PHYS 1441 – Section 002
Lecture #6

Monday, Sept. 27, 2010
Dr. Jaehoon Yu

- Components of the 2D Vector
- Understanding the 2 Dimensional Motion
- 2D Kinematic Equations of Motion
- Projectile Motion
- Maximum Range and Height

Today's homework is homework #4, due 10pm, Tuesday, Oc. 5!!

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Adhesion protein dynamics in cells migrating in the 2D and 3D microenviroment

Dr. Michelle Digman, Ph.D.

The University of California, Irvine

4:00 pm Wednesday September 29, 2010 room 101 SH

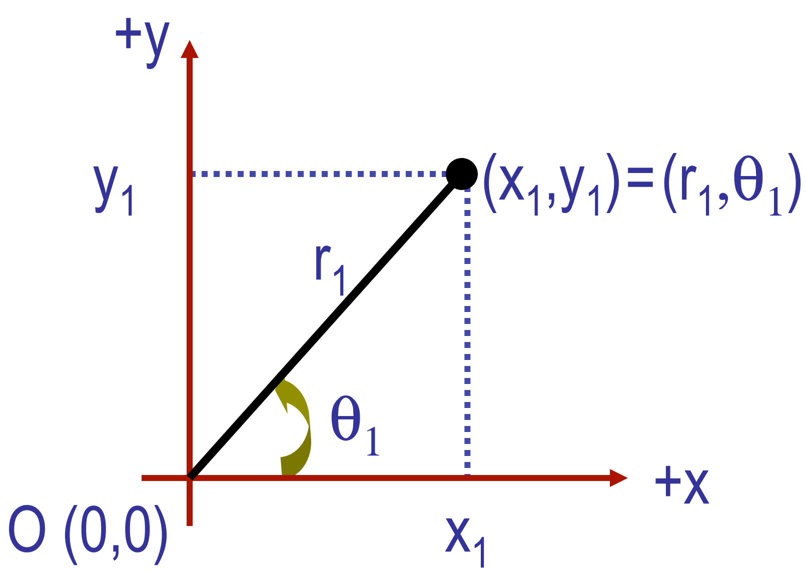
Abstract:

We have studied protein interaction which are central to adhesion formation, turnover and signaling during cell migration in 2D cell cultures. In 2 D, we used non-linear one and two photon fluorescence excitation microscopy in conjunction with correlation spectroscopy tools to detect proteins interacting during focal adhesion assembly and disassembly. Paxillin, FAK and vinculin binding partners were characterized using cross-raster image correlation spectroscopy (ccRICS) and provided maps of molecular diffusion and binding dynamics from fluorescence fluctuations in time and space. Detecting when and where these complexes form in the cell and quantifying their stoichiometry is an important goal of cell biology. Thus we developed the number and molecular brightness (N&B) method to determine protein aggregate sizes from the fluorescence amplitude fluctuations. Two-color cross-N&B detects the presence of molecular complexes and their stoichiometry. We have demonstrated that focal adhesions form in cell cultures in 2D by adding monomeric proteins at the growing edge and disassemble by the detachment of large protein clusters. Studying these molecular interactions directly in the tissue is technically challenging due to the spatial orientation and mobility of the cell in 3D. We are developing the modulation tracking method (MT) to image cell protrusions in 3D collagen matrices with nanometer and microsecond-millisecond resolution. The MT method uses a variant of circular tracking and high frequency modulation of the laser beam. Using the MT method we can also perform ccRICS and ccN&B with the orbital tracking technique and maintain focus on the cell protrusions while they are moving in 3D. These dynamics methods in conjunction with the MT provide unparalleled tools for image based tracking of compositionally heterogeneous complexes in viable cellular microenvironment and can be applied in live animal models.

Refreshments will be served at 3:30p.m in the Physics Library

2D Coordinate Systems

- They make it easy and consistent to express locations or positions
- Two commonly used systems, depending on convenience, are
  - Cartesian (Rectangular) Coordinate System
    - Coordinates are expressed in (x,y)
  - Polar Coordinate System
    - Coordinates are expressed in distance from the origin ® and the angle measured from the x-axis, θ (r,θ)
- Vectors become a lot easier to express and compute



How are Cartesian and Polar coordinates related?

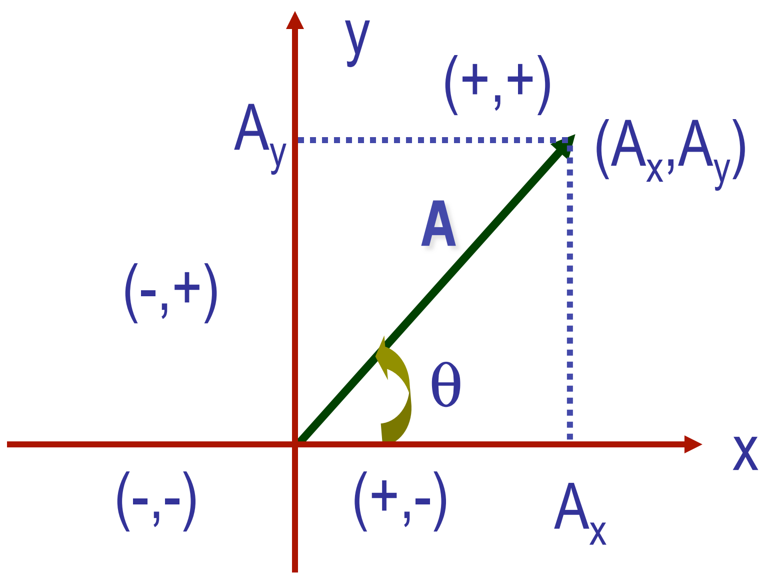
$$x_1 = r_1 \cos\theta_1 \qquad r_1 = \sqrt{\left(x_1^2 + y_1^2\right)}$$

$$y_1 = r_1 \sin\theta_1 \qquad \tan\theta_1 = \frac{y_1}{x_1}$$

$$\theta_1 = \tan^{-1}\left(\frac{y_1}{x_1}\right)$$

Components and Unit Vectors

Coordinate systems are useful in expressing vectors in their components



$$A_x = \left|\vec{A}\right|\cos\theta$$

$$A_y = \left|\vec{A}\right|\sin\theta$$

$$\left.\right\}\text{Components}$$

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2} \quad \}\ \text{Magnitude}$$

$$\left|\vec{A}\right| = \sqrt{\left(\left|\vec{A}\right|\cos\theta\right)^2 + \left(\left|\vec{A}\right|\sin\theta\right)^2}$$

$$= \sqrt{\left|\vec{A}\right|^2\left(\cos^2\theta + \sin^2\theta\right)} \quad = \left|\vec{A}\right|$$

