PHYS 1441 – Section 002
Lecture #2

Wednesday, Aug. 26, 2009

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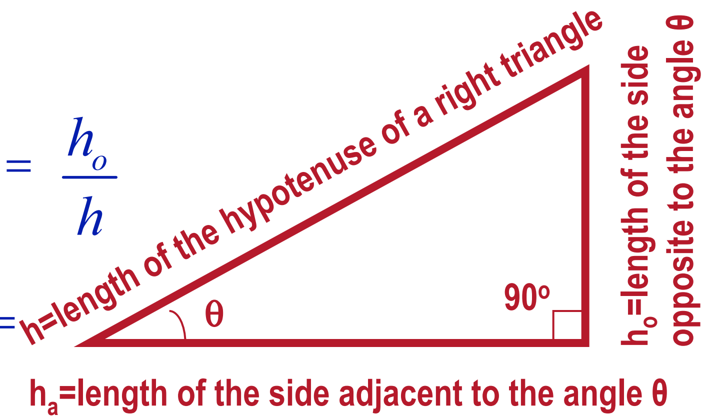
- Dimensional Analysis
- Some Fundamentals
- One Dimensional Motion
- Displacement
- Speed and Velocity
- Acceleration
- Motion under constant acceleration

Trigonometry Reminders

- Definitions of sinθ, cosθ and tanθ



$$\sin\theta = \frac{\text{Length of the opposite side to }\theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_o}{h}$$

$$\cos\theta = \frac{\text{Length of the adjacent side to }\theta}{\text{Length of the hypotenuse of the right triangle}} = \frac{h_a}{h}$$

$$\tan\theta = \frac{\text{Length of the opposite side to }\theta}{\text{Length of the adjacent side to }\theta} = \frac{h_o}{h_a}$$

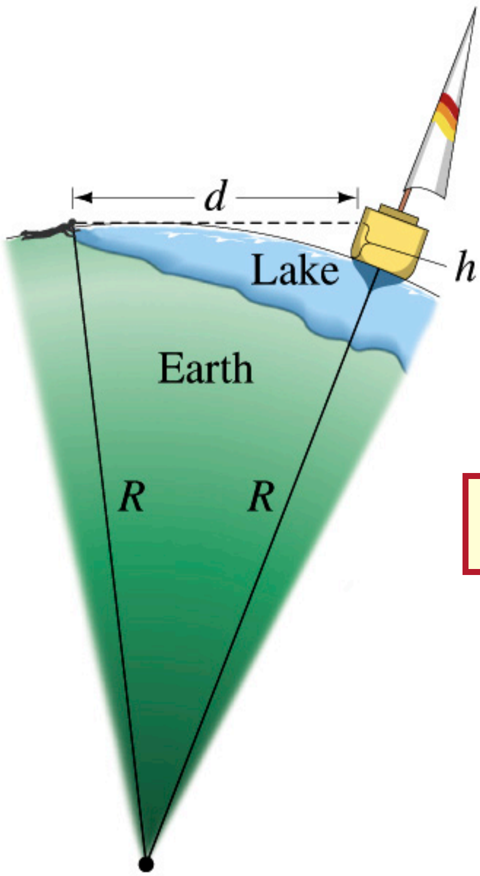
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{h_o}{h}}{\frac{h_a}{h}} = \frac{h_o}{h_a}$$

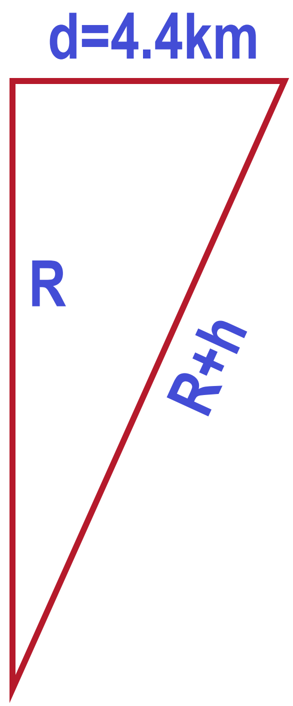
Pythagorian theorem: For right triangles

$$h^2 = h_o^2 + h_a^2 \Rightarrow h = \sqrt{h_o^2 + h_a^2}$$

Example for estimates using trig..

