# PHYS 1441 – Section 002
# Lecture #8

*Monday, Sept. 28, 2009*

*Dr. Jaehoon Yu*

- Projectile Motion
- Maximum Range and Height
- Newton's Law of Motion
- Forces
- Mass

Today's homework is homework #5, due 9pm, Tuesday, Oct. 6!!

# Announcements

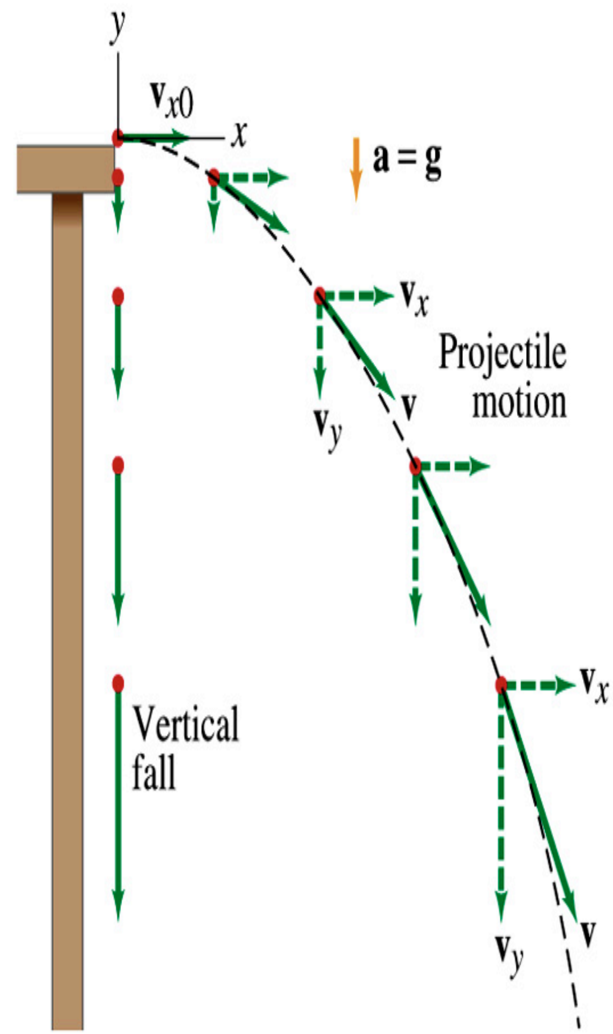
- Term exam results
  - Class Average: 53/98
    - Equivalent to 54/100
    - Not bad!!
  - Top score: 90/98
- Evaluation Policy
  - Homework: 30%
  - Final exam: 25%
  - Better of the two term exams: 20%
  - Lab: 15%
  - Quizzes: 10%
  - Extra Credit: 10%

