Vectors and Scalar
Properties and operations of vectors
Understanding a 2 Dimensional Motion
2D Kinematic Equations of Motion
Solving 2D Motion Problems Using Kinematics
Projectile Motion
Announcements

• First non-comprehensive term exam next Wednesday, Sept. 23!!
  – In the class, 1 – 2:20pm
  – Covers Appendices A1 – A8 and CH1 – CH3.6
  – Will be mixture of multiple choice and free response problems…
  – There will be a review session next Monday, Sept. 21 in the class
    • Be sure to bring problems to work out in the session

• Colloquium today
Physics Department
The University of Texas at Arlington

COLLOQUIUM

Transparent and red ZnO – Which Defect is Responsible for the Color?

Professor Marc Weber
Washington State University

Wednesday, September 16, 2009 at 4:00 pm in Room 101 SH

Abstract
Zinc oxide (ZnO) is a wide bandgap transparent semiconductor with tremendous potential in a broad range of applications. After more than 6 decades of research many obstacles remain. The largest challenge is to fabricate p-type material, an essential step towards electronic applications. I will discuss our recent work on this material and our view of oxygen vacancies and hydrogen impurities. Optical measurements and positron annihilation spectroscopy were applied to ZnO annealed in a range of conditions. Under oxygen poor conditions (sometimes referred to as zinc rich) ZnO may turn red. Methods to indirectly detect positively charged oxygen vacancies and to manipulate the hydrogen content will be discussed. A consistent, but controversial, picture emerges. Hydrogen occupying oxygen vacancies is the most likely defect responsible for the red color.

Refreshments will be served in the Physics Lounge at 3:30 p.m.
Vector and Scalar

Vector quantities have both magnitudes (sizes) and directions

- Force, gravitational acceleration, momentum

Normally denoted in **bold** letters, \( \mathbf{F} \), or a letter with arrow on top \( \vec{F} \)

Their sizes or magnitudes are denoted with normal letters, \( F \), or absolute values: \( |F| \) or \( \sqrt{F^2} \)

Scalar quantities have magnitudes only

Can be completely specified with a value and its unit

- Energy, heat, mass, time

Normally denoted in normal letters, \( E \)

Both have units!!!
Properties of Vectors

- Two vectors are the same if their sizes and the directions are the same, no matter where they are on a coordinate system!! ➔ You can move them around as you wish as long as their directions and sizes are kept the same.

Which ones are the same vectors?

A = B = E = D

Why aren’t the others?

C: The same magnitude but opposite direction: C = -A:

F: The same direction but different magnitude
Vector Operations

• Addition:
  – Triangular Method: One can add vectors by connecting the head of one vector to the tail of the other (head-to-tail)
  – Parallelogram method: Connect the tails of the two vectors and extend
  – Addition is commutative: Changing order of operation does not affect the results \( \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \), \( \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} + \mathbf{E} = \mathbf{E} + \mathbf{C} + \mathbf{A} + \mathbf{B} + \mathbf{D} \)

\[
\begin{align*}
\mathbf{A} + \mathbf{B} &= \mathbf{B} + \mathbf{A} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D} + \mathbf{E} = \mathbf{E} + \mathbf{C} + \mathbf{A} + \mathbf{B} + \mathbf{D} \\
\end{align*}
\]

• Subtraction:
  – The same as adding a negative vector: \( \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \)

\[
\begin{align*}
\mathbf{A} - \mathbf{B} &= \mathbf{A} + (-\mathbf{B}) \\
\end{align*}
\]

Since subtraction is the equivalent to adding a negative vector, subtraction is also commutative!!!

• Multiplication by a scalar is increasing the magnitude \( \mathbf{A}, \mathbf{B} = 2\mathbf{A} \)

\[
\begin{align*}
\mathbf{B} &= 2\mathbf{A} \\
\end{align*}
\]
Example for Vector Addition

A car travels 20.0 km due north followed by 35.0 km in a direction 60.0° west of north. Find the magnitude and direction of resultant displacement.

\[ r = \sqrt{(A + B \cos \theta)^2 + (B \sin \theta)^2} \]
\[ = \sqrt{A^2 + B^2 \left( \cos^2 \theta + \sin^2 \theta \right) + 2AB \cos \theta} \]
\[ = \sqrt{A^2 + B^2 + 2AB \cos \theta} \]
\[ = \sqrt{(20.0)^2 + (35.0)^2 + 2 \times 20.0 \times 35.0 \cos 60} \]
\[ = \sqrt{2325} = 48.2 (km) \]

\[ \theta = \tan^{-1} \frac{|\vec{B}| \sin 60}{|A| + |\vec{B}| \cos 60} \]
\[ = \tan^{-1} \frac{35.0 \sin 60}{20.0 + 35.0 \cos 60} \]
\[ = \tan^{-1} \frac{30.3}{37.5} = 38.9° \text{ to W wrt N} \]
Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components.