Unit Vectors

- Unit vectors are the ones that tells us the directions of the components
- **Dimensionless**
- **Magnitudes these vectors are exactly 1**
- Unit vectors are usually expressed in **i, j, k** or

$$\vec{i},\ \vec{\mathrm{j}},\ \vec{\mathrm{k}}$$

So a vector **A** can be expressed as

$$\boxed{\vec{A} = A_x\vec{i} + A_y\vec{j} = \left|\vec{A}\right|\cos\theta\,\vec{i} + \left|\vec{A}\right|\sin\theta\,\vec{j}}$$

Examples of Vector Operations

Find the resultant vector which is the sum of **A**=(2.0**i**+2.0**j**) and **B** =(2.0**i**-4.0**j**)

$$\vec{C} = \vec{A} + \vec{B} = \left(2.0\vec{i} + 2.0\vec{j}\right) + \left(2.0\vec{i} - 4.0\vec{j}\right)$$

$$= \left(2.0 + 2.0\right)\vec{i} + \left(2.0 - 4.0\right)\vec{j} = 4.0\vec{i} - 2.0\vec{j}\,(m)$$

$$\left|\vec{C}\right| = \sqrt{\left(4.0\right)^2 + \left(-2.0\right)^2}$$

$$= \sqrt{16 + 4.0} = \sqrt{20} = 4.5(m)$$

$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{-2.0}{4.0} = -27^\circ$$

Find the resultant displacement of three consecutive displacements: **d**$_1$=(15**i**+30**j** +12**k**)cm, **d**$_2$=(23**i**+14**j** -5.0**k**)cm, and **d**$_3$=(-13**i**+15**j**)cm

$$\overrightarrow{D} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = \left(15\vec{i} + 30\vec{j} + 12\vec{k}\right) + \left(23\vec{i} + 14\vec{j} - 5.0\vec{k}\right) + \left(-13\vec{i} + 15\vec{j}\right)$$

$$= \left(15 + 23 - 13\right)\vec{i} + \left(30 + 14 + 15\right)\vec{j} + \left(12 - 5.0\right)\vec{k} = 25\vec{i} + 59\vec{j} + 7.0\vec{k}\,(cm)$$

Magnitude

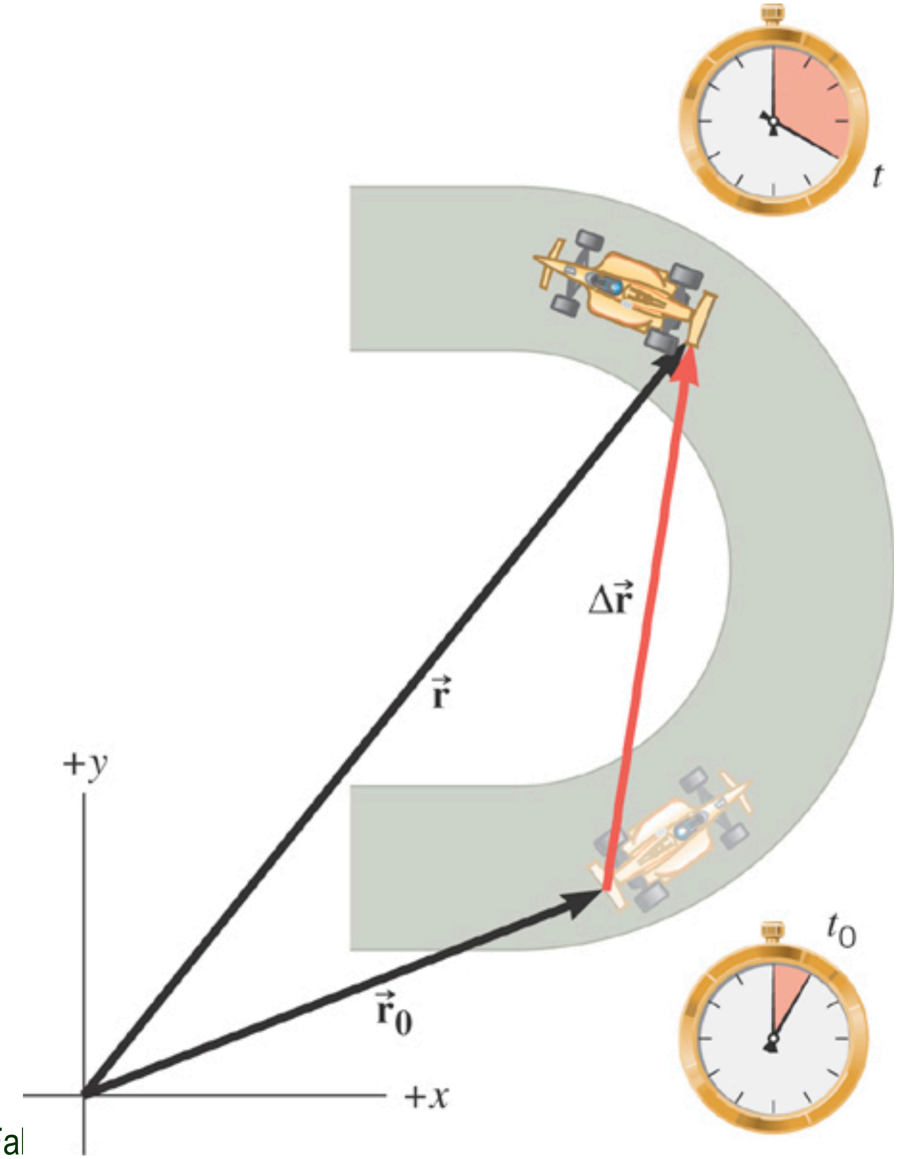
$$\left|\overrightarrow{D}\right| = \sqrt{\left(25\right)^2 + \left(59\right)^2 + \left(7.0\right)^2} = 65(cm)$$