Estimate the radius of the Earth using triangulation as shown in the picture when d=4.4km and h=1.5m.





Pythagorian theorem

$$(R+h)^2 \approx d^2 + R^2$$

$$R^2 + 2hR + h^2 \approx d^2 + R^2$$

Solving for R

$$R \approx \frac{d^2 - h^2}{2h}$$

$$= \frac{(4400m)^2 - (1.5m)^2}{2 \times 1.5m}$$

$$= 6500km$$

Dimension and Dimensional Analysis

- An extremely useful concept in solving physical problems
- Good to write physical laws in mathematical expressions
- No matter what units are used the base quantities are the same
  - *Length* (distance) is length whether meter or inch is used to express the size: Usually denoted as *[L]*
  - The same is true for *Mass ([M])* and *Time ([T])*
  - One can say "Dimension of Length, Mass or Time"
  - Dimensions are used as algebraic quantities: Can perform two algebraic operations; multiplication or division

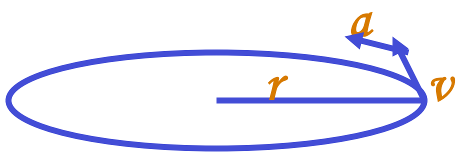
Dimension and Dimensional Analysis

- One can use dimensions only to check the validity of one's expression: Dimensional analysis
  - Eg: Speed $[v] = [L]/[T]=[L][T^{-1}]$
    - *Distance (L) traveled by a car running at the speed V in time T*
    - $L = V*T = [L/T]*[T]=[L]$
- More general expression of dimensional analysis is using exponents: eg. $[v]=[L^nT^m] =[L]\{T^{-1}]$ *where* $n = 1$ *and* $m = -1$

Examples

- Show that the expression $[v] = [at]$ is dimensionally correct
  - Speed: $[v]$ =L/T
  - Acceleration: $[a]$ =L/T^2
  - Thus, $[at]$ = (L/T^2)xT=L$T^{(-2+1)}$ =LT^{-1} =L/T= $[v]$

•Suppose the acceleration a of a circularly moving particle with speed v and radius r is proportional to r^n and v^m. What are n and m?



$$a = kr^n v^m$$

Dimensionless constant (k), Length (r), Speed (v)

$$L^1T^{-2} = (L)^n \left(\frac{L}{T}\right)^m = L^{n+m}T^{-m}$$

$$-m = -2 \Rightarrow m = 2$$

$$n + m = n + 2 \equiv 1 \Rightarrow n = -1$$

$$a = kr^{-1}v^2 = \frac{v^2}{r}$$

Some Fundamentals

- **Kinematics**: Description of Motion without understanding the cause of the motion
- **Dynamics**: Description of motion accompanied with understanding the cause of the motion
- Vector and Scalar quantities:
  - **Scalar**: Physical quantities that require magnitude but no direction
    - Speed, length, mass, height, volume, area, magnitude of a vector quantity, etc
  - **Vector**: Physical quantities that require both magnitude and direction
    - Velocity, Acceleration, Force, Momentum
    - It does not make sense to say "I ran with velocity of 10miles/hour."
- Objects can be treated as point-like if their sizes are smaller than the scale in the problem
  - Earth can be treated as a point like object (or a particle) in celestial problems
    - Simplification of the problem (The first step in setting up to solve a problem…)
  - Any other examples?

Some More Fundamentals

- **<u>Motions</u>**:Can be described as long as the position is known at any time (or position is expressed as a function of time)
  - Translation: Linear motion along a line
  - Rotation: Circular or elliptical motion
  - Vibration: Oscillation
- Dimensions
  - 0 dimension: A point
  - 1 dimension: Linear drag of a point, resulting in a line ➔ Motion in one-dimension is a motion on a straight line
  - 2 dimension: Linear drag of a line resulting in a surface
  - 3 dimension: Perpendicular Linear drag of a surface, resulting in a stereo object

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

A vector quantity

Displacement is the difference between initial and final potions of the motion and is a vector quantity. How is this different than distance?