# Special Project for Extra Credit

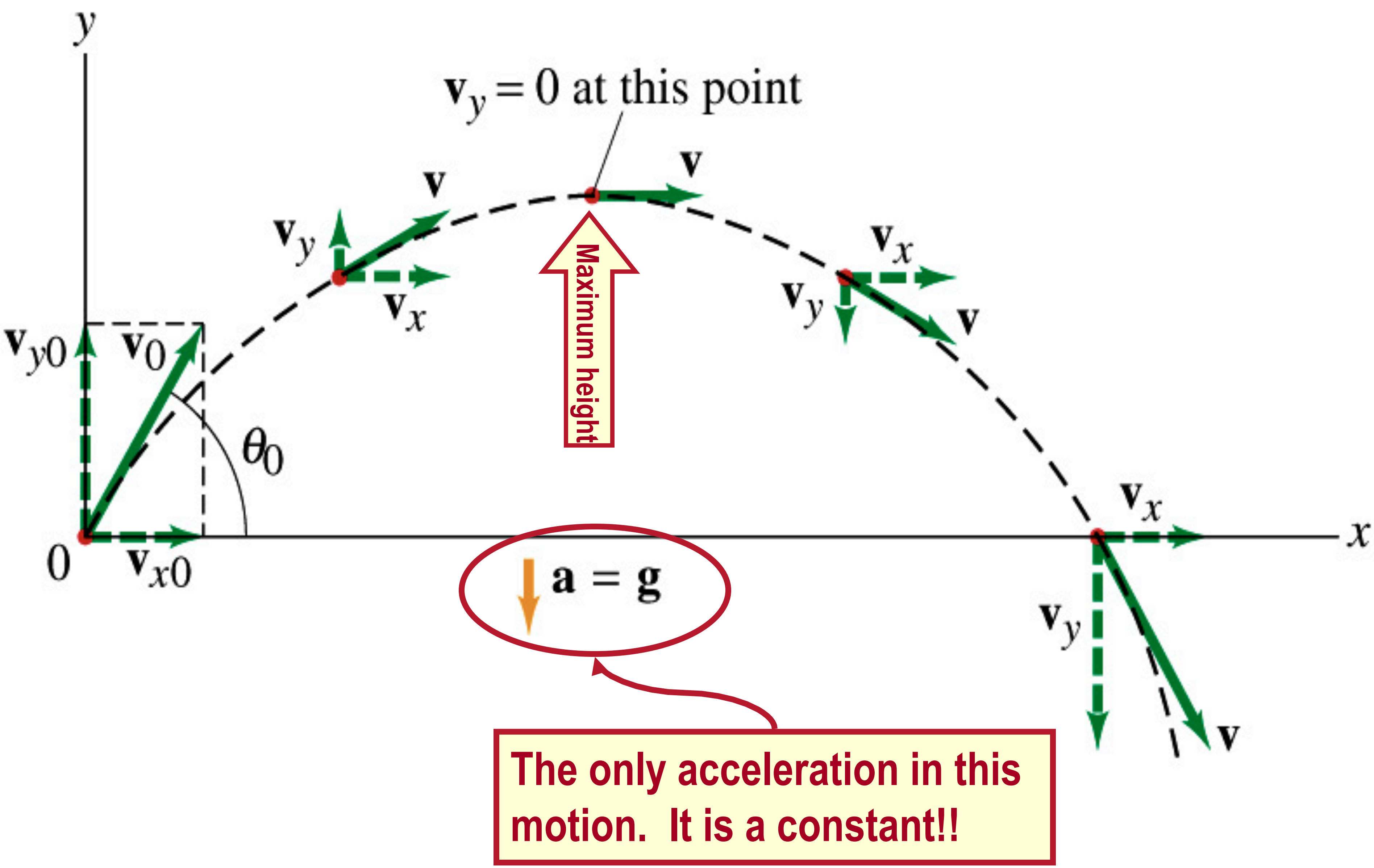
- Show that the trajectory of a projectile motion is a parabola!!
  - 20 points
  - Due: Monday, Oct. 5
  - You MUST show full details of your OWN computations to obtain any credit
    - Beyond what was covered in page 7 of this lecture note!!

# What is a Projectile Motion?

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, *g*, is constant over the range of the motion
    - $\vec{g} = -9.8\vec{j}\left(m/s^2\right)$
    - $a_x = 0\,m/s^2$ and $a_y = -9.8\,m/s^2$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity ( <u>no acceleration</u> ) $v_{xf} = v_{x0}$
  - Vertical motion under constant acceleration (*g*) $v_{yf} = v_{y0} + a_y t = v_{y0} + (-9.8)t$



# Projectile Motion



# Kinematic Equations for a projectile motion

**x-component**

$$a_x = 0$$

$$v_x = v_{xo}$$

$$\Delta x = v_{xo} t$$

$$v_{x0}^2 = v_{xo}^2$$

$$\Delta x = v_{xo} t$$

**y-component**

$$a_y = -\left|\vec{g}\right| = -9.8\, m/s^2$$

$$v_y = v_{yo} - gt$$

$$\Delta y = \tfrac{1}{2}\left(v_{yo} + v_y\right) t$$

$$v_y^2 = v_{yo}^2 - 2gy$$

$$\Delta y = v_{yo} t - \tfrac{1}{2} g t^2$$

## *Show that a projectile motion is a parabola!!!*

x-component $v_{xi} = v_i \cos\theta_i$

y-component $v_{yi} = v_i \sin\theta_i$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} = -g\vec{j}$$

In a projectile motion, the only acceleration is gravitational one whose direction is always toward the center of the earth (downward).

**$a_x$=0**

$$x_f = v_{xi}t = v_i \cos\theta_i t$$

$$t = \frac{x_f}{v_i \cos\theta_i}$$

$$y_f = v_{yi}t + \frac{1}{2}(-g)\,t^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

**Plug t into the above**

$$y_f = v_i \sin\theta_i \left(\frac{x_f}{v_i \cos\theta_i}\right) - \frac{1}{2}g\left(\frac{x_f}{v_i \cos\theta_i}\right)^2$$

$$y_f = x_f \tan\theta_i - \left(\frac{g}{2v_i^2 \cos^2\theta_i}\right)x_f^2$$

What kind of parabola is this?