2D Displacement

$\vec{\mathbf{r}}_o =$ initial position

$\vec{\mathbf{r}} =$ final position

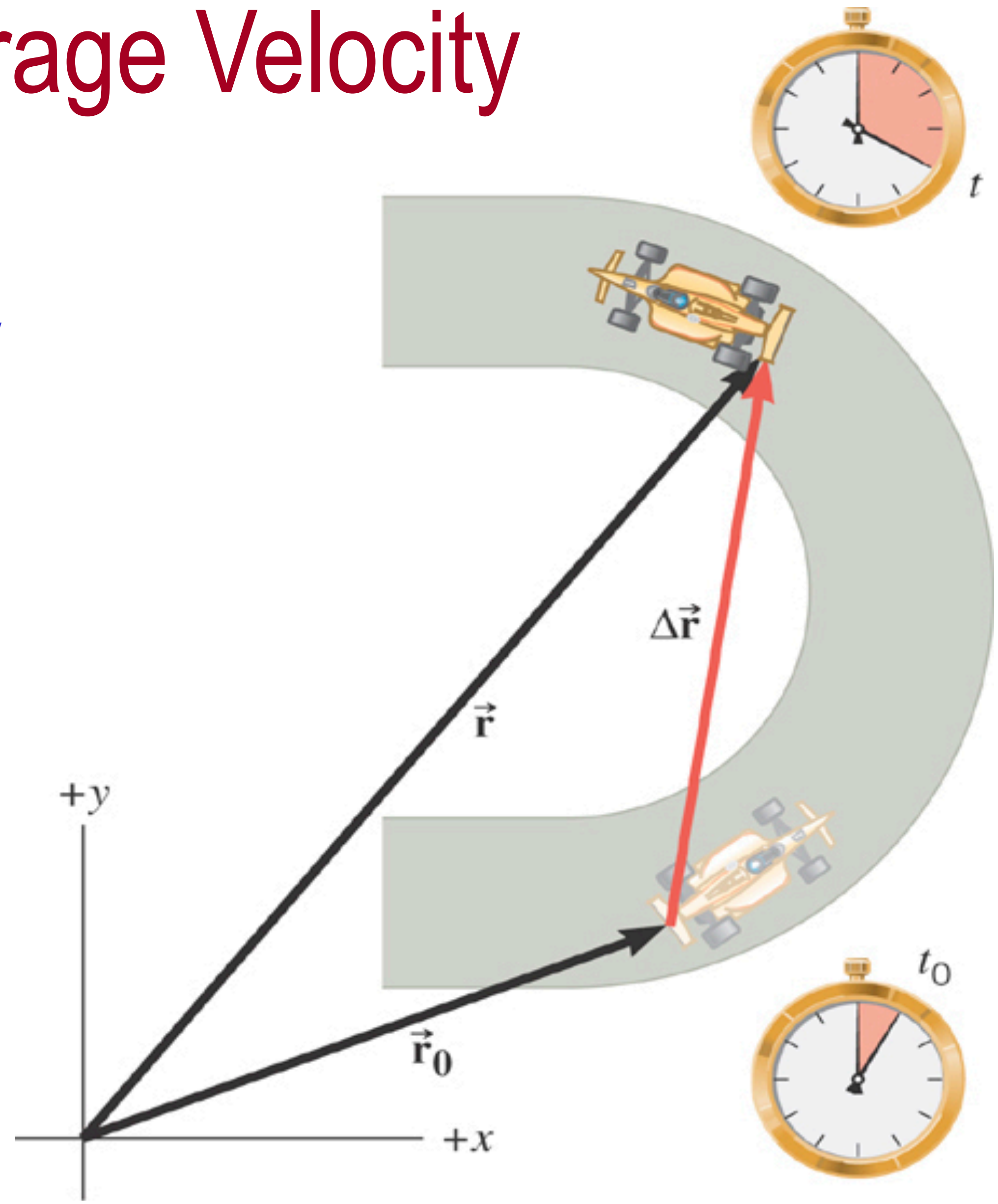
Displacement: $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o$



2D Average Velocity

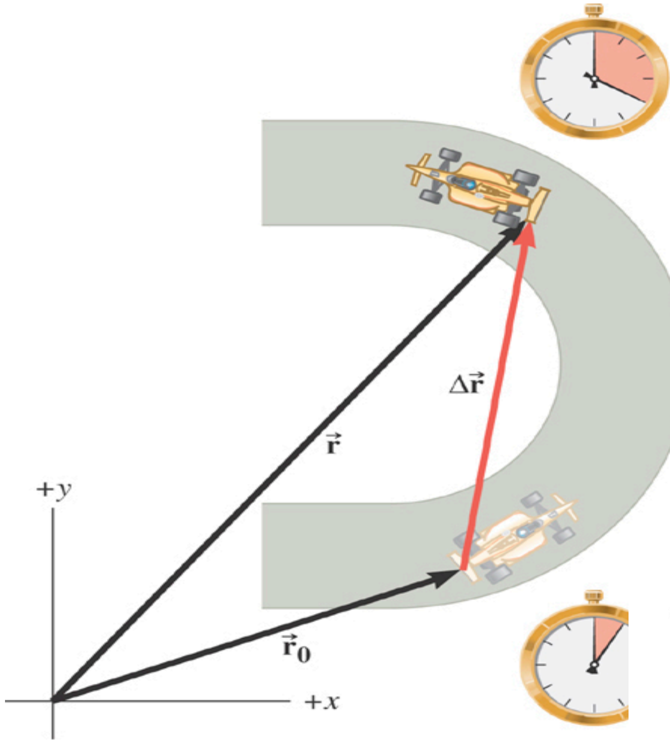
Average velocity is the displacement divided by the elapsed time.