Unit? **m**

The average velocity is defined as: $v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \equiv \frac{\text{Displacement}}{\text{Elapsed Time}}$

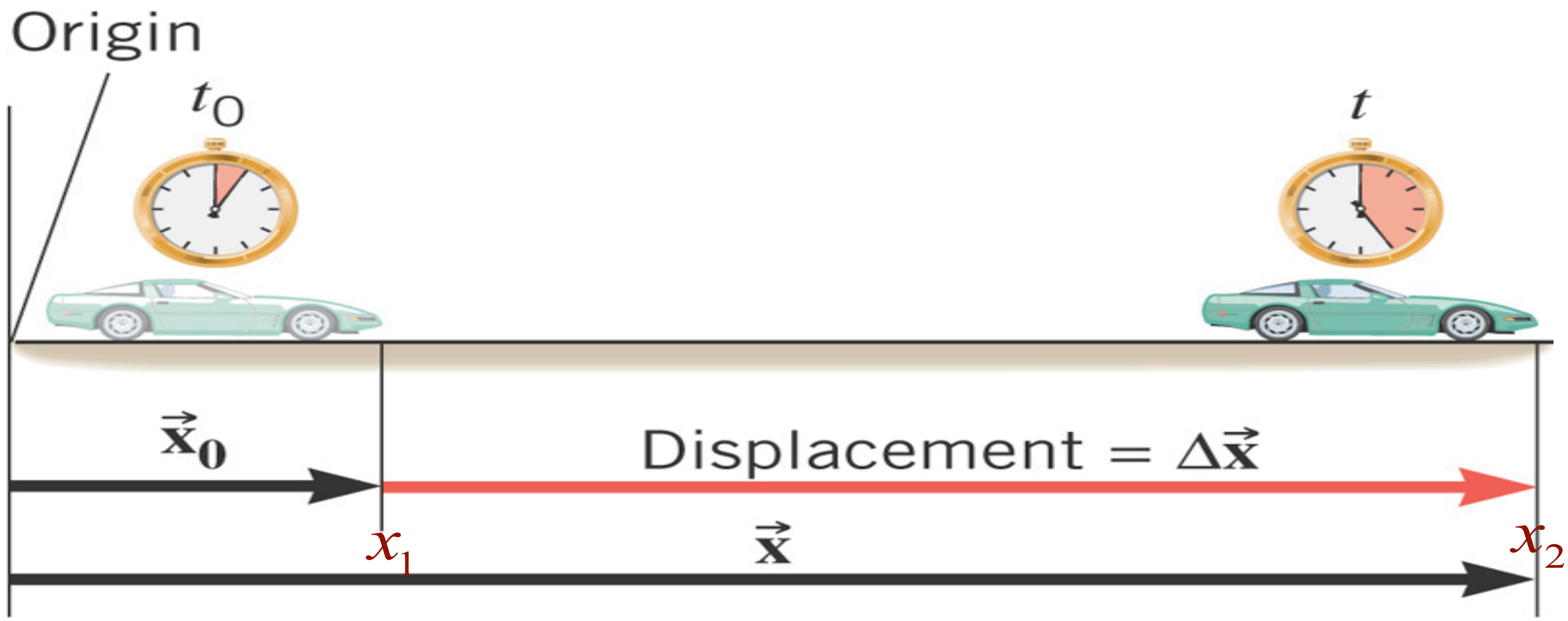
Unit? **m/s** A vector quantity

Displacement per unit time in the period throughout the motion

The average speed is defined as: $v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Elapsed Time}}$

Unit? **m/s**

A scalar quantity



What is the displacement? $\Delta x = x_2 - x_1$

How much is the elapsed time? $\Delta t = t - t_0$

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

Displacement is the difference between initial and final potions of the motion and is a vector quantity. How is this different than distance?