\[ A_x = |A| \cos \theta \]
\[ A_y = |A| \sin \theta \]
\[ |A| = \sqrt{A_x^2 + A_y^2} \]

\[ |A| = \sqrt{\left( |A| \cos \theta \right)^2 + \left( |A| \sin \theta \right)^2} \]
\[ = \sqrt{|A|^2 \left( \cos^2 \theta + \sin^2 \theta \right)} = |A| \]
Unit Vectors

- Unit vectors are the ones that tell us the directions of the components
- Dimensionless
- Magnitudes are exactly 1
- Unit vectors are usually expressed in $\vec{i}$, $\vec{j}$, $\vec{k}$

So the vector $\vec{A}$ can be re-written as

$$\vec{A} = A_x \vec{i} + A_y \vec{j} = |\vec{A}| \cos \theta \vec{i} + |\vec{A}| \sin \theta \vec{j}$$
Examples of Vector Operations

Find the resultant vector which is the sum of \( \vec{A} = (2.0i + 2.0j) \) and \( \vec{B} = (2.0i - 4.0j) \)

\[
\vec{C} = \vec{A} + \vec{B} = (2.0i + 2.0j) + (2.0i - 4.0j) = (2.0 + 2.0)i + (2.0 - 4.0)j = 4.0i - 2.0j \text{ (m)}
\]

\[
|\vec{C}| = \sqrt{(4.0)^2 + (-2.0)^2} = \sqrt{16 + 4.0} = \sqrt{20} = 4.5 \text{ (m)}
\]

\[
\theta = \tan^{-1} \frac{C_y}{C_x} = \tan^{-1} \frac{-2.0}{4.0} = -27^\circ
\]

Find the resultant displacement of three consecutive displacements: \( \vec{d}_1 = (15i + 30j + 12k) \text{ cm} \), \( \vec{d}_2 = (23i + 14j - 5.0k) \text{ cm} \), and \( \vec{d}_3 = (-13i + 15j) \text{ cm} \)

\[
\vec{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (15i + 30j + 12k) + (23i + 14j - 5.0k) + (-13i + 15j) = (15 + 23 - 13)i + (30 + 14 + 15)j + (12 - 5.0)k = 25i + 59j + 7.0k \text{ (cm)}
\]

Magnitude \( |\vec{D}| = \sqrt{(25)^2 + (59)^2 + (7.0)^2} = 65 \text{ (cm)} \)
2D Displacement

\[ \vec{r}_o = \text{initial position} \]

\[ \vec{r} = \text{final position} \]

\[ \Delta \vec{r} = \vec{r} - \vec{r}_o = \text{displacement} \]
Average velocity is the displacement divided by the elapsed time.

\[ \vec{V} = \frac{\vec{r} - \vec{r}_o}{t - t_o} = \frac{\Delta \vec{r}}{\Delta t} \]
The *instantaneous velocity* indicates how fast the car moves and the direction of motion at each instant of time.

\[
\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}
\]
2D Average Acceleration

\[ \vec{a} = \frac{\vec{v} - \vec{v}_o}{t - t_o} = \frac{\Delta \vec{v}}{\Delta t} \]
Displacement, Velocity, and Acceleration in 2-dim

• **Displacement:**
  \[ \vec{\Delta r} = \vec{r}_f - \vec{r}_i \]

• **Average Velocity:**
  \[ \vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} \]

• **Instantaneous Velocity:**
  \[ \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \]

• **Average Acceleration**
  \[ \vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} \]

• **Instantaneous Acceleration:**
  \[ \vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} \]

How is each of these quantities defined in 1-D?
## Kinematic Quantities in 1D and 2D

<table>
<thead>
<tr>
<th>Quantities</th>
<th>1 Dimension</th>
<th>2 Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>( \Delta x = x_f - x_i )</td>
<td>( \Delta r = r_f - r_i )</td>
</tr>
<tr>
<td>Average Velocity</td>
<td>( v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} )</td>
<td>( \vec{v} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i} )</td>
</tr>
<tr>
<td>Inst. Velocity</td>
<td>( v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} )</td>
<td>( \vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} )</td>
</tr>
<tr>
<td>Average Acc.</td>
<td>( a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} )</td>
<td>( \vec{a} \equiv \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} )</td>
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What is the difference between 1D and 2D quantities?
This is a motion that could be viewed as two motions combined into one. (superposition…)