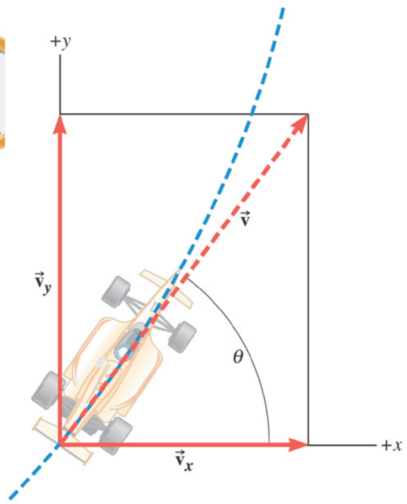
$$\overline{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$



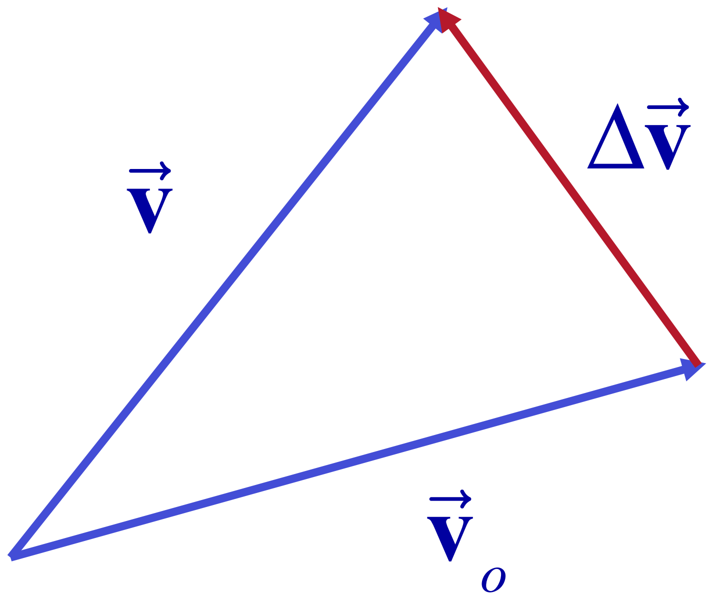
The ***instantaneous velocity*** indicates how fast the car moves and the direction of motion at each instant of time.

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$





2D Average Acceleration



$$\overline{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

Displacement, Velocity, and Acceleration in 2-dim

- Displacement: $$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$
- Average Velocity: $$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$
- Instantaneous Velocity: $$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$$
- Average Acceleration $$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$
- Instantaneous Acceleration: $$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$$

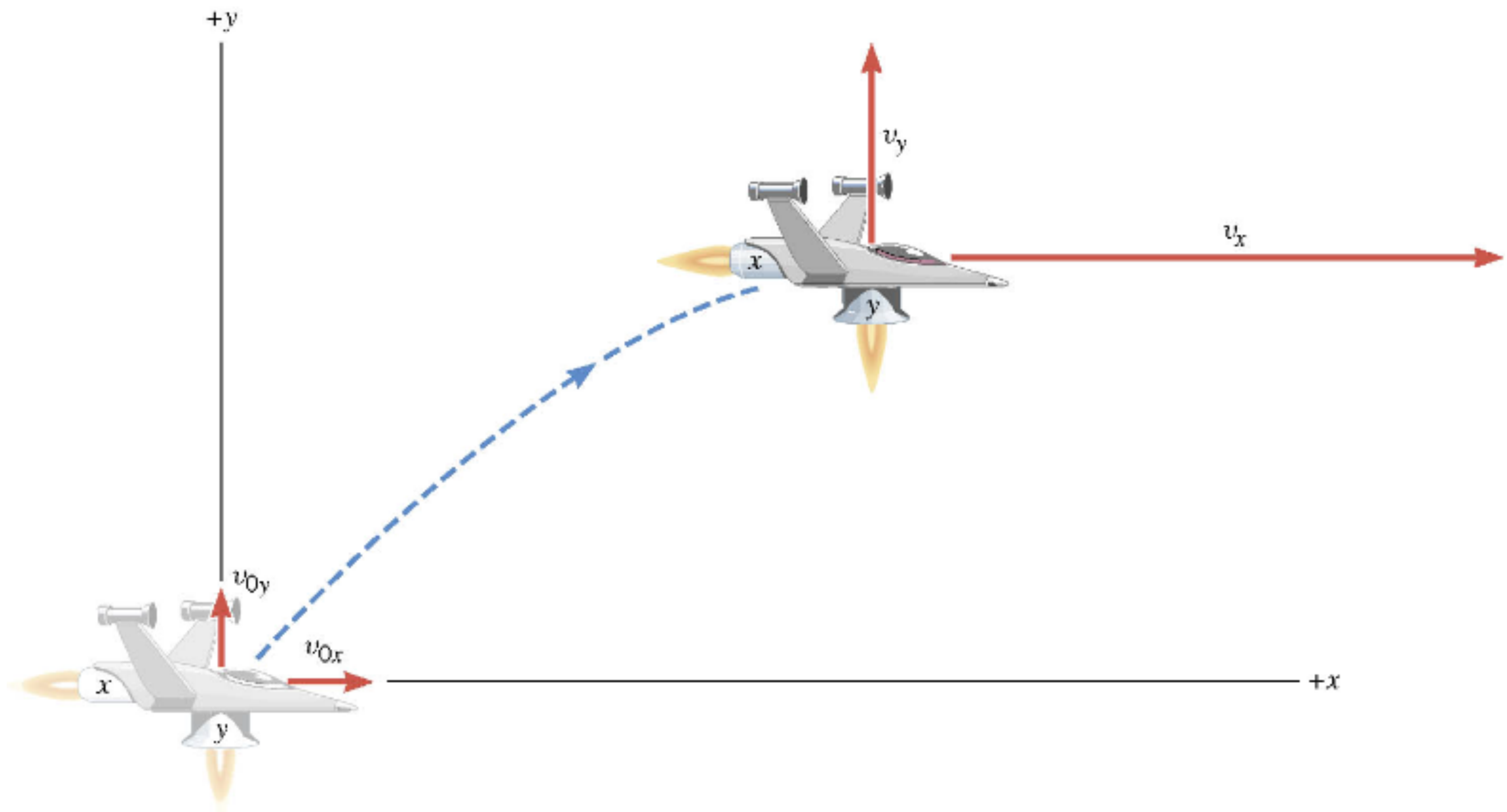
How is each of these quantities defined in 1-D?