# Example for Projectile Motion

A ball is thrown with an initial velocity **v**=(20**i**+40**j**)m/s. Estimate the time of flight and the distance the ball is from the original position when landed.

Which component determines the flight time and the distance?

Flight time is determined by the $y$ component, because the ball stops moving when it is on the ground after the flight.

$$y_f = 40t + \frac{1}{2}(-g)t^2 = 0m$$

$$t(80 - gt) = 0$$

So the possible solutions are…

$$\therefore t = 0 \text{ or } t = \frac{80}{g} \approx 8\sec$$

$$\therefore \ t \approx 8\sec$$

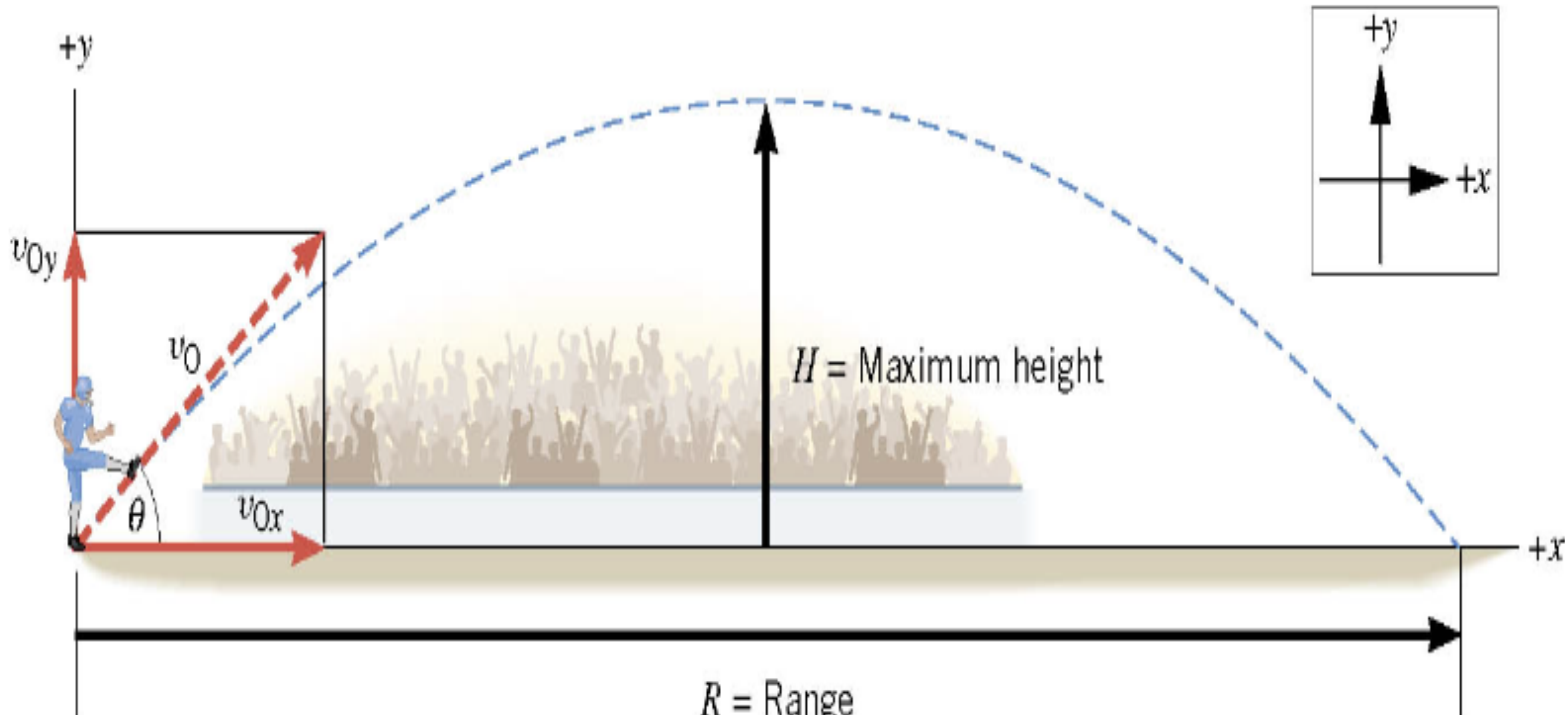
Why isn't 0 the solution?

Distance is determined by the $x$ component in 2-dim, because the ball is at $y=0$ position when it completed it's flight.