Unit? m

The average velocity is defined as: $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \equiv \frac{\text{Displacement}}{\text{Elapsed Time}}$$

Unit? m/s

Displacement per unit time in the period throughout the motion

The average speed is defined as: $$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Elapsed Time}}$$

Unit? m/s

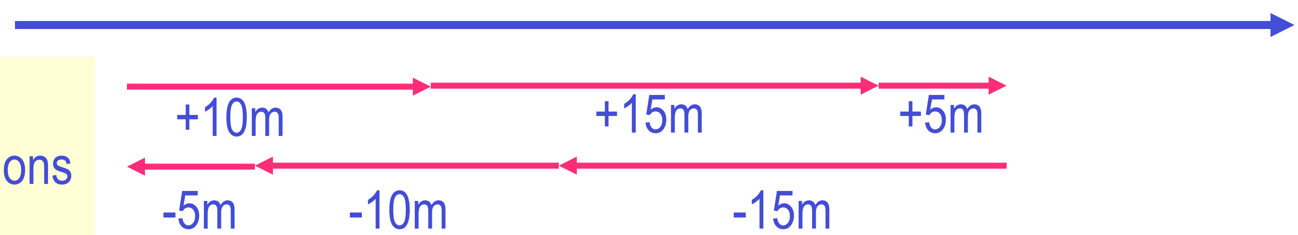
Can someone tell me what the difference between speed and velocity is?

Difference between Speed and Velocity

- Let's take a simple one dimensional translation that has many steps:

Let's call this line X-axis

Let's have a couple of motions in a total time interval of 20 sec.



Total Displacement: $\Delta x \equiv x_f - x_i = x_i - x_i = 0(m)$

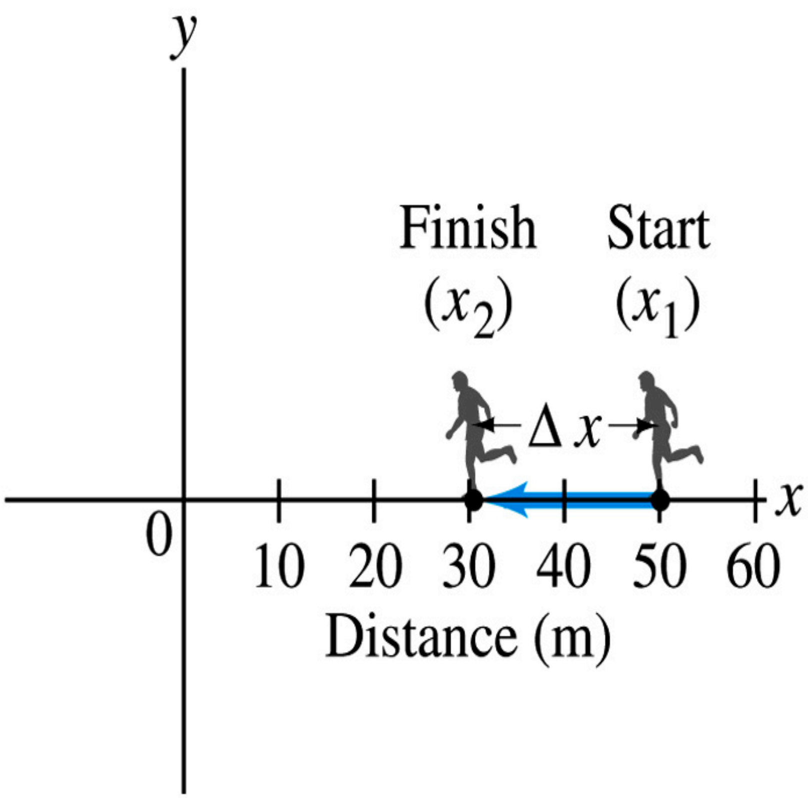
Average Velocity: $v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} = \frac{0}{20} = 0(m/s)$

Total Distance Traveled: $D = 10 + 15 + 5 + 15 + 10 + 5 = 60(m)$

Average Speed: $v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Elapsed Time}} = \frac{60}{20} = 3(m/s)$

Example 2.1

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from x_1=50.0m to x_2=30.5 m, as shown in the figure. What was the runner's average velocity? What was the average speed?