Kinematic Quantities in 1D and 2D

Quantities	1 Dimension	2 Dimension
Displacement	$\Delta x = x_f - x_i$	$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
Average Velocity	$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$	$\vec{v} \equiv \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$
Inst. Velocity	$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$	$\vec{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t}$
Average Acc.	$a_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$	$\vec{a} \equiv \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$
Inst. Acc.	$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$	$\vec{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t}$

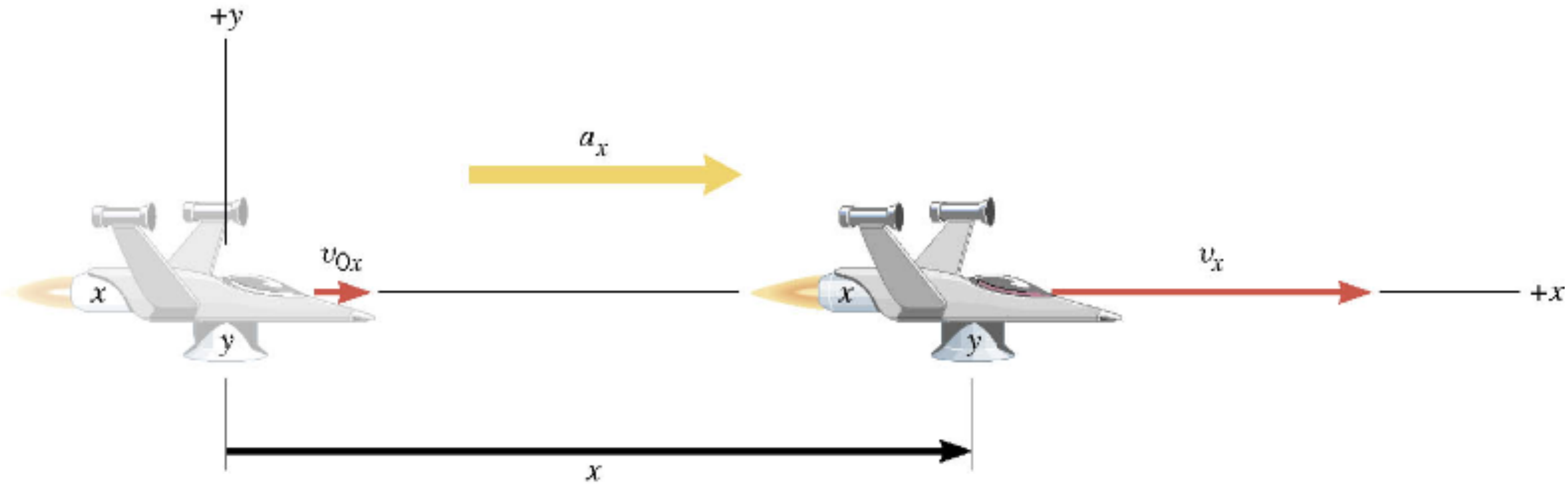
What is the difference between 1D and 2D quantities?

A Motion in 2 Dimension



This is a motion that could be viewed as two motions combined into one. (superposition…)

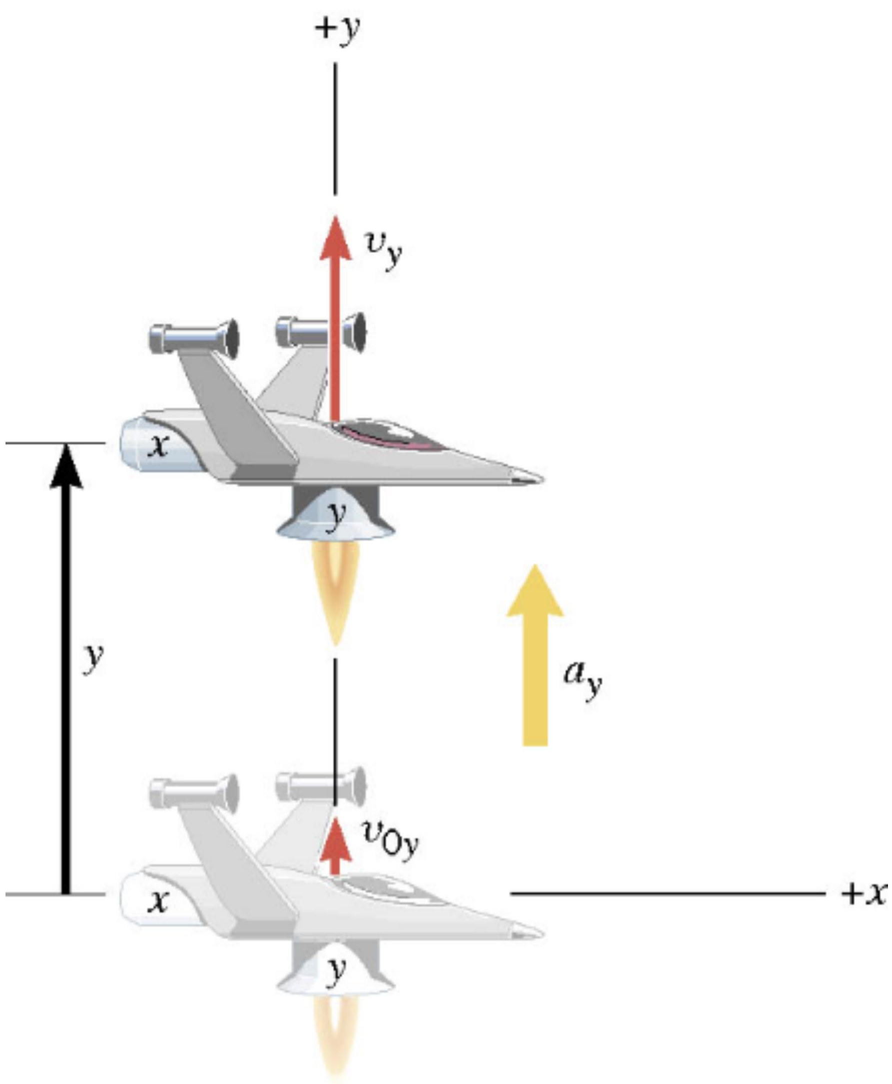
Motion in horizontal direction (x)



$$v_x = v_{xo} + a_x t \qquad x = \tfrac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x \qquad x = v_{xo}t + \tfrac{1}{2}a_x t^2$$

