PHYS 1441 – Section 002
Lecture #17

Monday, Nov. 9, 2009

Dr. Jaehoon Yu

- Linear Momentum
- Linear Momentum and Impulse
- Linear Momentum and Forces
- Linear Momentum Conservation
- Collisions
- Center of Mass

Today's homework is homework #10, due 9pm, Tuesday, Nov. 17!!

Announcements

- Quiz #5
  - Beginning of the class, Monday, Nov. 16
  - Covers: Ch. 7.1 – what we finish this Wednesday
- This Wednesday's colloquium is a triple extra credit colloquium

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Recent Results from Cosmic Ray Experiments

Dr. Eun-Suk Seo
University of Maryland

4:00p.m Wednesday November 11, 2009
Room 101SH

Abstract:

Balloon-borne and space based instruments configured with particle detectors have been flown to study cosmic-ray origin, acceleration and propagation. They were also used to search for exotic sources, such as dark matter and antimatter, and to explore a possible limit to particle acceleration in supernova. I will review recent results from cosmic-ray experiments, including the unexpected excess in electrons (and positrons) reported by the Advanced Thin Ionization Calorimeter (ATIC) and Payload for Antimatter Matter Exploration and Light-nuclei Astrophysics (PAMELA), along with their implications. The observed excess of leptons is attributed either to a relatively nearby, unidentified astrophysical object that accelerates electrons to those energies or to annihilation of dark matter. I will also discuss some highlights of results from our on-going Cosmic Ray Energetics And Mass (CREAM) experiment that constrain the conventional cosmic ray propagation model.

Refreshments will be served in the Physics Lounge at 3:30 pm

Linear Momentum

The principle of energy conservation can be used to solve problems that are harder to solve just using Newton's laws. It is used to describe motion of an object or a system of objects.

A new concept of linear momentum can also be used to solve physical problems, especially the problems involving collisions of objects.

Linear momentum of an object whose mass is m and is moving at the velocity of ***v*** *is defined as*

$$\boxed{\vec{p} \equiv m\vec{v}}$$

What can you tell from this definition about momentum?

1. *Momentum is a vector quantity.*
2. *The heavier the object the higher the momentum*
3. *The higher the velocity the higher the momentum*
4. *Its unit is kg.m/s*

What else can use see from the definition? Do you see force?

The change of momentum in a given time interval

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v} - m\vec{v}_0}{\Delta t} = \frac{m\left(\vec{v} - \vec{v}_0\right)}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a} = \sum\vec{F}$$

Impulse and Linear Momentum

Net force causes change of momentum → Newton's second law

$$\vec{\bar{F}} = \frac{\Delta\vec{p}}{\Delta t} \Rightarrow \Delta\vec{p} = \vec{\bar{F}}\Delta t$$

The quantity impulse is defined as the change of momentum

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_0$$

So what do you think an impulse is?

*Effect of the force **F** acting on an object over the time interval $\Delta t = t_f - t_i$ is equal to the change of the momentum of the object caused by that force. Impulse is the degree of which an external force changes an object's momentum.*

The above statement is called the impulse-momentum theorem and is equivalent to Newton's second law.

What are the dimension and unit of Impulse? What is the direction of an impulse vector?

Defining a time-averaged force

$$\vec{\bar{F}} \equiv \frac{1}{\Delta t}\sum_i \vec{F}_i \Delta t$$

Impulse can be rewritten

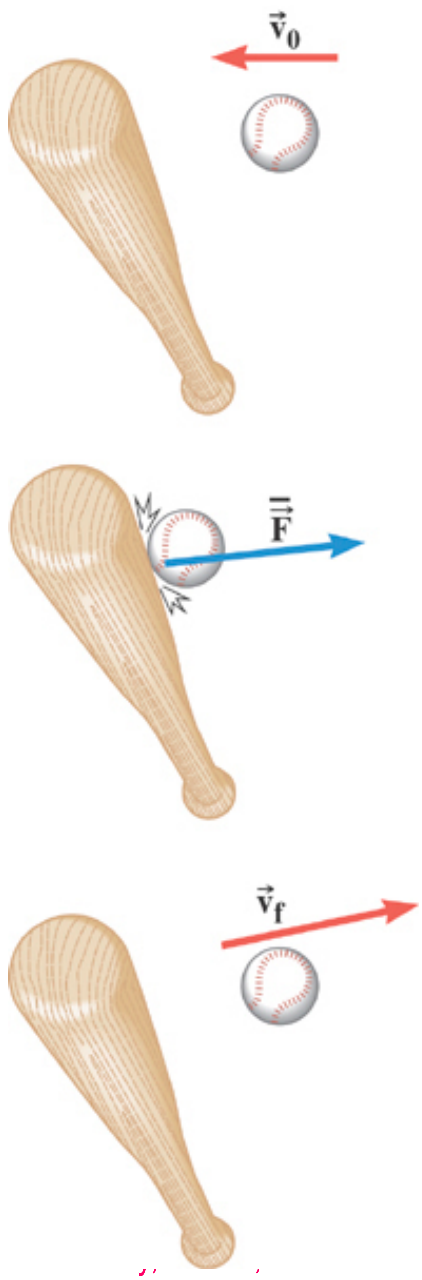
$$\vec{J} \equiv \vec{\bar{F}}\Delta t$$

