PHYS 1441 – Section 002
Lecture #12

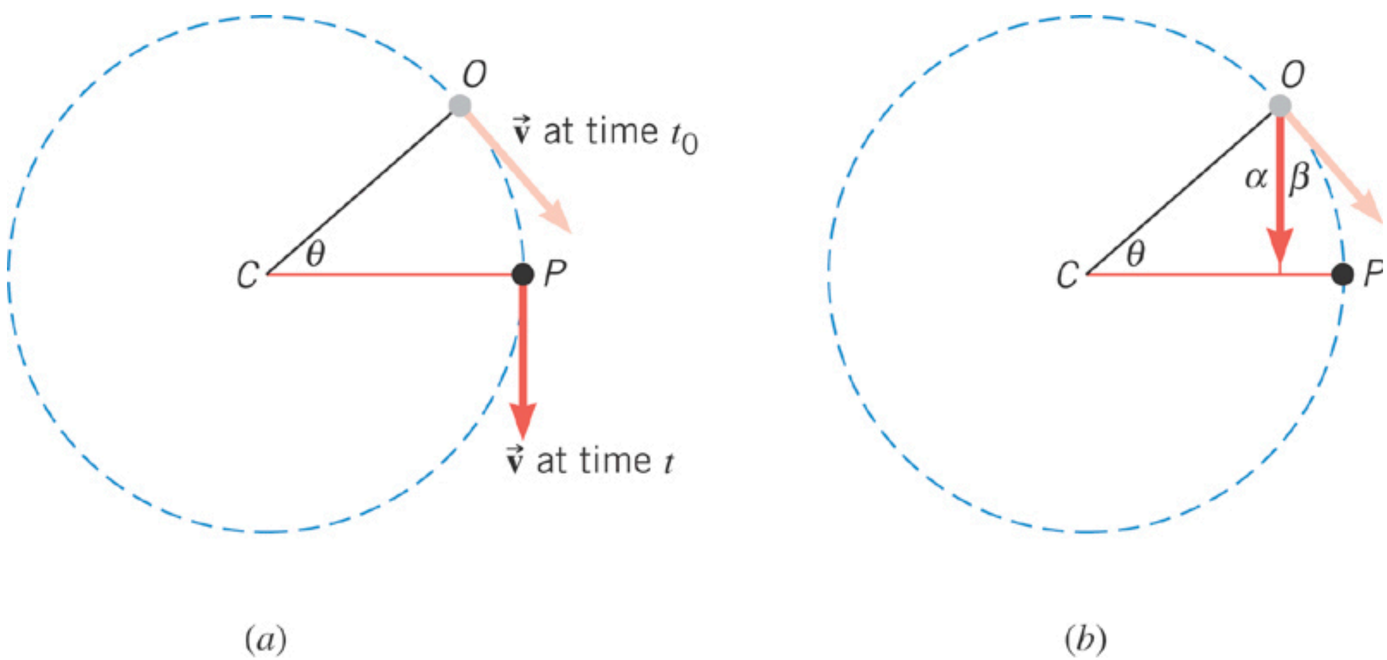
Monday, Oct. 12, 2009
Dr. Mark Sosebee
(Disguised as Dr. Yu)

- Centripetal Acceleration
- Newton's Law & Uniform Circular Motion
- Unbanked and Banked highways
- Newton's Law of Universal Gravitation
- Satellite Motion

Today's homework is homework #7, due 9pm, Tuesday, Oct. 20!!

Centripetal Acceleration

In uniform circular motion, the speed is constant, but the direction of the velocity vector is *not constant.*