Motion in vertical direction (y)



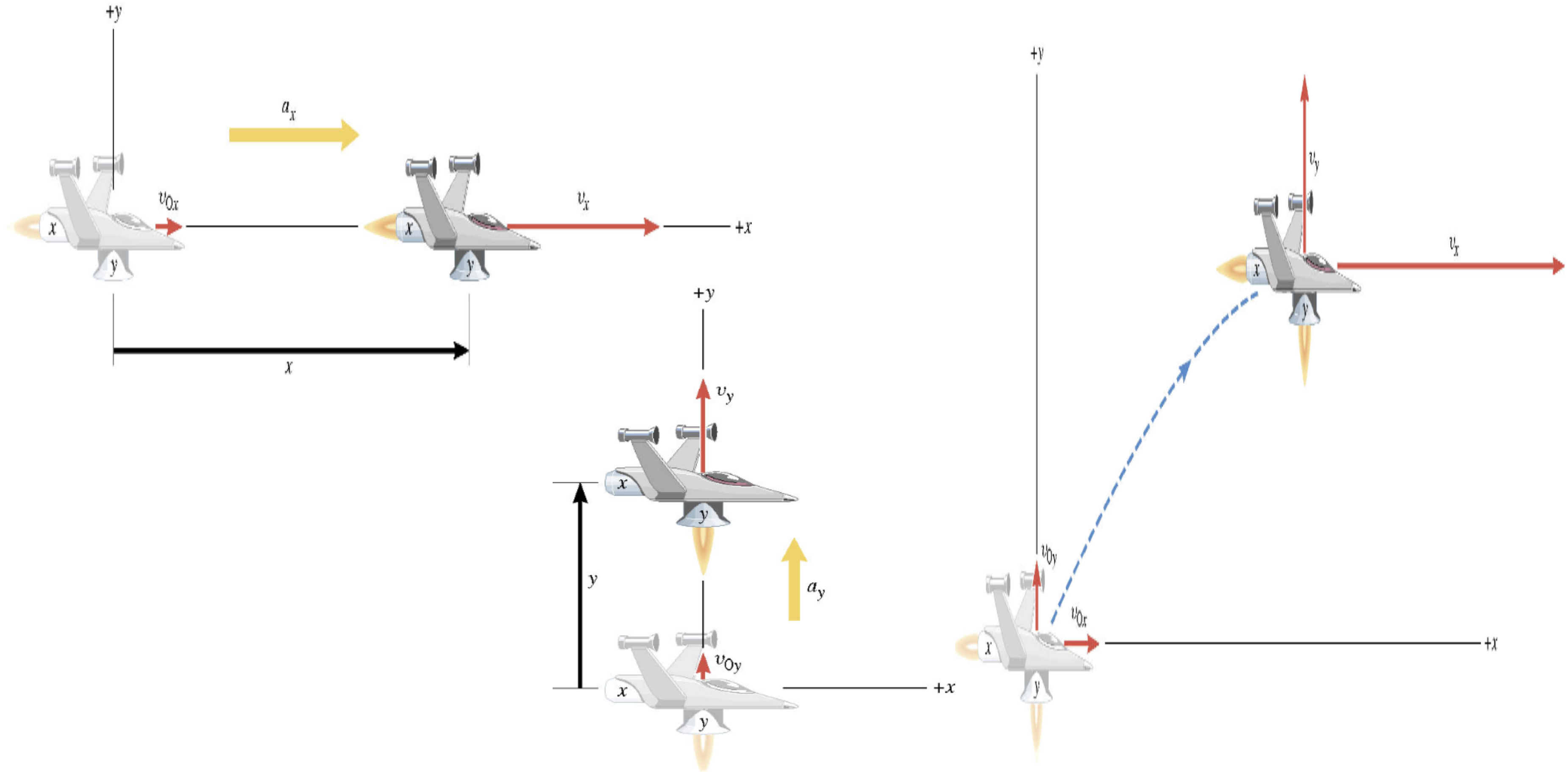
$$v_y = v_{yo} + a_y t$$

$$y = \frac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$y = v_{yo}t + \frac{1}{2}a_y t^2$$

A Motion in 2 Dimension



Imagine you add the two 1 dimensional motions on the left. It would make up a one 2 dimensional motion on the right.