- Displacement:

$$\Delta x \equiv x_f - x_i = x_2 - x_1 = 30.5 - 50.0 = -19.5(m)$$

- Average Velocity:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-19.5}{3.00} = -6.50(m/s)$$

- Average Speed:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}}$$

$$= \frac{50.0 - 30.5}{3.00} = \frac{+19.5}{3.00} = +6.50(m/s)$$

Example Distance Run by a Jogger

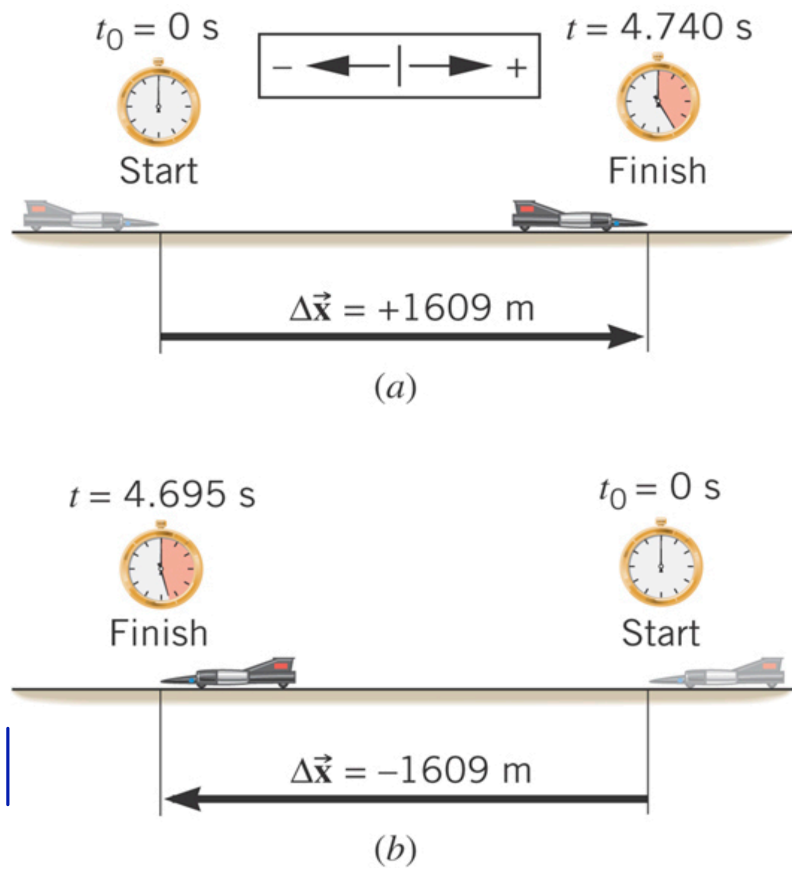
How far does a jogger run in 1.5 hours (5400 s) if his average speed is 2.22 m/s?

$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}}$$

$$\text{Distance} = (\text{Average speed})(\text{Elapsed time}) =$$
$$= (2.22 \text{ m/s})(5400 \text{ s}) = 12000 \text{ m}$$

Example The World's Fastest Jet-Engine Car

Andy Green in the car *ThrustSSC* set a world record of 341.1 m/s in 1997. To establish such a record, the driver makes two runs through the course, one in each direction to nullify wind effects. From the data, determine the average velocity for each run.



$$\bar{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{+1609\ \text{m}}{4.740\ \text{s}} = +339.5\ \text{m/s}$$

What is the speed? $v = \left|\bar{\vec{v}}\right| = 339.5\ \text{m/s}$

$$\bar{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t} = \frac{-1609\ \text{m}}{4.695\ \text{s}} = -342.7\ \text{m/s}$$