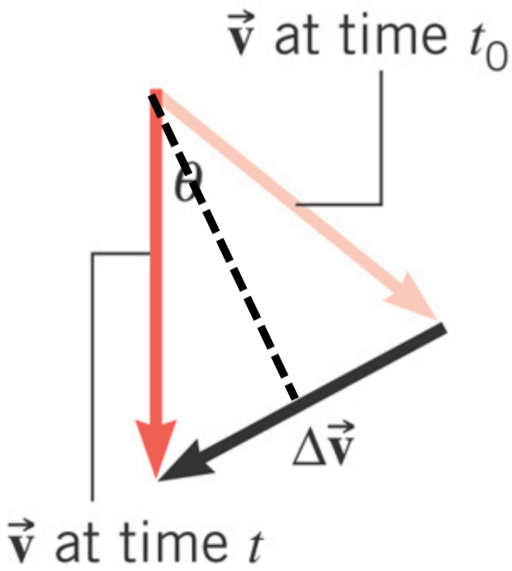
$$\alpha + \beta = 90^\circ$$

$$\alpha + \theta = 90^\circ$$

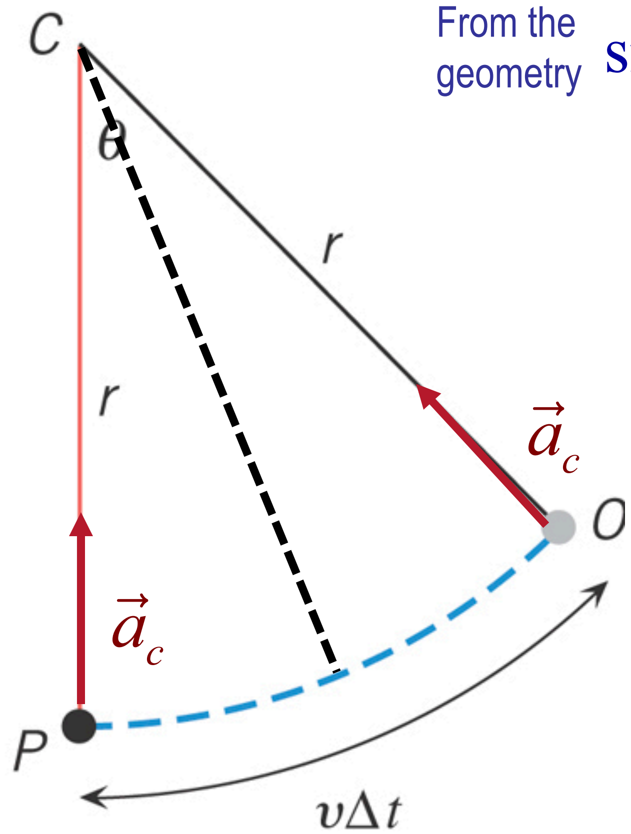
$$\beta - \theta = 0$$

$$\beta = \theta$$

Centripetal Acceleration



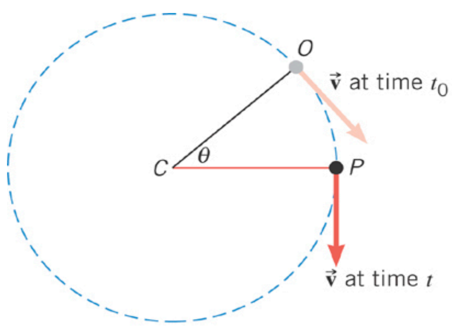
(a)



From the geometry $\sin\theta/2 = \dfrac{\Delta v}{2v} = \dfrac{v\Delta t}{2r}$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$a_c = \frac{v^2}{r}$$



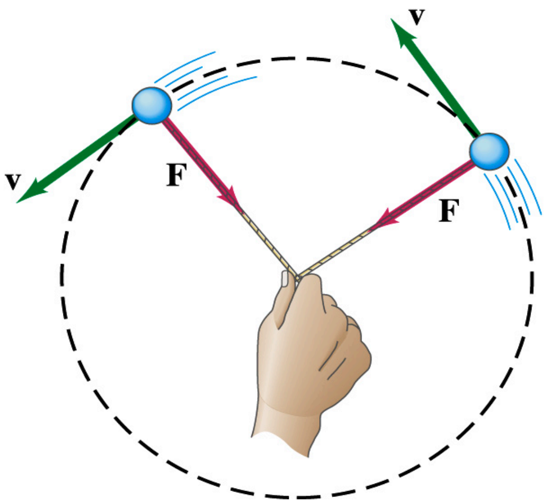
(a) (b)

What is the direction of a_c?

Always toward the center of circle!

Centripetal Acceleration

Newton's Second Law & Uniform Circular Motion



The ***centripetal**** *acceleration is always perpendicular to the velocity vector,* ***v****, and points to the center of the axis (radial direction) in a uniform circular motion.*

$$a_c = \frac{v^2}{r}$$

Are there forces in this motion? If so, what do they do?

The force that causes the centripetal acceleration acts toward the center of the circular path and causes the change in the direction of the velocity vector. This force is called the **centripetal force.**