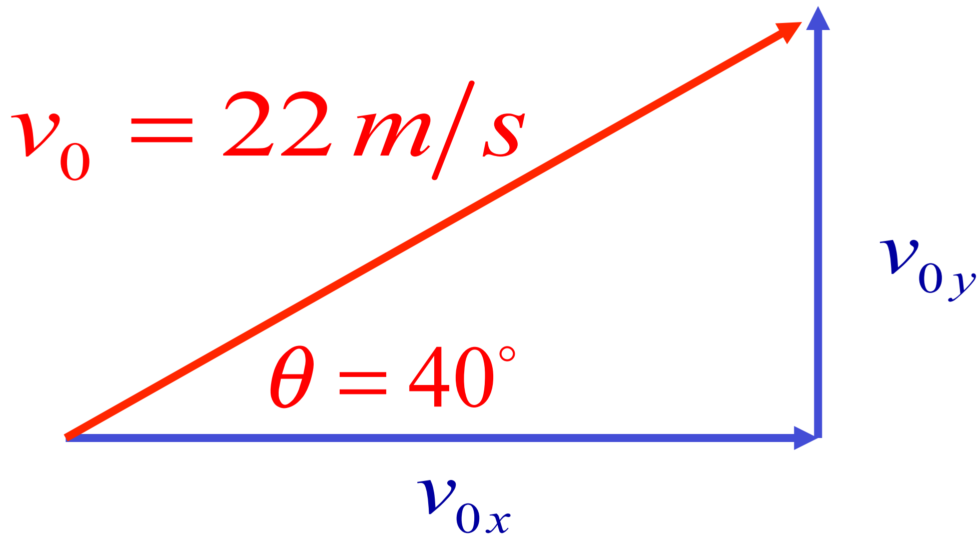
$$x_f = v_{xi}t = 20\times 8 = 160\,(m)$$

# *Ex.3.9* The Height of a Kickoff

A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.



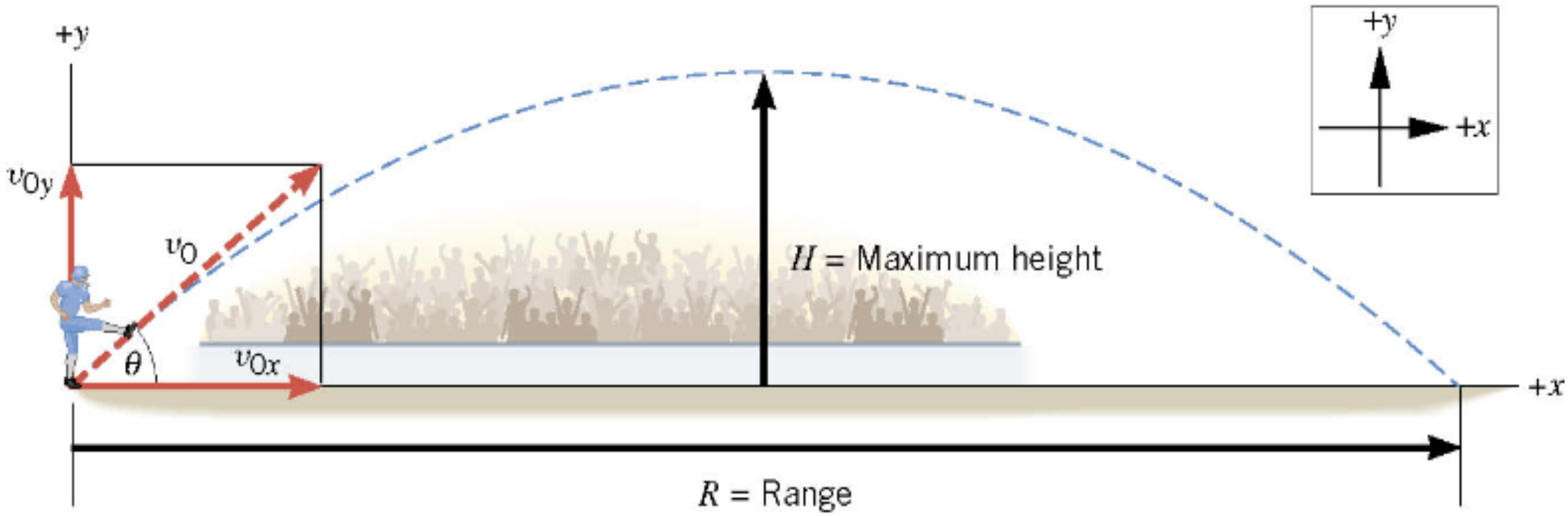
# First, the initial velocity components




$$v_{ox} = v_o \cos\theta = (22\,\mathrm{m/s})\cos 40^\circ = 17\,\mathrm{m/s}$$

$$v_{oy} = v_o \sin\theta = (22\,\mathrm{m/s})\sin 40^\circ = 14\,\mathrm{m/s}$$

# Motion in y-direction is of the interest..




| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| ? | -9.8 m/s$^2$ | 0 m/s | +14 m/s | |