Kinematic Equations in 2-Dim

x-component

$$v_x = v_{xo} + a_x t$$

$$x = \frac{1}{2}\left(v_{xo} + v_x\right)t$$

$$v_x^2 = v_{xo}^2 + 2a_x x$$

$$\Delta x = v_{xo}t + \frac{1}{2}a_x t^2$$

y-component

$$v_y = v_{yo} + a_y t$$

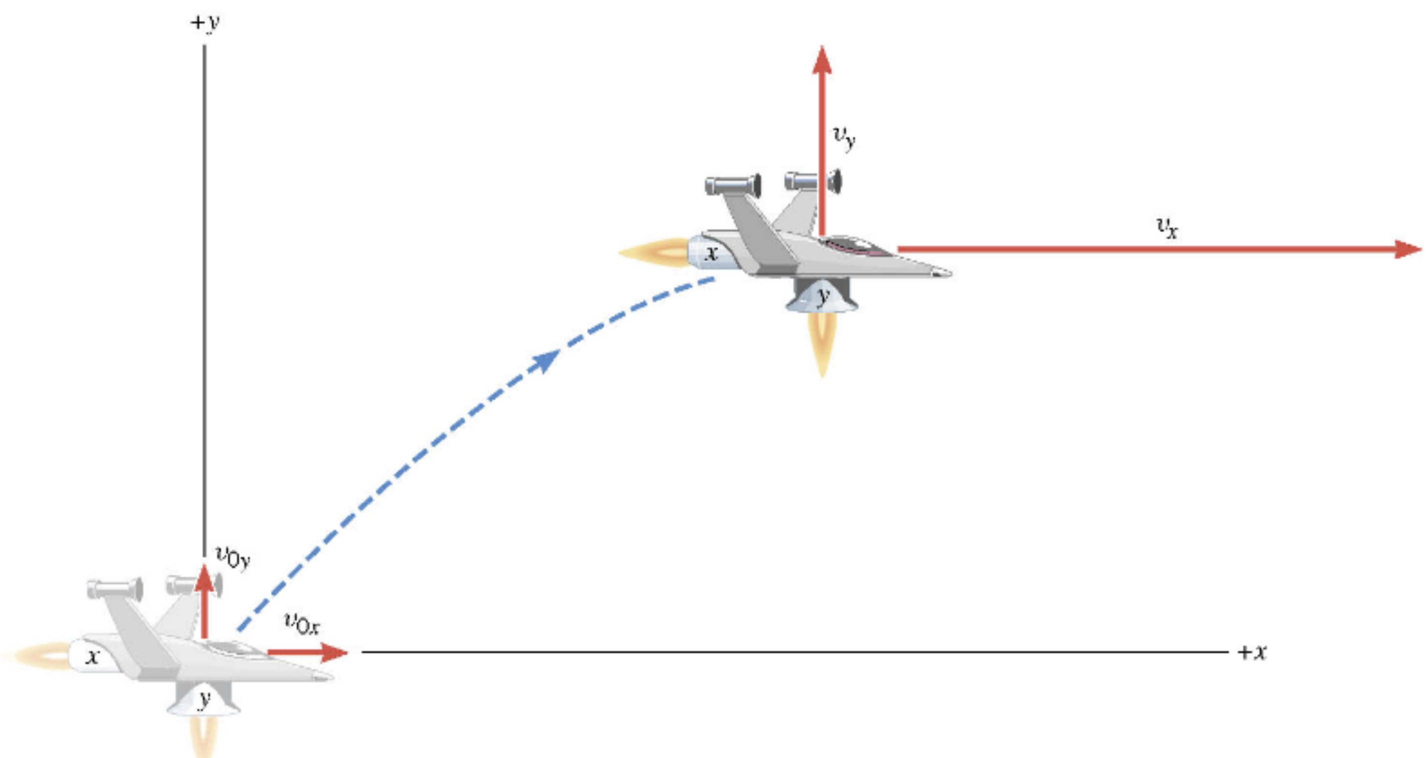
$$y = \frac{1}{2}\left(v_{yo} + v_y\right)t$$

$$v_y^2 = v_{yo}^2 + 2a_y y$$

$$\Delta y = v_{yo}t + \frac{1}{2}a_y t^2$$

Ex. A Moving Spacecraft

In the *x* direction, the spacecraft in zero-gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



How do we solve this problem?

1. Visualize the problem ➔ Draw a picture!
2. Decide which directions are to be called positive (+) and negative (-).
3. Write down the values that are given for any of the five kinematic variables associated with each direction.
4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y* *separately.* Select the appropriate equation.
5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.
6. Keep in mind that there may be two possible answers to a kinematics problem.

Ex. continued

In the *x* direction, the spacecraft in a zero gravity zone has an initial velocity component of +22 m/s and an acceleration of +24 m/s^2. In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s^2. Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22.0 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14.0 m/s	7.0 s

First, the motion in x-direciton…

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s^2	?	+22 m/s	7.0 s

$$\Delta x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

$$= (22\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(24\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +740\ \mathrm{m}$$

$$v_x = v_{ox} + a_x t$$

$$= (22\,\mathrm{m/s}) + (24\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +190\,\mathrm{m/s}$$

Now, the motion in y-direction…

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s^2	?	+14 m/s	7.0 s

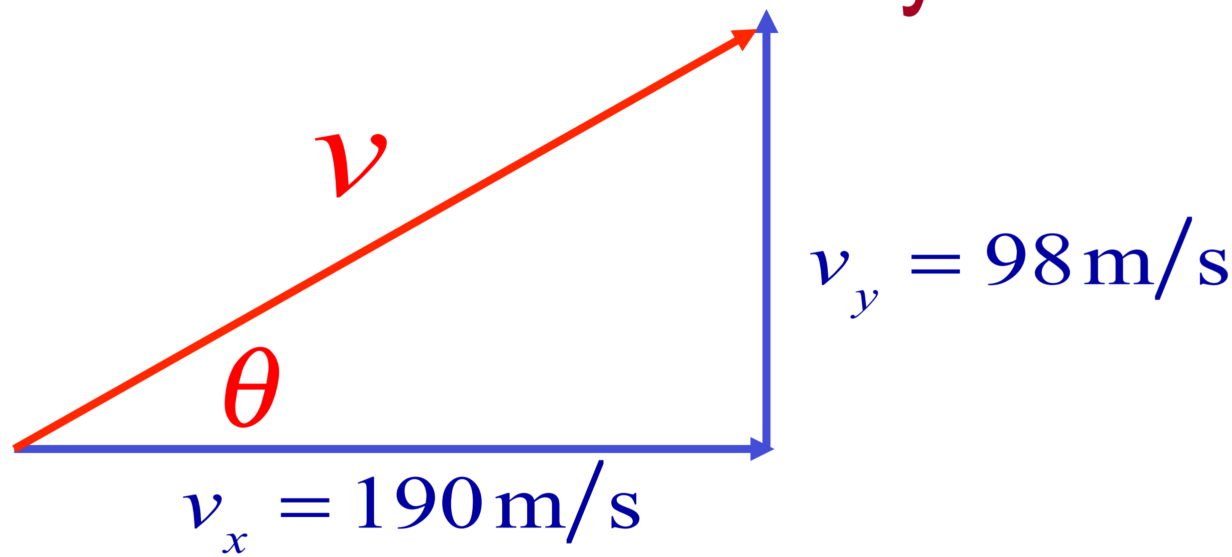
$$\Delta y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$= (14\,\mathrm{m/s})(7.0\ \mathrm{s}) + \tfrac{1}{2}(12\,\mathrm{m/s^2})(7.0\ \mathrm{s})^2 = +390\ \mathrm{m}$$

