PHYS 1443 – Section 002
Lecture #15

Monday, Nov. 3, 2008
Dr. Jae Yu

- Linear Momentum
- Conservation of Momentum
- Impulse
- Collisions – Elastic and Inelastic Collisions
- Exam Problem Solving Session

Today's homework is HW #9, due 9pm, Monday, Nov. 10!!

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at a velocity of v is defined as

$$\boxed{\vec{p} \equiv m\vec{v}}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m\left(\vec{v} - \vec{v}_0\right)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$

More on Conservation of Linear Momentum in a Two Body System

From the previous slide we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interactions

Mathematically this statement can be written as

$$\vec{p}_{2i} + \vec{p}_{1i} = \vec{p}_{2f} + \vec{p}_{1f}$$

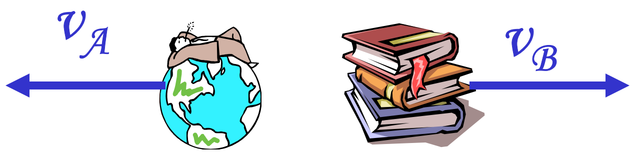
$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \qquad \sum_{system} P_{yi} = \sum_{system} P_{yf} \qquad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

This can be generalized into conservation of linear momentum in many particle systems.

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

Example for Linear Momentum Conservation

What is the astronaut's (M=70kg) resulting velocity after he throws his book (m=1kg, $\vec{v} = +20\vec{i}\,(m/s)$) in the space to move to the opposite direction?



From momentum conservation, we can write

$$\vec{p}_i = 0 = \vec{p}_f = m_A \vec{v}_A + m_B \vec{v}_B$$

Assuming the astronaut's mass is 70kg, and the book's mass is 1kg and using linear momentum conservation

$$\vec{v}_A = -\frac{m_B \vec{v}_B}{m_A} = -\frac{1}{70}\vec{v}_{\mathbf{B}}$$

Now if the book gained a velocity of 20 m/s in +x-direction, the Astronaut's velocity is

$$\vec{v}_A = -\frac{1}{70}\left(20\vec{i}\right) = -0.3\vec{i}\ \left(m/s\right)$$

Impulse and Linear Momentum

Net force causes change of momentum → Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \vec{F}dt$$

By integrating the above equation in a time interval t_i to t_f, one can obtain impulse I.

$$\int_{t_i}^{t_f} d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}dt = \vec{I}$$

So what do you think an impulse is?

The effect of a force F acting on an object over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the object caused by that force. Impulse is the degree of which an external force changes an object's momentum.

The above statement is called the impulse-momentum theorem and is equivalent to Newton's second law.

What are the dimension and unit of Impulse? What is the direction of an impulse vector?