# Now the nitty, gritty calculations…

| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| ? | -9.80 m/s$^2$ | 0 | 14 m/s | |

What happens at the maximum height?

The ball's velocity in y-direction becomes 0!!

And the ball's velocity in x-direction? Stays the same!! Why?

Because there is no acceleration in x-direction!!

Which kinematic formula would you like to use?

$$v_y^2 = v_{oy}^2 + 2a_y y$$

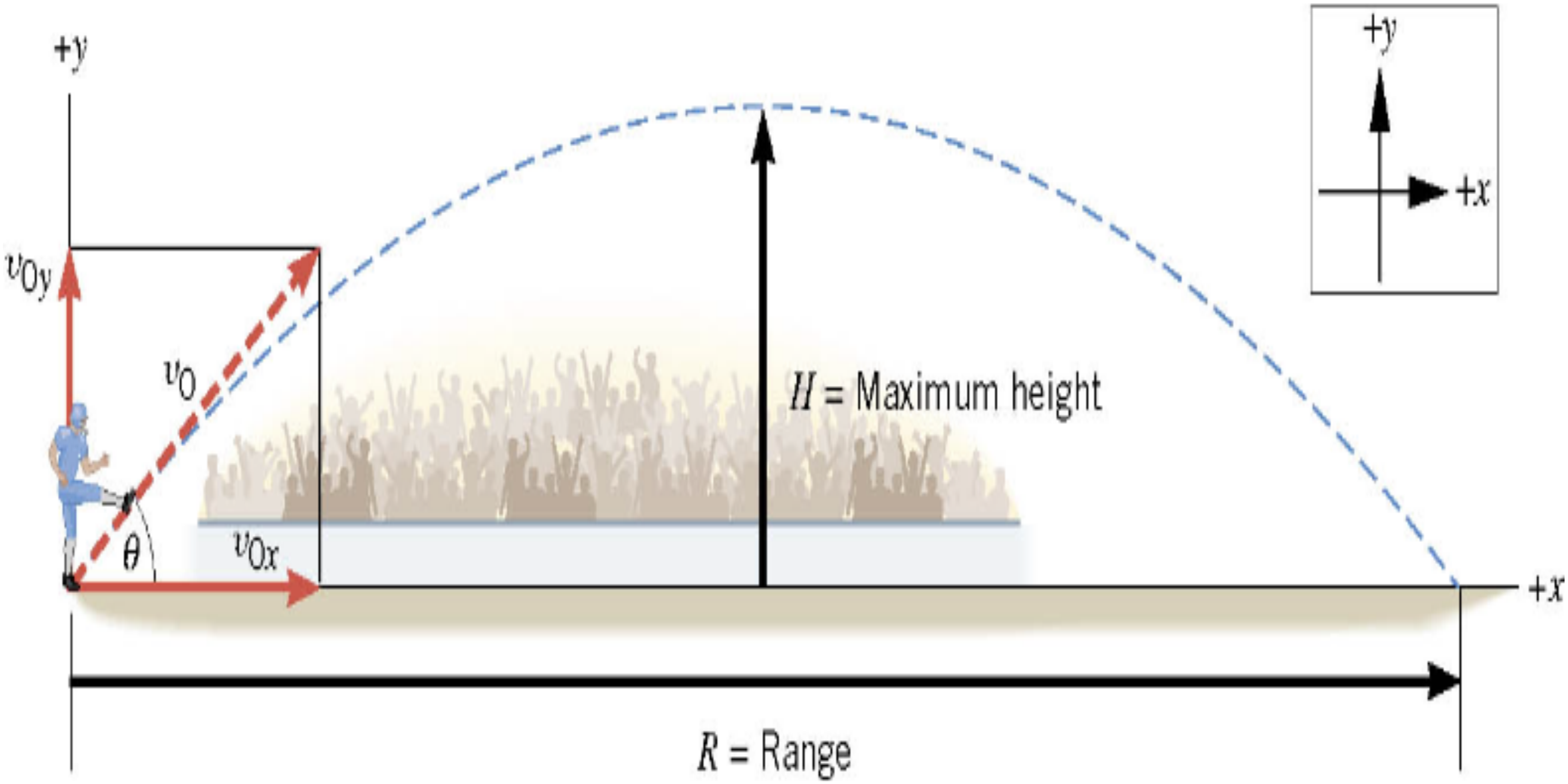
Solve for y

$$y = \frac{v_y^2 - v_{oy}^2}{2a_y}$$

