

PHYS 1443 – Section 002

Lecture #17

Monday, Nov. 10, 2008

Dr. Jae Yu

- Fundamentals of Rotational Motions
- Rotational Kinematics
- Rolling Motion of a Rigid Body

Today's homework is HW #10, due 9pm, Monday, Nov. 17!!



Announcements

- Third term exam
 - 1 – 2:20pm, Wednesday, Nov. 19, in SH103
 - Covers CH 9 – What we complete this Wednesday, Nov. 12
 - Jason will do a summary session on Monday, Nov. 17



Reminder: Extra-Credit Special Project

- Derive the formula for the final velocity of two objects which underwent an elastic collision as a function of known quantities m_1 , m_2 , v_{01} and v_{02} in page 6 of this lecture note. Must be done in far greater detail than what is covered in the lecture note.
 - 20 points extra credit
- Describe in detail what happens to the final velocities if $m_1=m_2$.
 - 5 point extra credit
- Due: Start of the class this Wednesday, Nov. 12

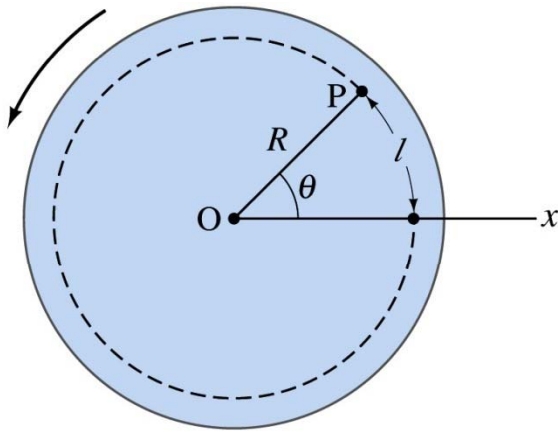


Fundamentals of Rotational Motions

Linear or translational motions can be described as the motion of the center of mass with all the mass of the object concentrated on it.

Is this still true for rotational motions?

No, because different parts of the object have different linear velocities and accelerations.



Consider a motion of a rigid body – an object that does not change its shape – rotating about the axis protruding out of the slide.

The arc length is $l = R\theta$

Therefore the angle, θ , is $\theta = \frac{l}{R}$. And the unit of the angle is in radian. It is dimensionless!!

One radian is the angle swept by an arc length equal to the radius of the arc.

Since the circumference of a circle is $2\pi r$, $360^\circ = 2\pi r / r = 2\pi$

The relationship between radian and degrees is $1 \text{ rad} = 360^\circ / 2\pi = 180^\circ / \pi$

Monday, Nov. 10, 2008



PHYS 1443-002, Fall 2008 Dr. Jaehoon Yu

$\cong 180^\circ / 3.14 \cong 57.3^\circ$ 4

Angular Displacement, Velocity, and Acceleration

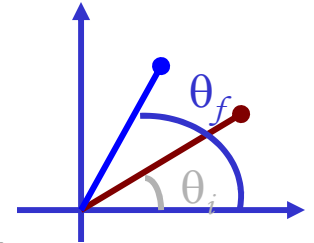
Using what we have learned in the previous slide, how would you define the angular displacement?

$$\Delta\theta = \theta_f - \theta_i$$

How about the average angular speed?

Unit? rad/s

$$\bar{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$



And the instantaneous angular speed?

Unit? rad/s

$$\omega \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

By the same token, the average angular acceleration is defined as...

Unit? rad/s²

$$\bar{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

And the instantaneous angular acceleration? Unit? rad/s²

$$\alpha \equiv \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.

Rotational Kinematics

The first type of motion we have learned in linear kinematics was under a constant acceleration. We will learn about the rotational motion under constant angular acceleration, because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular Speed under constant angular acceleration:

$$\omega_f = \omega_i + \alpha t$$

Angular displacement under constant angular acceleration:

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2$$

One can also obtain

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$



Example for Rotational Kinematics

A wheel rotates with a constant angular acceleration of 3.50 rad/s^2 . If the angular speed of the wheel is 2.00 rad/s at $t_i=0$, a) through what angle does the wheel rotate in 2.00s ?

Using the angular displacement formula in the previous slide, one gets

$$\begin{aligned}\theta_f - \theta_i &= \omega t + \frac{1}{2} \alpha t^2 \\ &= 2.00 \times 2.00 + \frac{1}{2} 3.50 \times (2.00)^2 = 11.0 \text{ rad} \\ &= \frac{11.0}{2\pi} \text{ rev.} = 1.75 \text{ rev.}\end{aligned}$$



Example for Rotational Kinematics cnt'd

What is the angular speed at $t=2.00\text{s}$?