$$\sum F_c = ma_c = m\frac{v^2}{r}$$

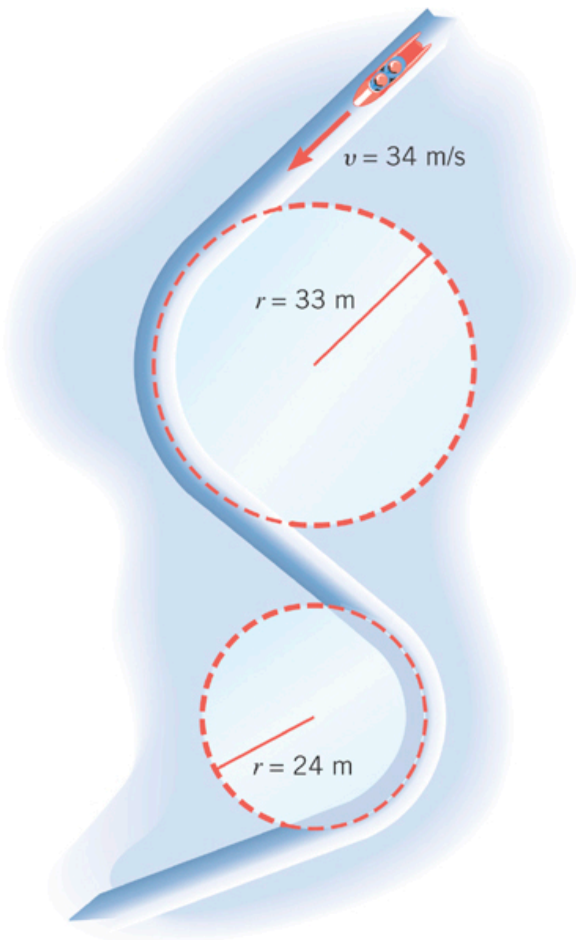
What do you think will happen to the ball if the string that holds the ball breaks?

The external force no longer exist. Therefore, based on Newton's 1st law, the ball will continue its motion without changing its velocity and will fly away following the tangential direction to the circle.

*Mirriam Webster: Proceeding or acting in a direction toward a center or axis

Ex. Effect of Radius on Centripetal Acceleration

The bobsled track at the 1994 Olympics in Lillehammer, Norway, contain turns with radii of 33m and 23m. Find the centripetal acceleration at each turn for a speed of 34m/s, a speed that was achieved in the two –man event. Express answers as multiples of g=9.8m/s^2.



Centripetal acceleration:

$$a_r = \frac{v^2}{r}$$

R=33m

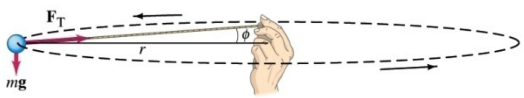
$$a_{r=33m} = \frac{(34)^2}{33} = 35\,m/s^2 = 3.6g$$

R=24m

$$a_{r=24m} = \frac{(34)^2}{24} = 48\,m/s^2 = 4.9g$$

Example of Uniform Circular Motion

A ball of mass 0.500kg is attached to the end of a 1.50m long cord. The ball is moving in a horizontal circle. If the string can withstand maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks?



Centripetal acceleration: $$a_r = \frac{v^2}{r}$$

When does the string break? $$\sum F_r = ma_r = m\frac{v^2}{r} > T$$

when the required centripetal force is greater than the sustainable tension.

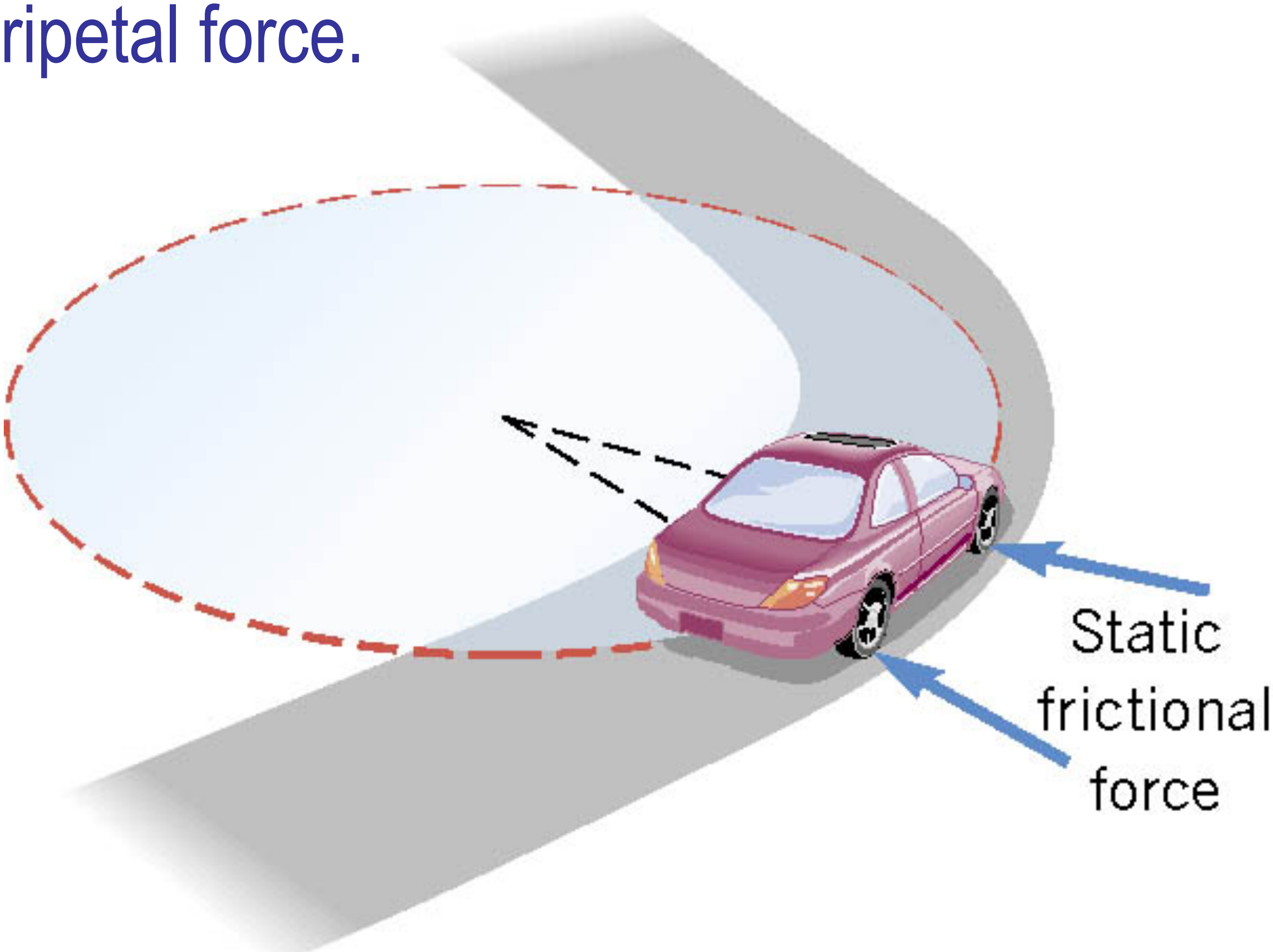
$$m\frac{v^2}{r} = T \qquad v = \sqrt{\frac{Tr}{m}} = \sqrt{\frac{50.0 \times 1.5}{0.500}} = 12.2\left(m/s\right)$$