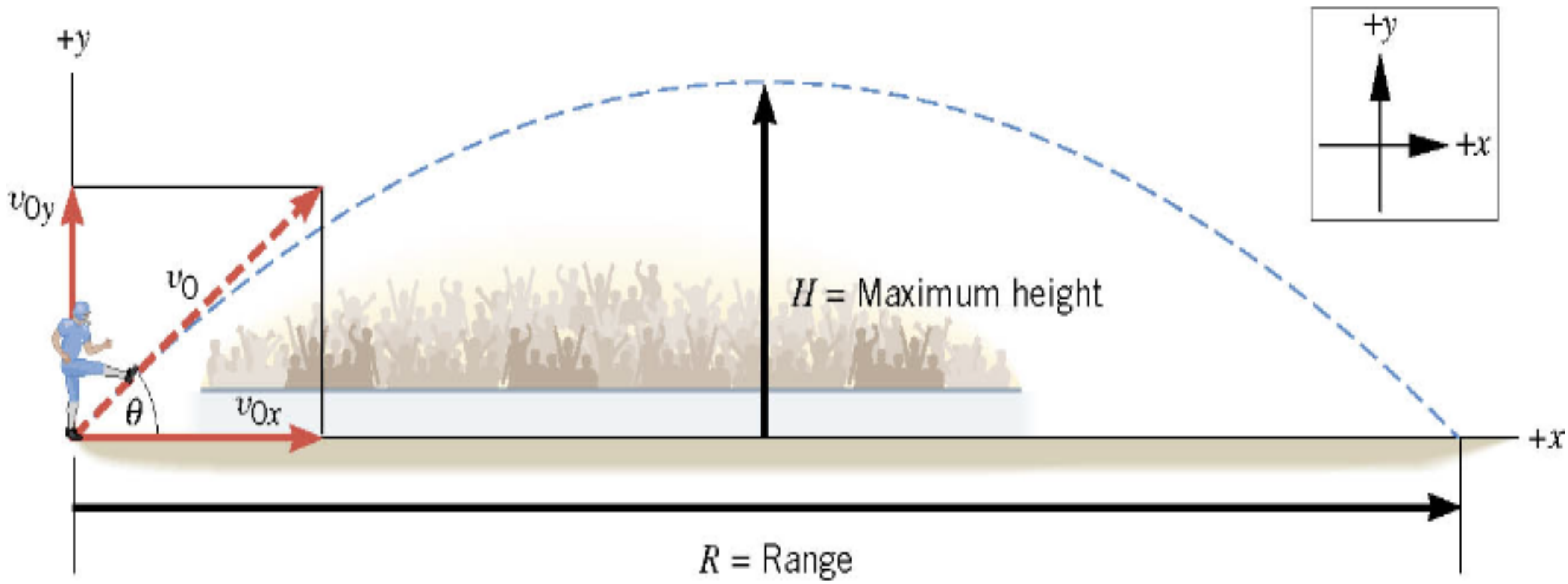
$$y = \frac{0 - (14\,\mathrm{m/s})^2}{2\left(-9.8\,\mathrm{m/s^2}\right)} = +10\ \mathrm{m}$$

# *Ex.3.9 extended:* The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



# What is y when it reached the max range?




| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| 0 m | -9.80 m/s$^2$ | | 14 m/s | ? |

# Now solve the kinematic equations in y direction!!

| $y$ | $a_y$ | $v_y$ | $v_{oy}$ | $t$ |
|---|---|---|---|---|
| 0 | -9.80 m/s$^2$ | | 14 m/s | ? |