What is the speed? $v = \left|\bar{\vec{v}}\right| = \left|-342.7\ \text{m/s}\right|$

$$= 342.7\ \text{m/s}$$

Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion?
- Instantaneous velocity is defined as:
  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Average velocity over a very short amount of time

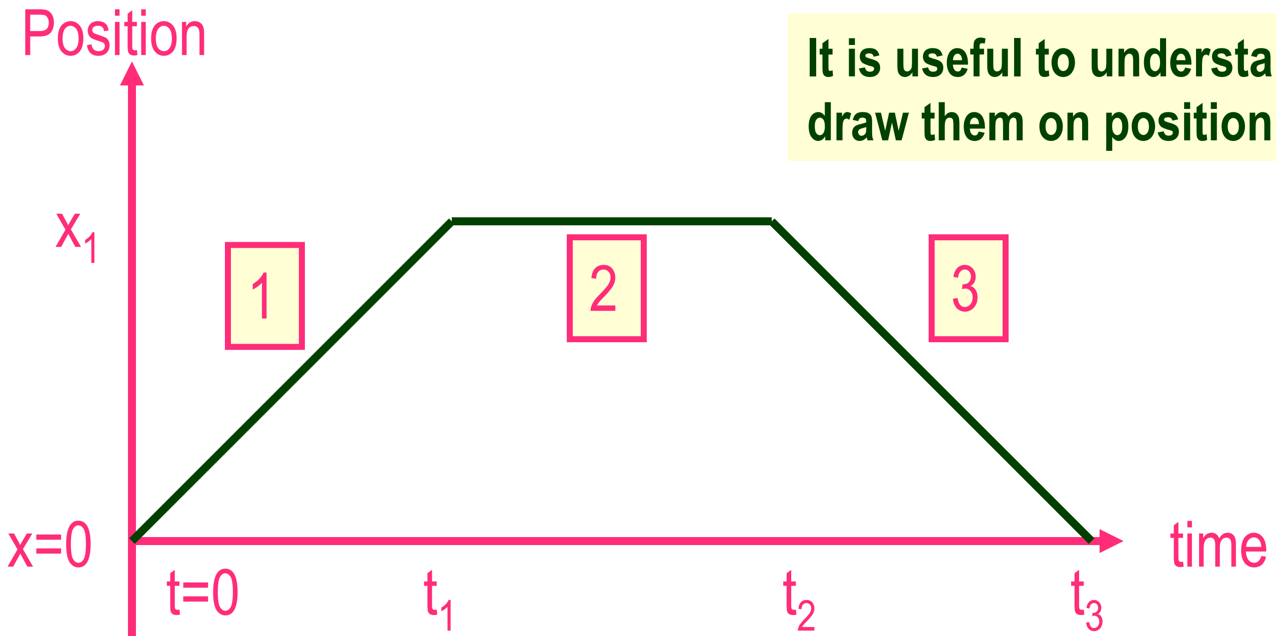
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- Instantaneous speed is the size (magnitude) of the velocity vector:

$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right|$$

*Magnitude of Vectors are Expressed in absolute values

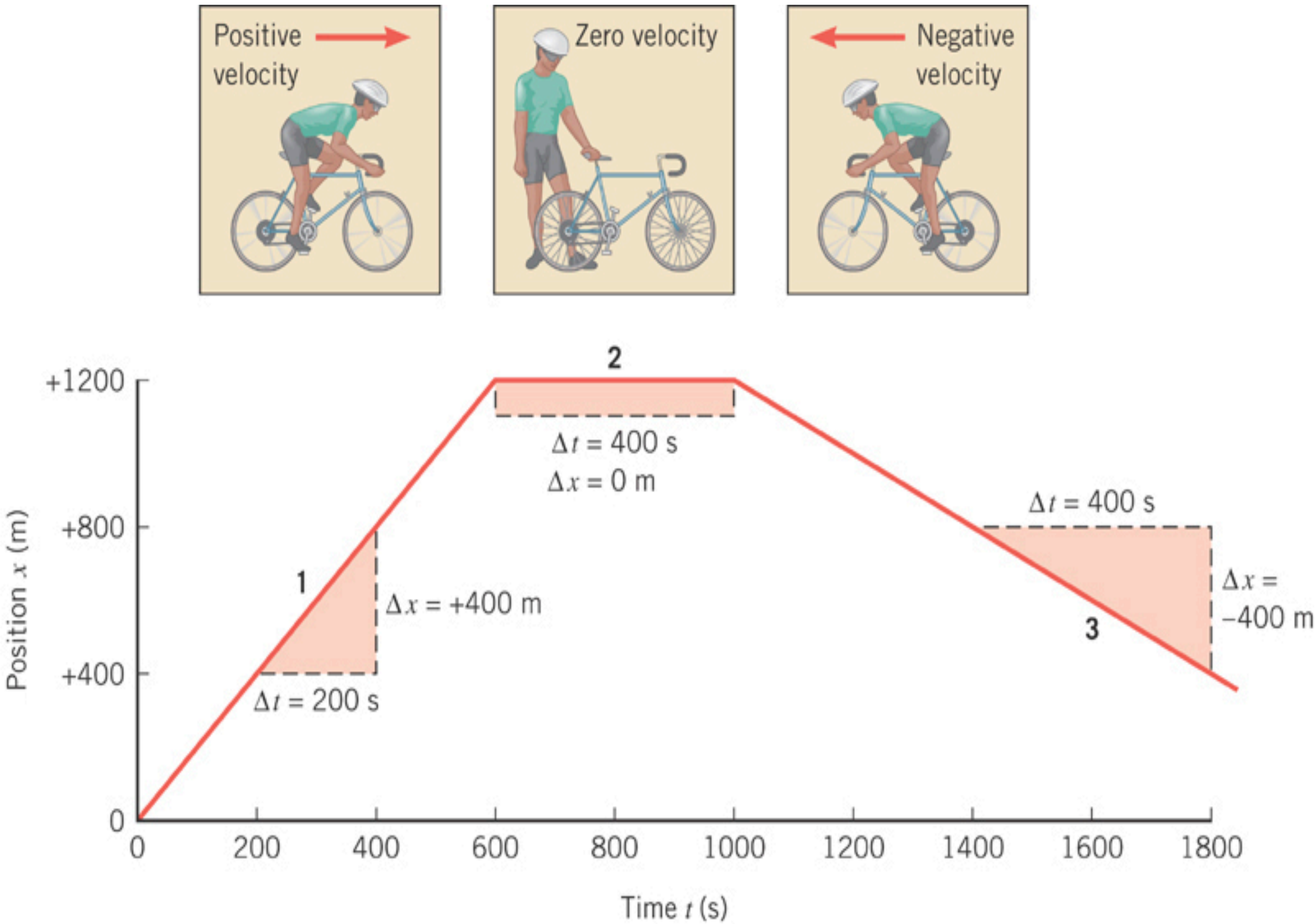
Position vs Time Plot



It is useful to understand motions to draw them on position vs time plots.