Calculate the tension of the cord when speed of the ball is 5.00m/s.

$$T = m\frac{v^2}{r} = 0.500 \times \frac{(5.00)^2}{1.5} = 8.33\left(N\right)$$

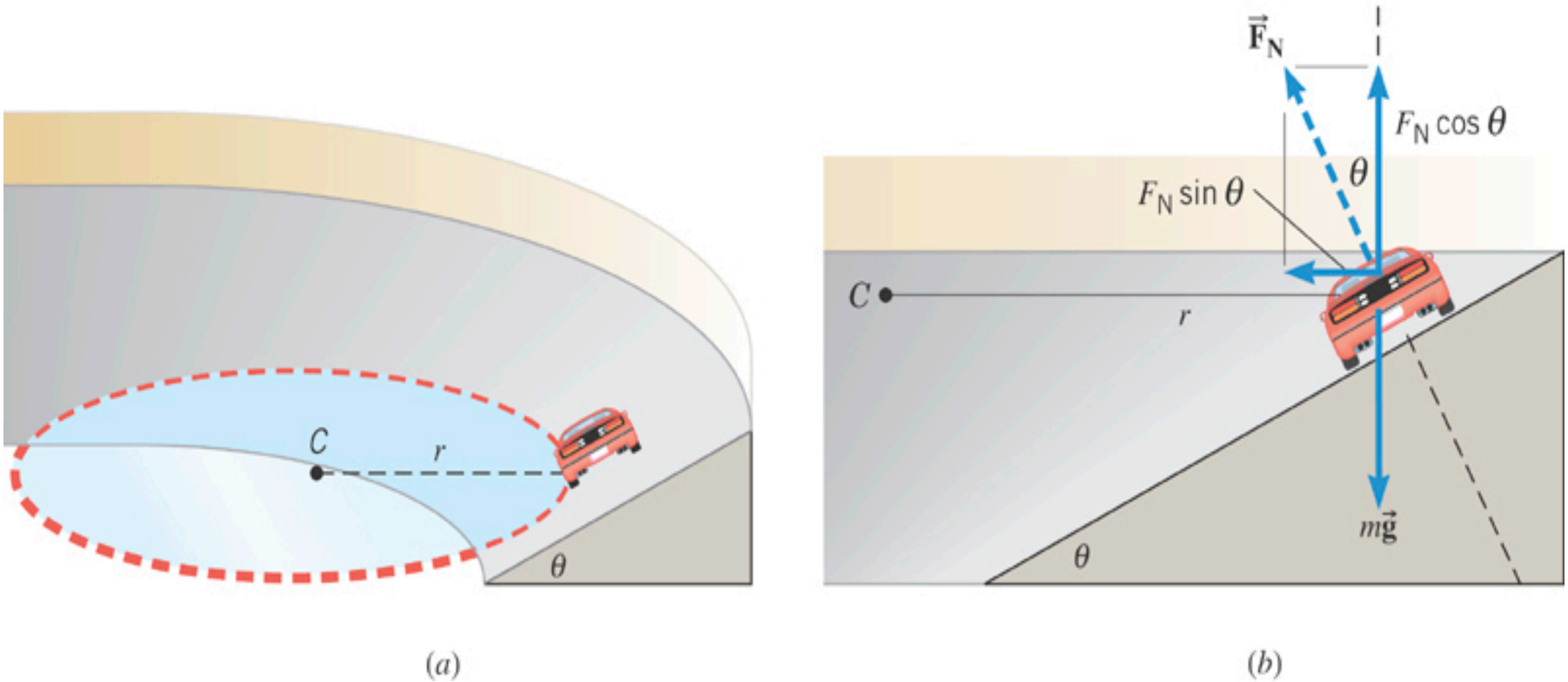
Unbanked Curve and Centripetal Force

On an unbanked curve, the static frictional force provides the centripetal force.



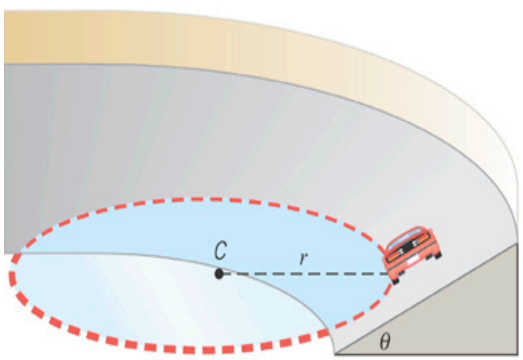
Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight.

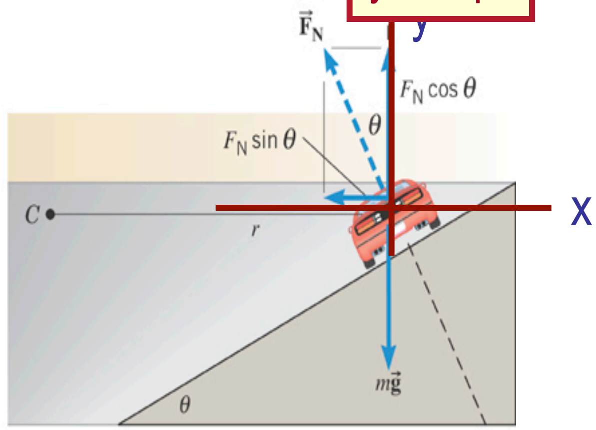


Ex. The Daytona 500

The Daytona 500 is the major event of the NASCAR season. It is held at the Daytona International Speedway in Daytona, Florida. The turns in this oval track have a maximum radius (at the top) of r=316m and are banked steeply, with θ=31°. Suppose these maximum radius turns were frictionless. At what speed would the cars have to travel around them?



(a)



(b)

x comp.

$$\sum F_x = F_N \sin\theta - m\frac{v^2}{r} = 0$$

y comp.

$$\sum F_y = F_N \cos\theta - mg = 0$$

$$\tan\theta = \frac{mv^2}{mgr} = \frac{v^2}{gr}$$

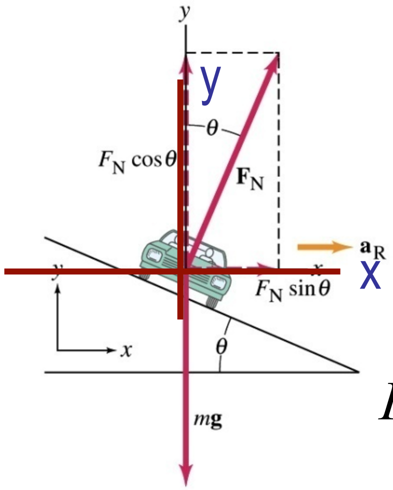
$$v^2 = gr\tan\theta$$

$$v = \sqrt{gr\tan\theta} =$$

$$\sqrt{9.8 \cdot 316 \tan\left(31^\circ\right)} = 43\,m/s = 96\,mi/hr$$

Ex. 5 – 7 Bank Angle

(a) For a car traveling with speed v around a curve of radius r, determine the formula for the angle at which the road should be banked so that no friction is required to keep the car from skidding.



x comp. $$\sum F_x = F_N \sin\theta - ma_r = F_N \sin\theta - \frac{mv^2}{r} = 0$$

$$F_N \sin\theta = \frac{mv^2}{r}$$

y comp. $$\sum F_y = F_N \cos\theta - mg = 0 \qquad F_N \cos\theta = mg$$

$$F_N = \frac{mg}{\cos\theta} \Rightarrow F_N \sin\theta = \frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{gr}$$

(b) What is this angle for an expressway off-ramp curve of radius 50m at a design speed of 50km/h?

$$v = 50km/hr = 14m/s \qquad \tan\theta = \frac{(14)^2}{50\times 9.8} = 0.4 \qquad \theta = \tan^{-1}(0.4) = 22°$$

Newton's Law of Universal Gravitation

People have been very curious about the stars in the sky, making observations for a long time. The data people collected, however, have not been explained until Newton has discovered the law of gravitation.