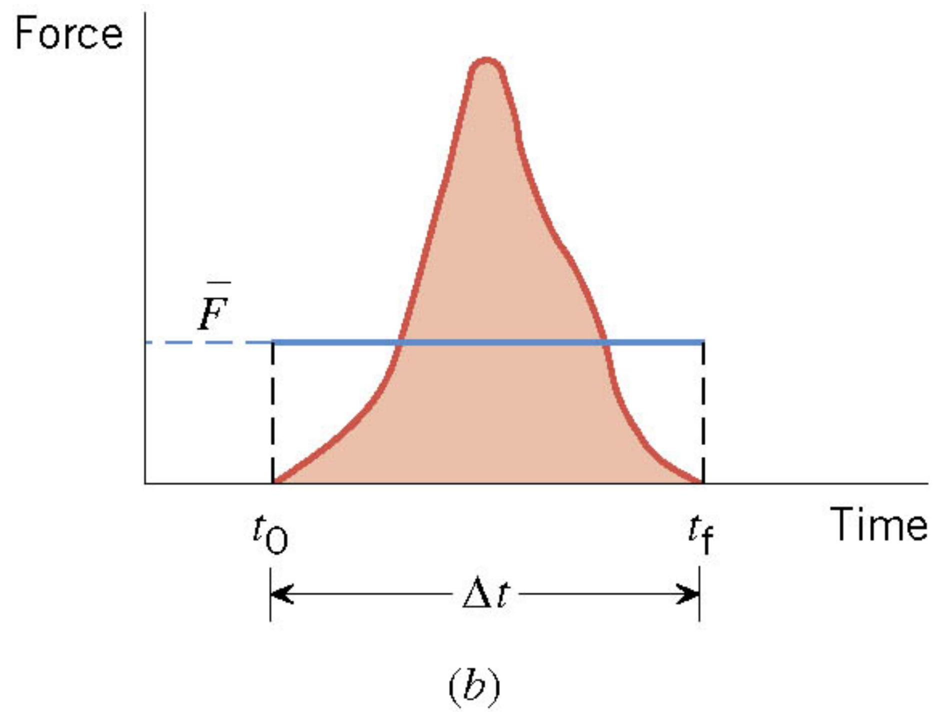
If force is constant

$$\vec{J} \equiv \vec{F}\Delta t$$

Impulse is a vector quantity!!

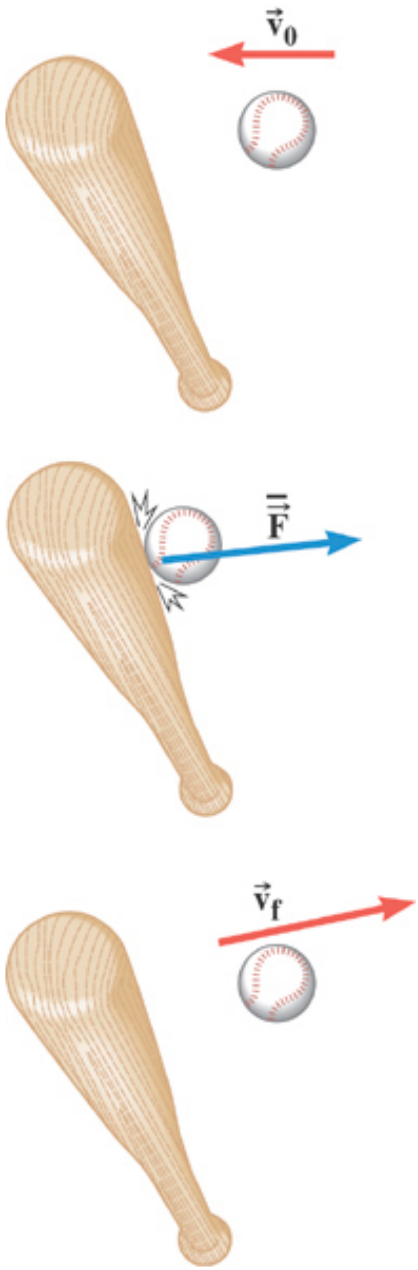
Impulse





There are many situations when the force on an object is not constant.