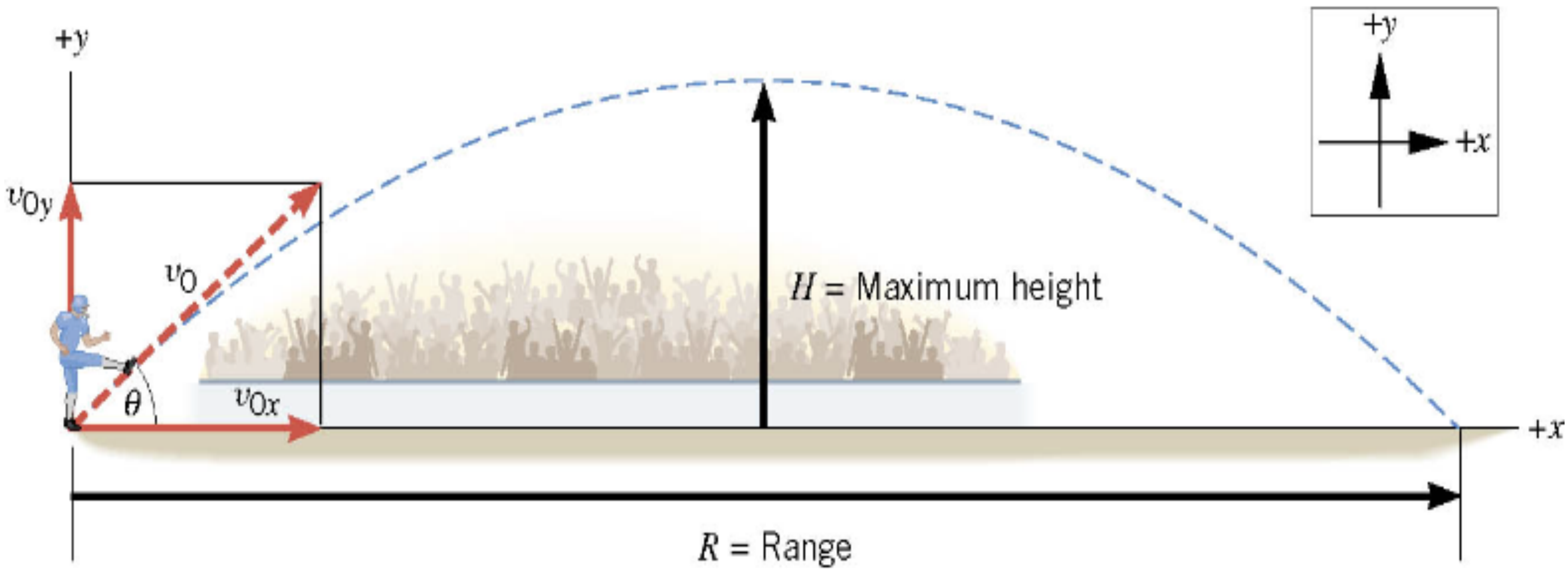
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2 \quad \xrightarrow{\text{Since } y=0} \quad 0 = v_{oy}t + \tfrac{1}{2}a_y t^2 = t\left(v_{oy} + \tfrac{1}{2}a_y t\right)$$

Two soultions $t = 0$ or

$$v_{oy} + \tfrac{1}{2}a_y t = 0 \quad \xrightarrow{\text{Solve for } t} \quad t = \frac{-v_{oy}}{\frac{1}{2}a_y} = \frac{-2v_{oy}}{a_y} = \frac{-2 \cdot 14}{-9.8} = 2.9s$$

# Ex.3.9 The Range of a Kickoff

## Calculate the range R of the projectile.




$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t = (17\,\text{m/s})(2.9\ \text{s}) = +49\ \text{m}$$

# Example for a Projectile Motion

- A stone was thrown upward from the top of a cliff at an angle of 37° to horizontal with initial speed of 65.0m/s. If the height of the cliff is 125.0m, how long is it before the stone hits the ground?

$$v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yi} = v_i \sin\theta_i = 65.0 \times \sin 37^\circ = 39.1 m/s$$