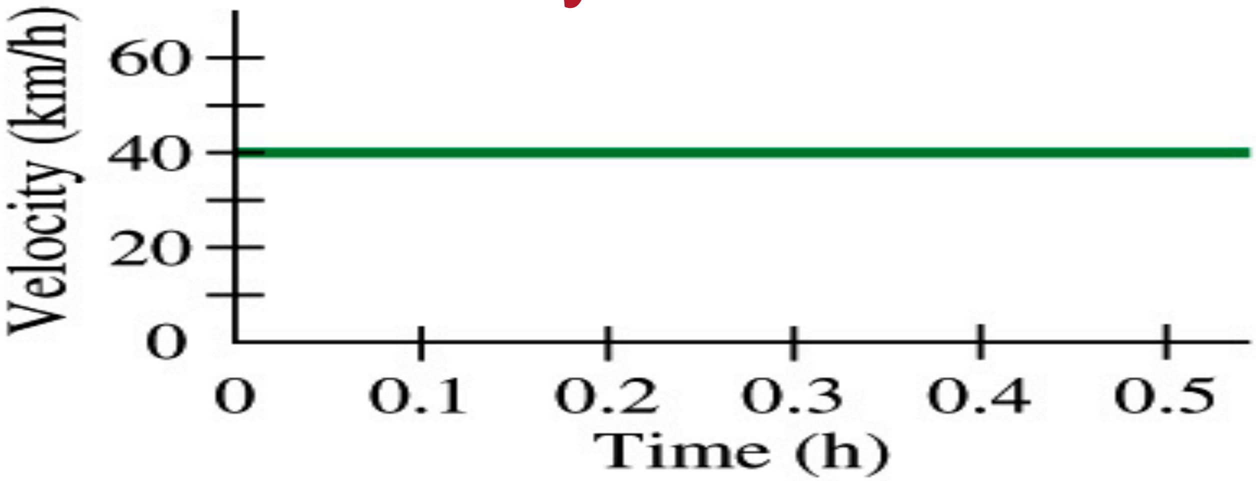
1. Running at a constant velocity (go from $x=0$ to $x=x_1$ in t_1, Displacement is $+ x_1$ in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to $x=0$ in t_3-t_2 time interval, Displacement is $- x_1$ in t_3-t_2 time interval)

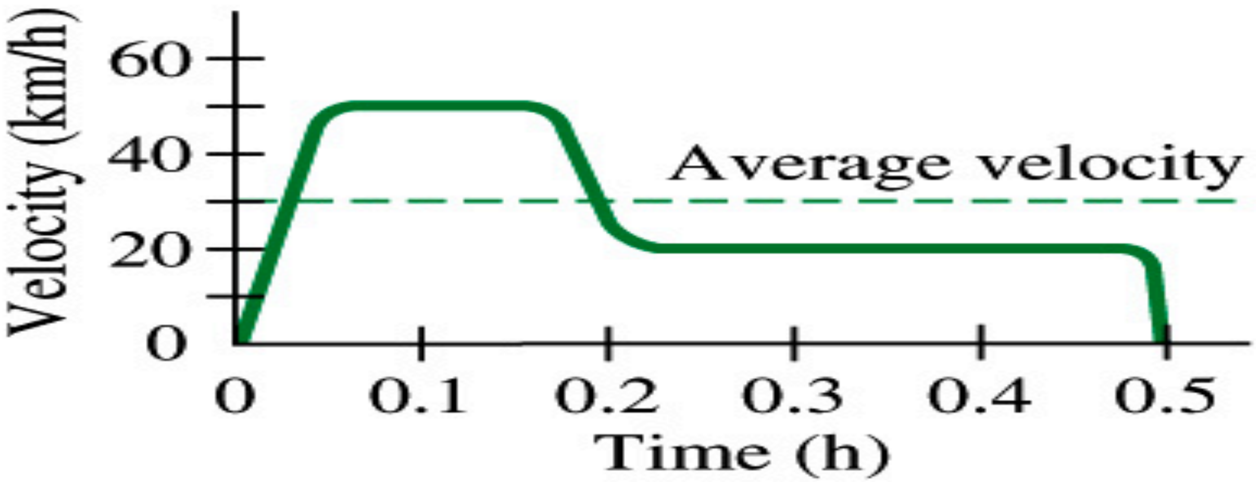
Does this motion physically make sense?



Positive velocity
Zero velocity
Negative velocity
2
1
3
Δt = 400 s
Δx = 0 m
Δx = +400 m
Δt = 200 s
Δt = 400 s
Δx = −400 m
+1200
+800
+400
0
0
200
400
600
800
1000
1200
1400
1600
1800
Position x (m)
Time t (s)

Velocity vs Time Plot





Displacement, Velocity and Speed

Displacement $$\Delta x \equiv x_f - x_i$$

Average velocity $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed $$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity $$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Instantaneous speed $$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right|$$