Using the angular speed and acceleration relationship

$$\omega_f = \omega_i + \alpha t = 2.00 + 3.50 \times 2.00 = 9.00 \text{ rad/s}$$

Find the angle through which the wheel rotates between $t=2.00\text{s}$ and $t=3.00\text{s}$.

Using the angular kinematic formula

$$\theta_f - \theta_i = \omega t + \frac{1}{2} \alpha t^2$$

At $t=2.00\text{s}$

$$\theta_{t=2} = 2.00 \times 2.00 + \frac{1}{2} 3.50 \times 2.00^2 = 11.0 \text{ rad}$$

At $t=3.00\text{s}$

$$\theta_{t=3} = 2.00 \times 3.00 + \frac{1}{2} 3.50 \times (3.00)^2 = 21.8 \text{ rad}$$

Angular displacement

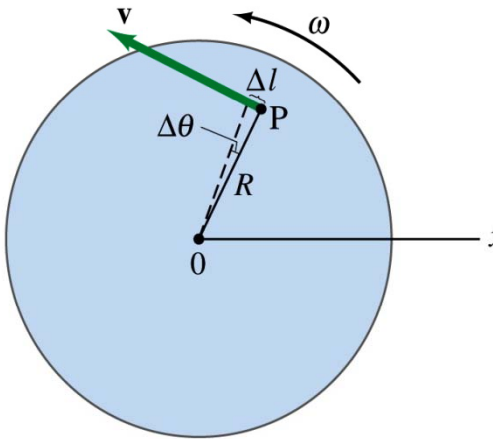
$$\Delta \theta = \theta_3 - \theta_2 = 10.8 \text{ rad} = \frac{10.8}{2\pi} \text{ rev.} = 1.72 \text{ rev.}$$



Relationship Between Angular and Linear Quantities

What do we know about a rigid object that rotates about a fixed axis of rotation?

Every particle (or masslet) in the object moves in a circle centered at the same axis of rotation.



When a point rotates, it has both the linear and angular components in its motion.

What is the linear component of the motion you see?

Linear velocity along the tangential direction.

How do we relate these linear quantities of the motion with angular quantities?

The direction of ω follows a right-hand rule.

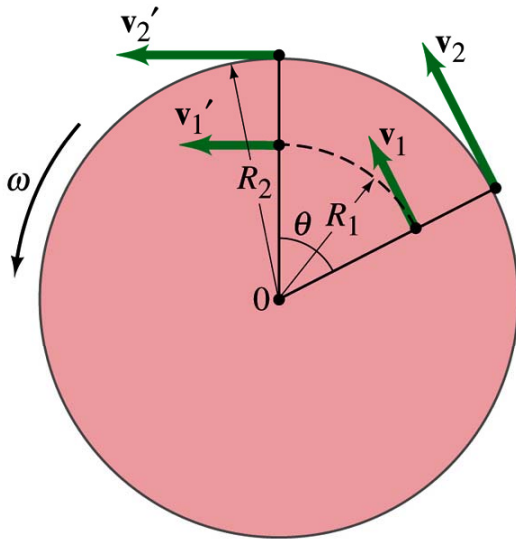
The arc-length is $l = r\theta$ So the tangential speed v is $v = \frac{dl}{dt} = \frac{d}{dt}(r\theta) = r \frac{d\theta}{dt} = r\omega$

What does this relationship tell you about the tangential speed of the points in the object and their angular speed?

Although every particle in the object has the same angular speed, its tangential speed differs and is proportional to its distance from the axis of rotation.

Is the lion faster than the horse?

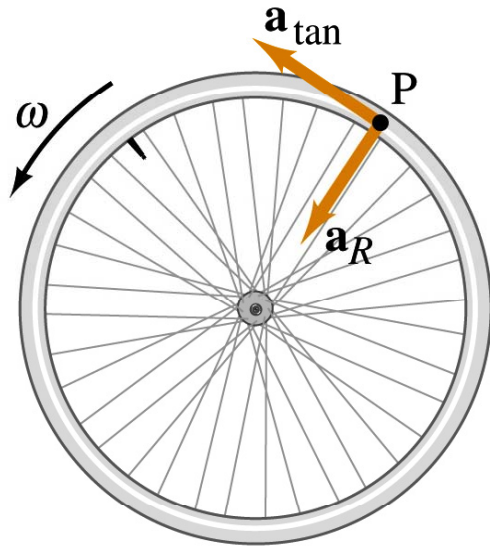
A rotating carousel has one child sitting on a horse near the outer edge and another child on a lion halfway out from the center. (a) Which child has the greater linear speed? (b) Which child has the greater angular speed?