Every particle in the Universe attracts every other particle with a force that is directly **proportional to the product of their masses** and **inversely proportional to the square of the distance** between them.

How would you write this law mathematically?

$$F_g \propto \frac{m_1 m_2}{r_{12}^2}$$

With G

$$F_g = G\frac{m_1 m_2}{r_{12}^2}$$

G is the universal gravitational constant, and its value is

$$G = 6.673 \times 10^{-11}$$

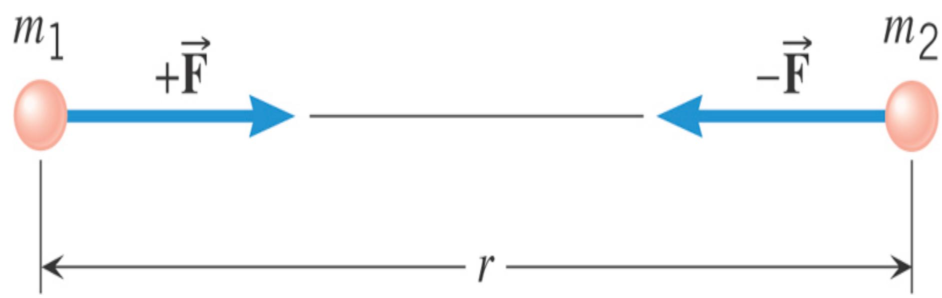
Unit? $N \cdot m^2 / kg^2$

This constant is not given by the theory but must be measured by experiments.

This form of forces is known as **the inverse-square law**, because the magnitude of the force is inversely proportional to the square of the distances between the objects.

Ex. Gravitational Attraction

What is the magnitude of the gravitational force that acts on each particle in the figure, assuming m_1=12kg, m_2=25kg, and r=1.2m?

