent legged landing: $\Delta t = \frac{d}{\bar{v}} = \frac{0.50\text{m}}{3.8\text{m / s}} = 0.13\text{s}$

$$\bar{F} = \frac{540}{0.13} = 4.1 \times 10^3\text{N} = 5.9\text{Weight}$$



Another Example for Impulse

In a crash test, an automobile of mass 1500kg collides with a wall. The initial and final velocities of the automobile are $\vec{v}_i = -15.0\vec{i}$ m/s and $\vec{v}_f = 2.60\vec{i}$ m/s. If the collision lasts for 0.150 seconds, what would be the impulse caused by the collision and the average force exerted on the automobile?

Let's assume that the force involved in the collision is a lot larger than any other forces in the system during the collision. From the problem, the initial and final momentum of the automobile before and after the collision is

$$\vec{p}_i = m\vec{v}_i = 1500 \times (-15.0)\vec{i} = -22500\vec{i} \text{ kg} \cdot \text{m} / \text{s}$$

$$\vec{p}_f = m\vec{v}_f = 1500 \times (2.60)\vec{i} = 3900\vec{i} \text{ kg} \cdot \text{m} / \text{s}$$

Therefore the impulse on the automobile due to the collision is

$$\begin{aligned}\vec{I} &= \Delta \vec{p} = \vec{p}_f - \vec{p}_i = (3900 + 22500)\vec{i} \text{ kg} \cdot \text{m} / \text{s} \\ &= 26400\vec{i} \text{ kg} \cdot \text{m} / \text{s} = 2.64 \times 10^4 \vec{i} \text{ kg} \cdot \text{m} / \text{s}\end{aligned}$$

The average force exerted on the automobile during the collision is

$$\begin{aligned}\vec{F} &= \frac{\Delta \vec{p}}{\Delta t} = \frac{2.64 \times 10^4}{0.150} \vec{i} \\ &= 1.76 \times 10^5 \vec{i} \text{ kg} \cdot \text{m} / \text{s}^2 = 1.76 \times 10^5 \vec{i} \text{ N}\end{aligned}$$

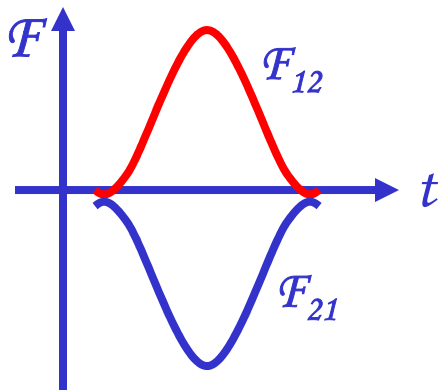


Collisions

Generalized collisions must cover not only the physical contact but also the collisions without physical contact such as that of electromagnetic ones in a microscopic scale.

Consider a case of a collision between a proton and a helium ion.

The collisions of these ions never involve physical contact because the electromagnetic repulsive force between these two become great as they get closer causing a collision.



Assuming no external forces, the force exerted on particle 1 by particle 2, F_{21} , changes the momentum of particle 1 by

$$d\vec{p}_1 = \vec{F}_{21} dt$$

Likewise for particle 2 by particle 1

$$d\vec{p}_2 = \vec{F}_{12} dt$$

Using Newton's 3rd law we obtain

$$d\vec{p}_2 = \vec{F}_{12} dt = -\vec{F}_{21} dt = -d\vec{p}_1$$

So the momentum change of the system in the collision is 0, and the momentum is conserved

$$\begin{aligned} d\vec{p} &= d\vec{p}_1 + d\vec{p}_2 = 0 \\ \vec{p}_{\text{system}} &= \vec{p}_1 + \vec{p}_2 = \text{constant} \end{aligned}$$

Elastic and Inelastic Collisions

Momentum is conserved in any collisions as long as external forces are negligible.

*Collisions are classified as **elastic** or **inelastic** based on whether the kinetic energy is conserved, meaning whether it is the same before and after the collision.*

**Elastic
Collision**

A collision in which the total kinetic energy and momentum are the same before and after the collision.

**Inelastic
Collision**

A collision in which the total kinetic energy is not the same before and after the collision, but momentum is.

Two types of inelastic collisions: Perfectly inelastic and inelastic

Perfectly Inelastic: *Two objects stick together after the collision, moving together at a certain velocity.*

Inelastic: *Colliding objects do not stick together after the collision but some kinetic energy is lost.*

Note: Momentum is constant in all collisions but kinetic energy is only in elastic collisions.

Elastic and Perfectly Inelastic Collisions

In perfectly Inelastic collisions, the objects *stick together* after the collision, moving together.

Momentum is conserved in this collision, so the final velocity of the stuck system is

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f$$

$$\vec{v}_f = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{(m_1 + m_2)}$$

How about elastic collisions?

In elastic collisions, both the momentum and the *kinetic energy* are conserved. Therefore, the final speeds in an elastic collision can be obtained in terms of initial speeds as

$$m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 (v_{1i}^2 - v_{1f}^2) = m_2 (v_{2i}^2 - v_{2f}^2)$$

$$m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 (v_{2i} - v_{2f})(v_{2i} + v_{2f})$$

From momentum conservation above

$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2i} - v_{2f})$$

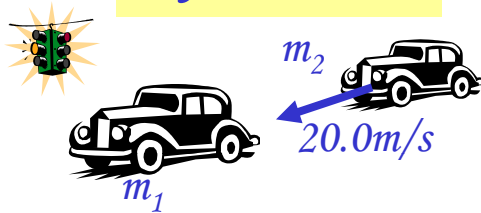
$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{2i}$$

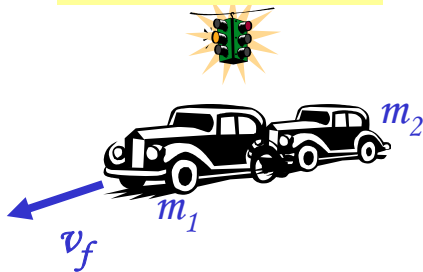
Example for Collisions

A car of mass 1800kg stopped at a traffic light is rear-ended by a 900kg car, and the two become entangled. If the lighter car was moving at 20.0m/s before the collision what is the velocity of the entangled cars after the collision?

Before collision



After collision



The momenta before and after the collision are

$$\vec{p}_i = m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = 0 + m_2 \vec{v}_{2i}$$

$$\vec{p}_f = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} = (m_1 + m_2) \vec{v}_f$$

Since momentum of the system must be conserved

$$\vec{p}_i = \vec{p}_f \quad \Rightarrow \quad (m_1 + m_2) \vec{v}_f = m_2 \vec{v}_{2i}$$

$$\vec{v}_f = \frac{m_2 \vec{v}_{2i}}{(m_1 + m_2)} = \frac{900 \times 20.0 \vec{i}}{900 + 1800} = 6.67 \vec{i} \text{ m/s}$$

What can we learn from these equations on the direction and magnitude of the velocity before and after the collision?

The cars are moving in the same direction as the lighter car's original direction to conserve momentum.

The magnitude is inversely proportional to its own mass.