Ball Hit by a Bat



$$\bar{\vec{a}} = \frac{\vec{v}_f - \vec{v}_o}{\Delta t}$$

$$\sum \bar{\vec{F}} = m\bar{\vec{a}}$$

$$\sum \bar{\vec{F}} = \frac{m\vec{v}_f - m\vec{v}_o}{\Delta t}$$

Multiply either side by Δt

$$\left(\sum \bar{\vec{F}}\right)\Delta t = m\vec{v}_f - m\vec{v}_o = \vec{J}$$

Ex. A Well-Hit Ball

A baseball (m=0.14kg) has an initial velocity of $\mathbf{v_0}$=-38m/s as it approaches a bat. We have chosen the direction of approach as the negative direction. The bat applies an average force F that is much larger than the weight of the ball, and the ball departs from the bat with a final velocity of $\mathbf{v_f}$=+58m/s. (a) determine the impulse applied to the ball by the bat. (b) Assuming that the time of contact is $\Delta t=1.6\text{x}10^{-3}$s, find the average force exerted on the ball by the bat.

What are the forces involved in this motion? The force by the bat and the force by the gravity. Since the force by the bat is much greater than the weight, we ignore the ball's weight.

(a) Using the impulse-momentum theorem

$$\vec{J} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_0$$
$$= 0.14\times 58 - 0.14\times(-38) = +13.4 kg\cdot m/s$$

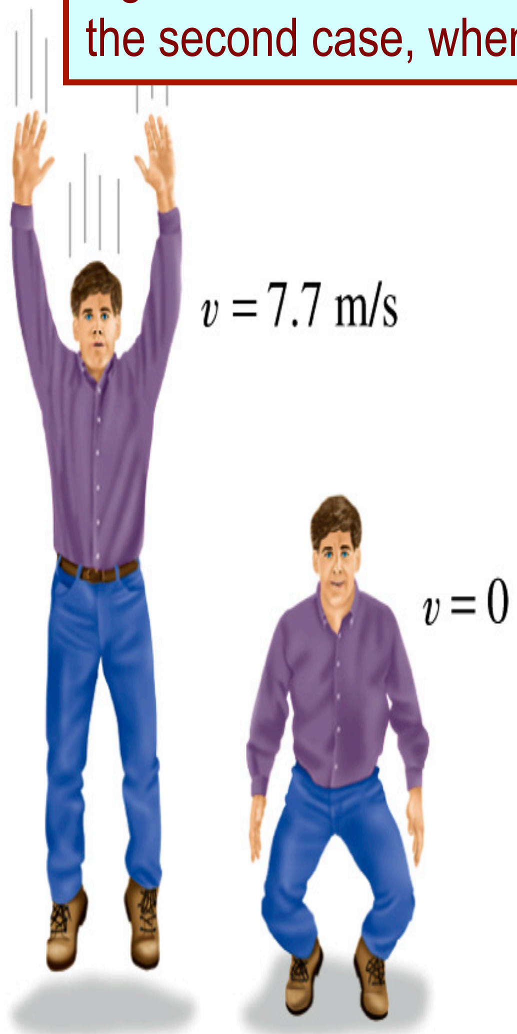
(b)Since the impulse is known and the time during which the contact occurs are know, we can compute the average force exerted on the ball during the contact

$$\vec{J} = \bar{\vec{F}}\Delta t \Rightarrow \bar{\vec{F}} = \frac{\vec{J}}{\Delta t} = \frac{+13.4}{1.6\times 10^{-3}} = +8400N$$

How large is this force? $\left|\vec{W}\right| = mg = 0.14\cdot 9.8 = 1.37N \Rightarrow \left|\bar{\vec{F}}\right| = \frac{8400}{1.37}\left|\vec{W}\right| = 6131\left|\vec{W}\right|$

Example 7.6 for Impulse

(a) Calculate the impulse experienced when a 70 kg person lands on firm ground after jumping from a height of 3.0 m. Then estimate the average force exerted on the person's feet by the ground, if the landing is (b) stiff-legged and (c) with bent legs. In the former case, assume the body moves 1.0cm during the impact, and in the second case, when the legs are bent, about 50 cm.