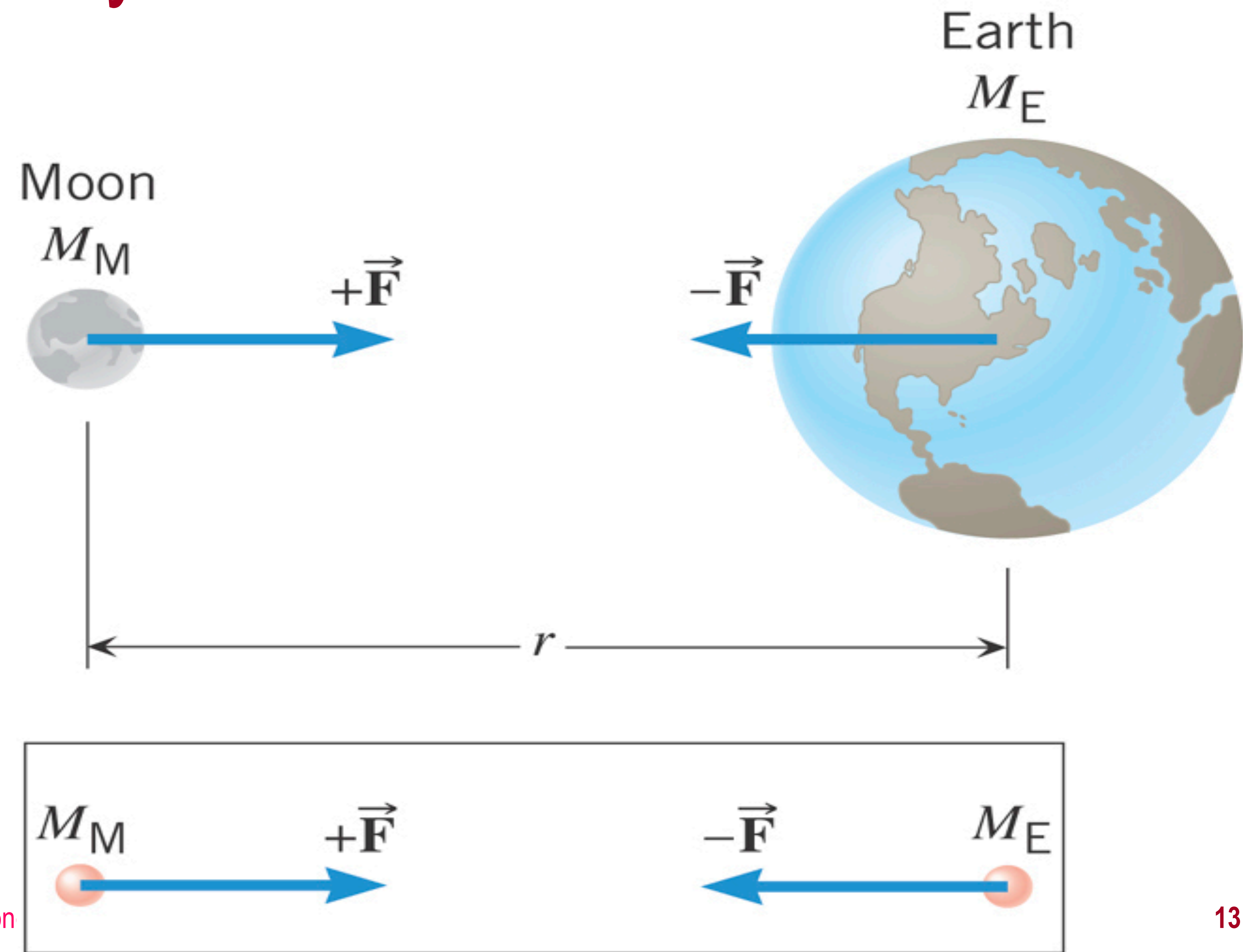


$$F = G\frac{m_1 m_2}{r^2}$$

$$= \left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\frac{(12\ \mathrm{kg})(25\ \mathrm{kg})}{(1.2\ \mathrm{m})^2}$$

$$= 1.4\times10^{-8}\ \mathrm{N}$$

Why does the Moon orbit the Earth?



Gravitational Force and Weight

Gravitational Force, $\mathcal{F}_g$

The attractive force exerted on an object by the Earth

$$\overrightarrow{F}_G = m\vec{a} = m\overline{g}$$

Weight of an object with mass M is

$$W = \left|\overrightarrow{F}_G\right| = M\left|\vec{g}\right| = Mg$$

What is the SI unit of weight?

N

*Since weight depends on the magnitude of gravitational acceleration, **g**, it varies depending on geographical location.*

By measuring the forces one can determine masses. This is why you can measure mass using the spring scale.

Gravitational Acceleration

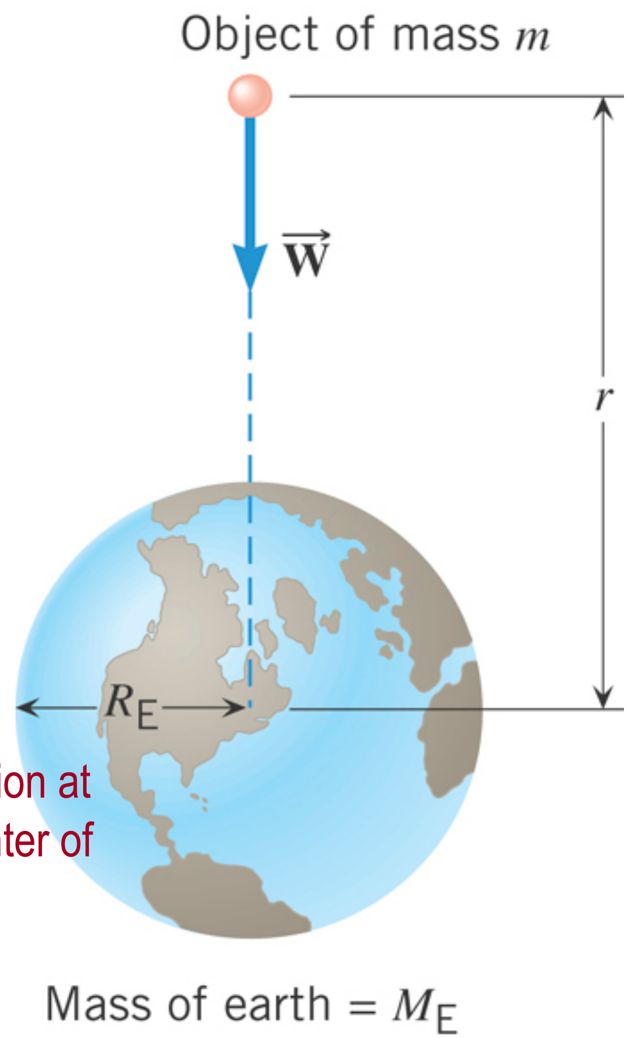
$$W = G\frac{M_E m}{r^2}$$

$$W = mg$$

$$\not{m}g = G\frac{M_E \not{m}}{r^2}$$

$$g = G\frac{M_E}{r^2}$$

Gravitational acceleration at distance r from the center of the earth!