Defining a time-averaged force

$$\overline{\vec{F}} \equiv \frac{1}{\Delta t}\sum_i \vec{F}_i \Delta t$$

Impulse can be rewritten

$$\vec{I} \equiv \overline{\vec{F}}\Delta t$$

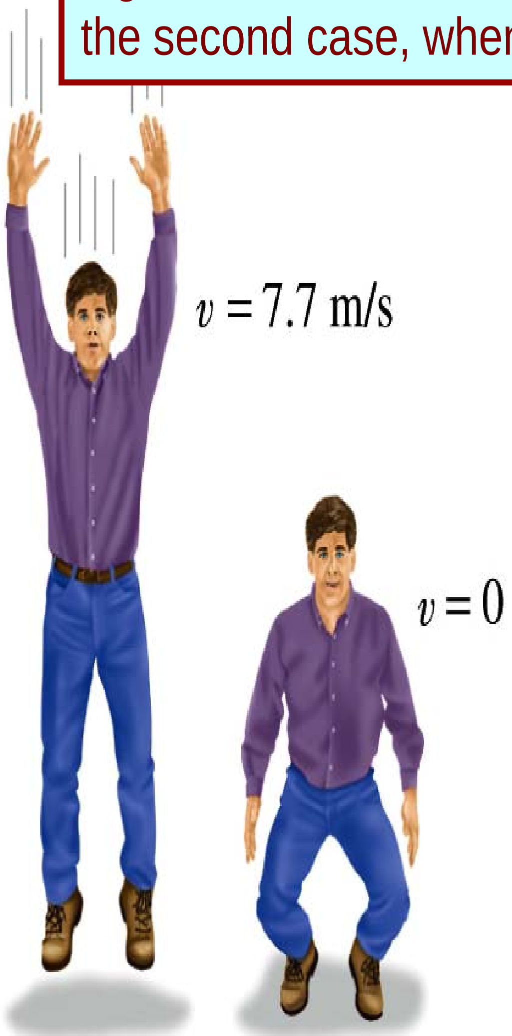
If force is constant

$$\vec{I} \equiv \vec{F}\Delta t$$

It is generally assumed that the impulse force acts on a short time but much greater than any other forces present.

Example for Impulse

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2 \cdot 9.8 \cdot 3} = 7.7m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly, giving the impulse

$$\vec{I} = \vec{\overline{F}}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - m\vec{v} =$$

$$= -70kg \cdot 7.7m/s\,\vec{j} = -540\,\vec{j}N \cdot s$$

Example cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $$\bar{v} = \frac{0+v_i}{2} = \frac{7.7}{2} = 3.8m/s$$

The time period the collision lasts is $$\Delta t = \frac{d}{\bar{v}} = \frac{0.01m}{3.8m/s} = 2.6\times 10^{-3}s$$

Since the magnitude of impulse is $$\left|\vec{I}\right| = \left|\vec{\bar{F}}\Delta t\right| = 540N\cdot s$$

The average force on the feet during this landing is $$\bar{F} = \frac{I}{\Delta t} = \frac{540}{2.6\times 10^{-3}} = 2.1\times 10^{5}N$$

How large is this average force? $$Weight = 70kg\cdot 9.8m/s^{2} = 6.9\times 10^{2}N$$

$$\bar{F} = 2.1\times 10^{5}N = 304\times 6.9\times 10^{2}N = 304\times Weight$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

For bent legged landing: $$\Delta t = \frac{d}{\bar{v}} = \frac{0.50m}{3.8m/s} = 0.13s$$

$$\bar{F} = \frac{540}{0.13} = 4.1\times 10^{3}N = 5.9Weight$$

Another Example for Impulse

In a crash test, an automobile of mass 1500kg collides with a wall. The initial and final velocities of the automobile are v_i= -15.0i m/s and v_f=2.60i m/s. If the collision lasts for 0.150 seconds, what would be the impulse caused by the collision and the average force exerted on the automobile?

assume that the force involved in the collision is a lot larger than any other forces in the system during the collision. From thLet's e problem, the initial and final momentum of the automobile before and after the collision is

$$\vec{p}_i = m\vec{v}_i = 1500 \times (-15.0)\vec{i} = -22500\vec{i}\ kg \cdot m/s$$

$$\vec{p}_f = m\vec{v}_f = 1500 \times (2.60)\vec{i} = 3900\vec{i}\ kg \cdot m/s$$

Therefore the impulse on the automobile due to the collision is

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3900 + 22500)\vec{i}\ kg \cdot m/s$$
$$= 26400\vec{i}\ kg \cdot m/s = 2.64 \times 10^4\vec{i}\ kg \cdot m/s$$

The average force exerted on the automobile during the collision is

$$\overline{\vec{F}} = \frac{\Delta\vec{p}}{\Delta t} = \frac{2.64 \times 10^4}{0.150}\vec{i}$$
$$= 1.76 \times 10^5\vec{i}\ kg \cdot m/s^2 = 1.76 \times 10^5\vec{i}\ \mathrm{N}$$