We don't know the force. How do we do this?

Obtain velocity of the person before striking the ground.

$$KE = -\Delta PE \qquad \frac{1}{2}mv^2 = -mg\left(y - y_i\right) = mgy_i$$

Solving the above for velocity v, we obtain

$$v = \sqrt{2gy_i} = \sqrt{2 \cdot 9.8 \cdot 3} = 7.7 m/s$$

Then as the person strikes the ground, the momentum becomes 0 quickly giving the impulse

$$\vec{J} = \vec{\overline{F}}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = 0 - m\vec{v} =$$

$$= -70kg \cdot 7.7 m/s\vec{j} = -540\vec{j}\, N \cdot s$$

Example 7.6 cont'd

In coming to rest, the body decelerates from 7.7m/s to 0m/s in a distance d=1.0cm=0.01m.

The average speed during this period is $$\bar{v} = \frac{0+v_i}{2} = \frac{7.7}{2} = 3.8m/s$$

The time period the collision lasts is $$\Delta t = \frac{d}{\bar{v}} = \frac{0.01m}{3.8m/s} = 2.6\times10^{-3}s$$

Since the magnitude of impulse is $$\left|\vec{J}\right| = \left|\vec{\bar{F}}\Delta t\right| = 540N\cdot s$$

The average force on the feet during this landing is $$\bar{F} = \frac{J}{\Delta t} = \frac{540}{2.6\times10^{-3}} = 2.1\times10^{5}N$$

How large is this average force? $$Weight = 70kg\cdot 9.8m/s^2 = 6.9\times10^2 N$$

$$\bar{F} = 2.1\times10^5 N = 304\times6.9\times10^2 N = 304\times Weight$$

If landed in stiff legged, the feet must sustain 300 times the body weight. The person will likely break his leg.

For bent legged landing: $$\Delta t = \frac{d}{\bar{v}} = \frac{0.50m}{3.8m/s} = 0.13s$$

$$\bar{F} = \frac{540}{0.13} = 4.1\times10^3 N = 5.9Weight$$

Linear Momentum and Forces

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

What can we learn from this force-momentum relationship?

- The rate of the change of particle's momentum is the same as the net force exerted on it.
- When the net force is 0, the particle's linear momentum is a constant as a function of time.
- If a particle is isolated, the particle experiences no net force. Therefore its momentum does not change and is conserved.

Something else we can do with this relationship. What do you think it is?

The relationship can be used to study the case where the mass changes as a function of time.

Can you think of a few cases like this?

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\Delta (m\vec{v})}{\Delta t} = \frac{\Delta m}{\Delta t}\vec{v} + m\frac{\Delta \vec{v}}{\Delta t}$$

Motion of a meteorite

Motion of a rocket

Conservation of Linear Momentum in a Two Particle System

Consider an isolated system with two particles that do not have any external forces exerting on it. What is the impact of Newton's 3rd Law?

If particle #1 exerts force on particle #2, there must be another force that the particle #2 exerts on #1 as the reaction force. Both the forces are internal forces, and the net force in the entire SYSTEM is ***still 0.***

Now how would the momenta of these particles look like?

Let say that the particle #1 has momentum $\mathbf{p_1}$ *and #2 has* $\mathbf{p_2}$ *at some point of time.*