$$y_f = -125.0 = v_{yi}t - \frac{1}{2}gt^2$$

**Becomes**

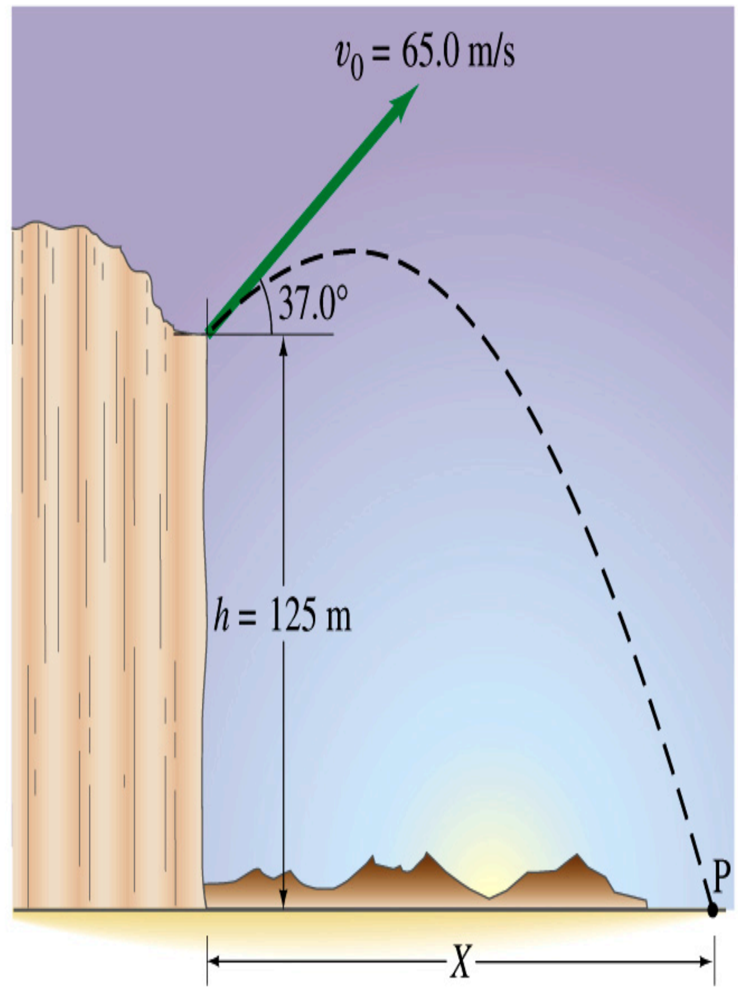
$$gt^2 - 78.2t - 250 = 9.80t^2 - 78.2t - 250 = 0$$

$$t = \frac{78.2 \pm \sqrt{(-78.2)^2 - 4 \times 9.80 \times (-250)}}{2 \times 9.80}$$

$$t = -2.43s \quad \text{or} \quad t = 10.4s$$

$$t = 10.4s$$

Since negative time does not exist.



# Example cont'd

- What is the speed of the stone just before it hits the ground?

$$v_{xf} = v_{xi} = v_i \cos\theta_i = 65.0 \times \cos 37^\circ = 51.9 m/s$$

$$v_{yf} = v_{yi} - gt = v_i \sin\theta_i - gt = 39.1 - 9.80 \times 10.4 = -62.8 m/s$$

$$|v| = \sqrt{v_{xf}^{\ 2} + v_{yf}^{\ 2}} = \sqrt{51.9^2 + (-62.8)^2} = 81.5 m/s$$

- What are the maximum height and the maximum range of the stone?

**Do these yourselves at home for fun!!!**

# Horizontal Range and Max Height

- Based on what we have learned in the previous pages, one can analyze a projectile motion in more detail
  - Maximum height an object can reach
  - Maximum range

What happens at the maximum height?

At the maximum height the object's vertical motion stops to turn around!!

$$v_{yf} = v_{0y} + a_y t = v_0 \sin\theta_0 - g t_A = 0$$

Solve for $t_A$

$$\therefore t_A = \frac{v_0 \sin\theta_0}{g}$$

**Time to reach to the maximum height!!**

# Horizontal Range and Max Height

Since no acceleration is in x direction, it still flies even if $v_y$=0.

$$R = v_{0x}t = v_{0x}\left(2t_A\right) = 2v_0 \cos\theta_0\left(\frac{v_0 \sin\theta_0}{g}\right)$$

**Range**

$$R = \left(\frac{v_0^2 \sin 2\theta_0}{g}\right)$$

$$y_f = h = v_{0y}t + \frac{1}{2}(-g)t^2 = v_0 \sin\theta_0\left(\frac{v_0 \sin\theta_0}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta_0}{g}\right)^2$$

**Height**

$$y_f = h = \left(\frac{v_0^2 \sin^2\theta_0}{2g}\right)$$

# Maximum Range and Height

- What are the conditions that give maximum height and range of a projectile motion?

$$h = \left( \frac{v_0^{\,2} \sin^2 \theta_0}{2g} \right)$$

This formula tells us that the maximum height can be achieved when $\theta_0$=90°!!!

$$R = \left( \frac{v_0^{\,2} \sin 2\theta_0}{g} \right)$$

This formula tells us that the maximum range can be achieved when $2\theta_0$=90°, i.e., $\theta_0$=45°!!!