What is the SI unit of g? m/s^2

Magnitude of the gravitational acceleration on the surface of the Earth

Gravitational force on the surface of the earth:

$$F_G = G\frac{M_E m}{r^2} = G\frac{M_E \not{m}}{R_E^2} = \not{m}g$$

$$g = G\frac{M_E}{R_E^2}$$

$G = 6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}$

$M_E = 5.98\times10^{24}\ \mathrm{kg};\quad R_E = 6.38\times10^{6}\ m$

$$= \left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\frac{\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\left(6.38\times10^{6}\ \mathrm{m}\right)^2}$$

$$= 9.80\ \mathrm{m/s^2}$$

Example for Universal Gravitation

Using the fact that g=9.80m/s^2 on the Earth's surface, find the average density of the Earth.

Since the gravitational acceleration is

$$F_g = G\frac{M_E m}{R_E^2} = mg$$

Solving for g

$$g = G\frac{M_E}{R_E^{\,2}} = 6.67\times10^{-11}\frac{M_E}{R_E^{\,2}}$$

Solving for M_E

$$M_E = \frac{R_E^{\,2} g}{G}$$

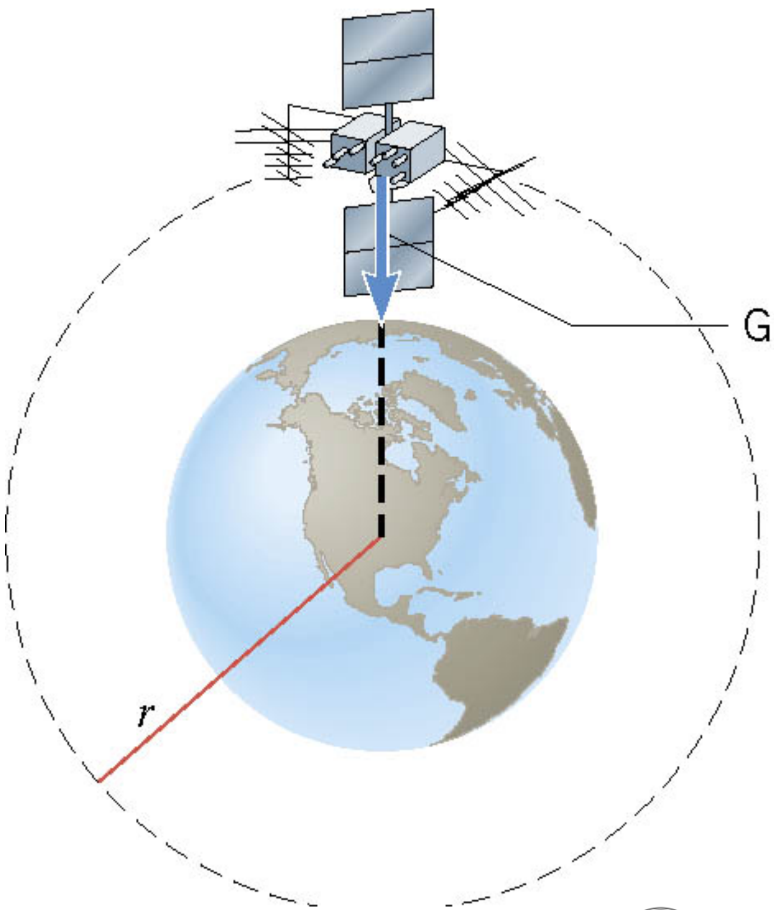
Therefore the density of the Earth is

$$\rho = \frac{M_E}{V_E} = \frac{\frac{R_E^{\,2} g}{G}}{\frac{4\pi}{3}R_E^{\,3}} = \frac{3g}{4\pi G R_E}$$

$$= \frac{3\times9.80}{4\pi\times6.67\times10^{-11}\times6.37\times10^{6}} = 5.50\times10^{3}\,kg/m^{3}$$

Satellite in Circular Orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.



What is the centripetal force?

The gravitational force of the earth pulling the satellite!

$$F_c = G\frac{mM_E}{r^2} = m\frac{v^2}{r}$$

$$v^2 = \frac{GM_E}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

Ex. Orbital Speed of the Hubble Space Telescope

Determine the speed of the Hubble Space Telescope orbiting at a height of 598 km above the earth's surface.

$$v = \sqrt{\frac{GM_E}{r}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{6.38\times10^{6}\,\mathrm{m} + 598\times10^{3}\,\mathrm{m}}}$$

$$= 7.56\times10^{3}\,\mathrm{m/s} \qquad (16900\,\mathrm{mi/h})$$

Period of a Satellite in an Orbit

Speed of a satellite $v=\sqrt{\frac{GM_E}{r}}=\frac{2\pi r}{T}$

$$\frac{GM_E}{r}=\left(\frac{2\pi r}{T}\right)^2$$

Square either side and solve for T2

$$T^2=\frac{(2\pi)^2 r^3}{GM_E}$$

Period of a satellite $T=\frac{2\pi r^{3/2}}{\sqrt{GM_E}}$

Kepler's 3rd Law

This is applicable to any satellite or even for planets and moons.