$$v_y = v_{oy} + a_y t$$

$$= (14\,\mathrm{m/s}) + (12\,\mathrm{m/s^2})(7.0\ \mathrm{s}) = +98\,\mathrm{m/s}$$

The final velocity…



$$v = \sqrt{(190\,\mathrm{m/s})^2 + (98\,\mathrm{m/s})^2} = 210\,\mathrm{m/s}$$

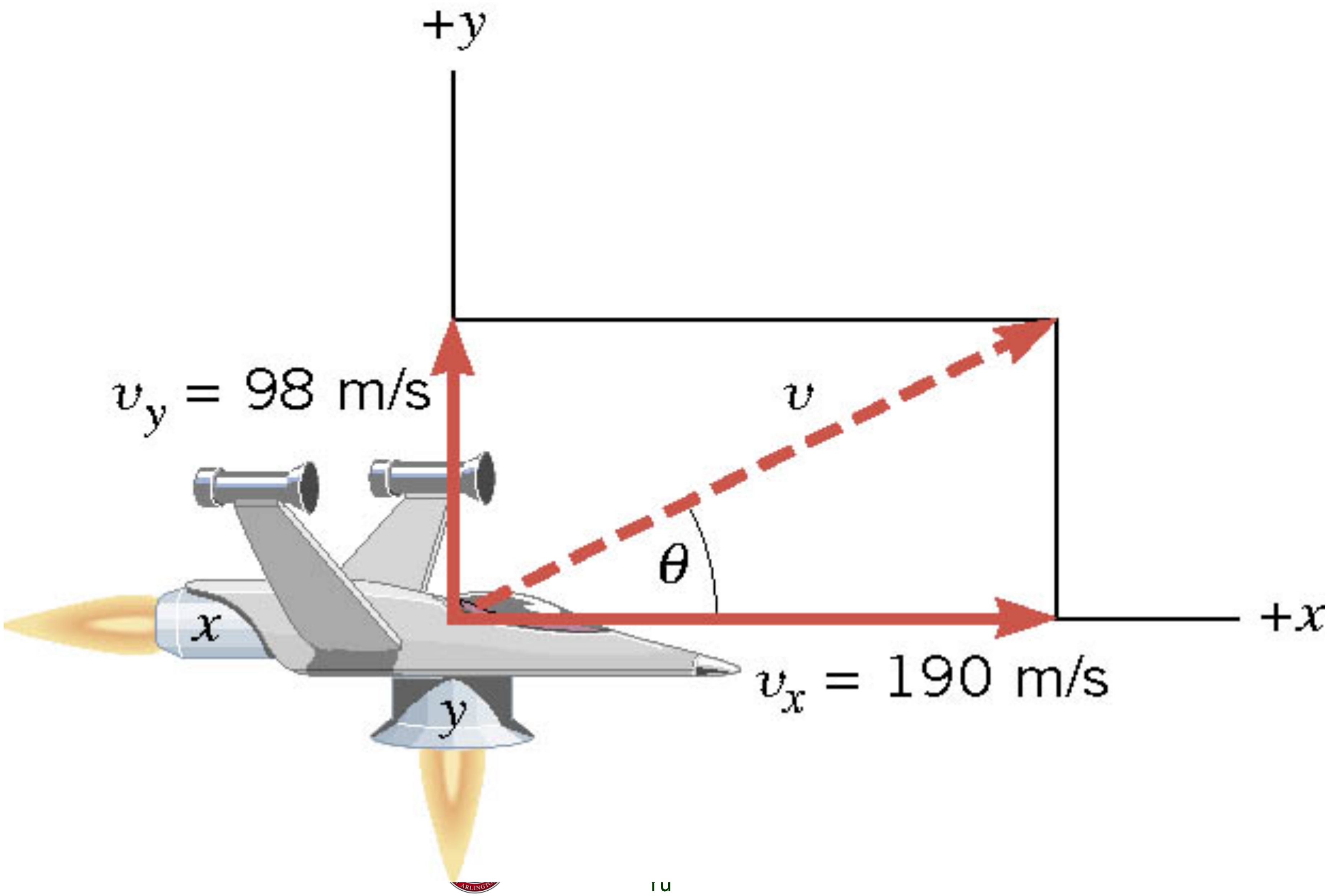
$$\theta = \tan^{-1}\left(98/190\right) = 27^\circ$$

A vector can be fully described when the magnitude and the direction are given. Any other way to describe it?

Yes, you are right! Using components and unit vectors!!

$$\vec{v} = v_x\vec{i} + v_y\vec{j} = \left(190\vec{i} + 98\vec{j}\right) m/s$$

If we visualize the motion…



What is a Projectile Motion?

- A 2-dim motion of an object under the gravitational acceleration with the following assumptions
  - Free fall acceleration, g, is constant over the range of the motion
    - $\vec{g} = -9.8\vec{j}\left(m/s^2\right)$
    - $a_x = 0\,m/s^2$ and $a_y = -9.8\,m/s^2$
  - Air resistance and other effects are negligible
- A motion under constant acceleration!!!! ➔ Superposition of two motions
  - Horizontal motion with constant velocity (<u>no acceleration</u>) $v_{xf} = v_{x0}$
  - Vertical motion under constant acceleration (g)

$$v_{yf} = v_{y0} + a_y t = v_{y0} + \left(-9.8\right)t$$