A Motion in 2 Dimension
Motion in horizontal direction (x)

\[ v_x = v_{xo} + a_x t \]
\[ x = \frac{1}{2} (v_{xo} + v_x) t \]
\[ v_x^2 = v_{xo}^2 + 2a_x x \]
\[ x = v_{xo} t + \frac{1}{2} a_x t^2 \]
Motion in vertical direction \((y)\)

\[ v_y = v_{yo} + a_y t \]

\[ y = \frac{1}{2} (v_{yo} + v_y) t \]

\[ v_y^2 = v_{yo}^2 + 2a_y y \]

\[ y = v_{yo} t + \frac{1}{2} a_y t^2 \]
Imagine you add the two 1 dimensional motions on the left. It would make up a one 2 dimensional motion on the right.
Kinematic Equations in 2-Dim

**x-component**

\[ v_x = v_{xo} + a_x t \]

\[ x = \frac{1}{2} (v_{xo} + v_x) t \]

\[ v_x^2 = v_{xo}^2 + 2a_x x \]

\[ x = v_{xo} t + \frac{1}{2} a_x t^2 \]

**y-component**

\[ v_y = v_{yo} + a_y t \]

\[ y = \frac{1}{2} (v_{yo} + v_y) t \]

\[ v_y^2 = v_{yo}^2 + 2a_y y \]

\[ y = v_{yo} t + \frac{1}{2} a_y t^2 \]
Ex. A Moving Spacecraft

In the \( x \) direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s\(^2\). In the \( y \) direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s\(^2\). Find (a) \( x \) and \( v_x \), (b) \( y \) and \( v_y \), and (c) the final velocity of the spacecraft at time 7.0 s.
How do we solve this problem?

1. Visualize the problem ➔ Make a drawing.
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for $x$ and $y$ separately. Select the appropriate equation.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.
**Ex. continued**

In the $x$ direction, the spacecraft has an initial velocity component of $+22$ m/s and an acceleration of $+24$ m/s$^2$. In the $y$ direction, the analogous quantities are $+14$ m/s and an acceleration of $+12$ m/s$^2$. Find (a) $x$ and $v_x$, (b) $y$ and $v_y$, and (c) the final velocity of the spacecraft at time $7.0$ s.

<table>
<thead>
<tr>
<th></th>
<th>$a_x$</th>
<th>$v_x$</th>
<th>$v_{ox}$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$+24.0$ m/s$^2$</td>
<td>?</td>
<td>$+22.0$ m/s</td>
<td>$7.0$ s</td>
</tr>
</tbody>
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<table>
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<tr>
<th></th>
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<th>$v_y$</th>
<th>$v_{oy}$</th>
<th>$t$</th>
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<tbody>
<tr>
<td>$y$</td>
<td>$+12.0$ m/s$^2$</td>
<td>?</td>
<td>$+14.0$ m/s</td>
<td>$7.0$ s</td>
</tr>
</tbody>
</table>
First, the motion in x-direction...

\[ x = v_{ox} t + \frac{1}{2} a_x t^2 \]

\[ = (22 \text{ m/s})(7.0 \text{ s}) + \frac{1}{2} (24 \text{ m/s}^2)(7.0 \text{ s})^2 = +740 \text{ m} \]

\[ v_x = v_{ox} + a_x t \]

\[ = (22 \text{ m/s}) + (24 \text{ m/s}^2)(7.0 \text{ s}) = +190 \text{ m/s} \]
Now, the motion in y-direction...

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tr>
<td>$y$</td>
<td>$a_y$</td>
<td>$v_y$</td>
<td>$v_{oy}$</td>
<td>$t$</td>
</tr>
<tr>
<td>?</td>
<td>+12.0 m/s²</td>
<td>?</td>
<td>+14 m/s</td>
<td>7.0 s</td>
</tr>
</tbody>
</table>

\[
y = v_{oy}t + \frac{1}{2} a_y t^2
\]

\[
= (14 \text{ m/s})(7.0 \text{ s}) + \frac{1}{2} (12 \text{ m/s}^2)(7.0 \text{ s})^2 = +390 \text{ m}
\]

\[
v_y = v_{oy} + a_y t
\]

\[
= (14 \text{ m/s}) + (12 \text{ m/s}^2)(7.0 \text{ s}) = +98 \text{ m/s}
\]
The final velocity...

\[ \vec{v} = \vec{v}_x \hat{i} + \vec{v}_y \hat{j} = (190 \hat{i} + 98 \hat{j}) \text{ m/s} \]

\[ \nu = \sqrt{(190 \text{ m/s})^2 + (98 \text{ m/s})^2} = 210 \text{ m/s} \]

\[ \theta = \tan^{-1}\left(\frac{98}{190}\right) = 27^\circ \]

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using components and unit vectors!!

\[ \nu_x = 190 \text{ m/s} \]

\[ \nu_y = 98 \text{ m/s} \]
If we visualize the motion...

\[ v_y = 98 \text{ m/s} \]

\[ v_{x} = 190 \text{ m/s} \]