Using momentum-force relationship

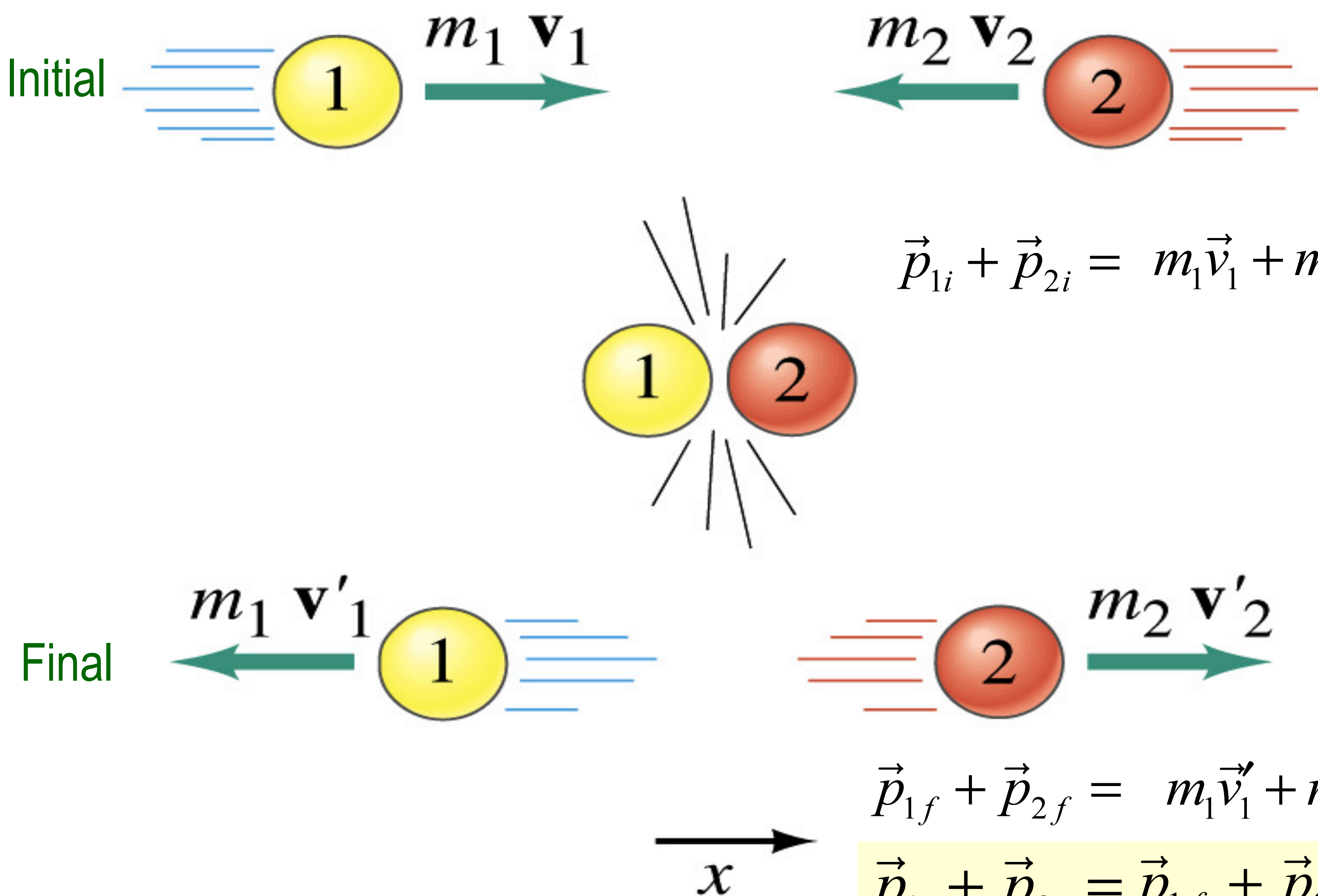
$$\vec{F}_{21} = \frac{\Delta\vec{p}_1}{\Delta t} \quad \text{and} \quad \vec{F}_{12} = \frac{\Delta\vec{p}_2}{\Delta t}$$

And since net force of this system is 0

$$\sum\vec{F} = \vec{F}_{12} + \vec{F}_{21} = \frac{\Delta\vec{p}_2}{\Delta t} + \frac{\Delta\vec{p}_1}{\Delta t} = \frac{\Delta}{\Delta t}\left(\vec{p}_2 + \vec{p}_1\right) = 0$$

Therefore $\vec{p}_2 + \vec{p}_1 = const$ *The total linear momentum of the system is conserved!!!*

Linear Momentum Conservation



$$\vec{p}_{1i} + \vec{p}_{2i} = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{p}_{1f} + \vec{p}_{2f} = m_1\vec{v}'_1 + m_2\vec{v}'_2$$

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$$

More on Conservation of Linear Momentum in a Two Body System

From the previous slide we've learned that the total momentum of the system is conserved if no external forces are exerted on the system.

$$\sum \vec{p} = \vec{p}_2 + \vec{p}_1 = const$$

What does this mean?

As in the case of energy conservation, this means that the total vector sum of all momenta in the system is the same before and after any interactions

Mathematically this statement can be written as

$$\vec{p}_{2i} + \vec{p}_{1i} = \vec{p}_{2f} + \vec{p}_{1f}$$

$$\sum_{system} P_{xi} = \sum_{system} P_{xf} \qquad \sum_{system} P_{yi} = \sum_{system} P_{yf} \qquad \sum_{system} P_{zi} = \sum_{system} P_{zf}$$

This can be generalized into conservation of linear momentum in many particle systems.

Whenever two or more particles in an ***isolated system*** *interact, the total momentum of the system remains constant.*