(a) Linear speed is the distance traveled divided by the time interval. So the child sitting at the outer edge travels more distance within the given time than the child sitting closer to the center. Thus, the horse is faster than the lion.

(b) Angular speed is the angle traveled divided by the time interval. The angle both the children travel in the given time interval is the same. Thus, both the horse and the lion have the same angular speed.

How about the acceleration?



How many different linear acceleration components do you see in a circular motion and what are they? **Two**

Tangential, a_t , and the radial acceleration, a_r

Since the tangential speed v is $v = r\omega$

The magnitude of tangential acceleration a_t is $a_t = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r \frac{d\omega}{dt} = r\alpha$

What does this relationship tell you?

Although every particle in the object has the same angular acceleration, its tangential acceleration differs proportional to its distance from the axis of rotation.

The radial or centripetal acceleration a_r is $a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$

What does this tell you?

The farther away the particle is from the rotation axis, the more radial acceleration it receives. In other words, it receives more centripetal force.

Total linear acceleration is $a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$

Example

(a) What is the linear speed of a child seated 1.2m from the center of a steadily rotating merry-go-around that makes one complete revolution in 4.0s? (b) What is her total linear acceleration?

First, figure out what the angular speed of the merry-go-around is.

$$\omega = \frac{1 \text{ rev}}{4.0 \text{ s}} = \frac{2\pi}{4.0 \text{ s}} = 1.6 \text{ rad / s}$$

Using the formula for linear speed

$$v = r\omega = 1.2 \text{ m} \times 1.6 \text{ rad / s} = 1.9 \text{ m / s}$$

Since the angular speed is constant, there is no angular acceleration.

Tangential acceleration is

$$a_t = r\alpha = 1.2 \text{ m} \times 0 \text{ rad / s}^2 = 0 \text{ m / s}^2$$

Radial acceleration is

$$a_r = r\omega^2 = 1.2 \text{ m} \times (1.6 \text{ rad / s})^2 = 3.1 \text{ m / s}^2$$

Thus the total acceleration is

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{0 + (3.1)^2} = 3.1 \text{ m / s}^2$$

Example for Rotational Motion

Audio information on compact discs are transmitted digitally through the readout system consisting of laser and lenses. The digital information on the disc are stored by the pits and flat areas on the track. Since the speed of readout system is constant, it reads out the same number of pits and flats in the same time interval. In other words, the linear speed is the same no matter which track is played. a) Assuming the linear speed is 1.3 m/s, find the angular speed of the disc in revolutions per minute when the inner most ($r=23\text{mm}$) and outer most tracks ($r=58\text{mm}$) are read.

Using the relationship between angular and tangential speed $V=r\omega$

$$\begin{aligned} r = 23\text{mm} \quad \omega &= \frac{v}{r} = \frac{1.3\text{m/s}}{23\text{mm}} = \frac{1.3}{23 \times 10^{-3}} = 56.5\text{rad/s} \\ &= 9.00\text{rev/s} = 5.4 \times 10^2\text{rev/min} \end{aligned}$$

$$\begin{aligned} r = 58\text{mm} \quad \omega &= \frac{1.3\text{m/s}}{58\text{mm}} = \frac{1.3}{58 \times 10^{-3}} = 22.4\text{rad/s} \\ &= 2.1 \times 10^2\text{rev/min} \end{aligned}$$



b) The maximum playing time of a standard music CD is 74 minutes and 33 seconds. How many revolutions does the disk make during that time?

$$\bar{\omega} = \frac{(\omega_i + \omega_f)}{2} = \frac{(540 + 210) \text{ rev/min}}{2} = 375 \text{ rev/min}$$

$$\theta_f = \theta_i + \bar{\omega} t = 0 + \frac{375}{60} \text{ rev/s} \times 4473 \text{ s} = 2.8 \times 10^4 \text{ rev}$$

c) What is the total length of the track past through the readout mechanism?

$$l = v_t \Delta t = 1.3 \text{ m/s} \times 4473 \text{ s} = 5.8 \times 10^3 \text{ m}$$

d) What is the angular acceleration of the CD over the 4473s time interval, assuming constant α ?

$$\alpha = \frac{(\omega_f - \omega_i)}{\Delta t} = \frac{(22.4 - 56.5) \text{ rad/s}}{4473 \text{ s}} = 7.6 \times 10^{-3} \text{ rad/s}^2$$