PHYS 1441 – Section 002
Lecture #19

Monday, Nov. 22, 2010

Dr. Jaehoon Yu

- Equations of Rotational Kinematics

- Relationship Between Angular and Linear Quantities

- Rolling Motion of a Rigid Body

Today's homework is homework #11, due 10pm, Tuesday, Nov. 30!!

Announcements

- ## Quiz 4 results
  - – Class average: 30/70
    - • Equivalent to: 43/100
    - • Previous quizzes: 53/100, 56/100 and 64/100
  - – Top score: 69/70
- ## This Wednesday
  - – Review with problem solving exercise

Reminder: Extra-Credit Special Project

- Derive the formula for the final velocity of two objects which underwent an elastic collision as a function of known quantities m_1, m_2, v_{01} and v_{02} in page 12 of this lecture note in a far greater detail than the note.

  – 20 points extra credit

- Show mathematically what happens to the final velocities if $m_1 = m_2$ and describe in words the resulting motion.

  – 5 point extra credit

- Due: Start of the class Monday, Nov. 29

Angular Displacement, Velocity, and Acceleration

Angular displacement is defined as

$$\Delta\theta = \theta_f - \theta_i$$
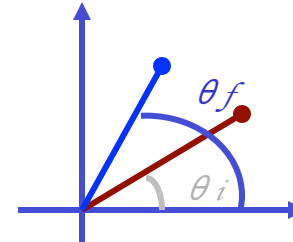
How about the average angular velocity, the rate of change of angular displacement?

$$\overline{\omega} \equiv \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Unit?   rad/s   Dimension? $[T^{-1}]$

By the same token, the average angular acceleration, rate of change of the angular velocity, is defined as…

$$\overline{\alpha} \equiv \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Unit?   rad/s^2 Dimension? $[T^{-2}]$

When rotating about a fixed axis, every particle on a rigid object rotates through the same angle and has the same angular speed and angular acceleration.

Ex. Gymnast on a High Bar

A gymnast on a high bar swings through two revolutions in a time of 1.90 s. Find the average angular velocity of the gymnast.

What is the angular displacement?

$$\Delta\theta = -2.00 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$$
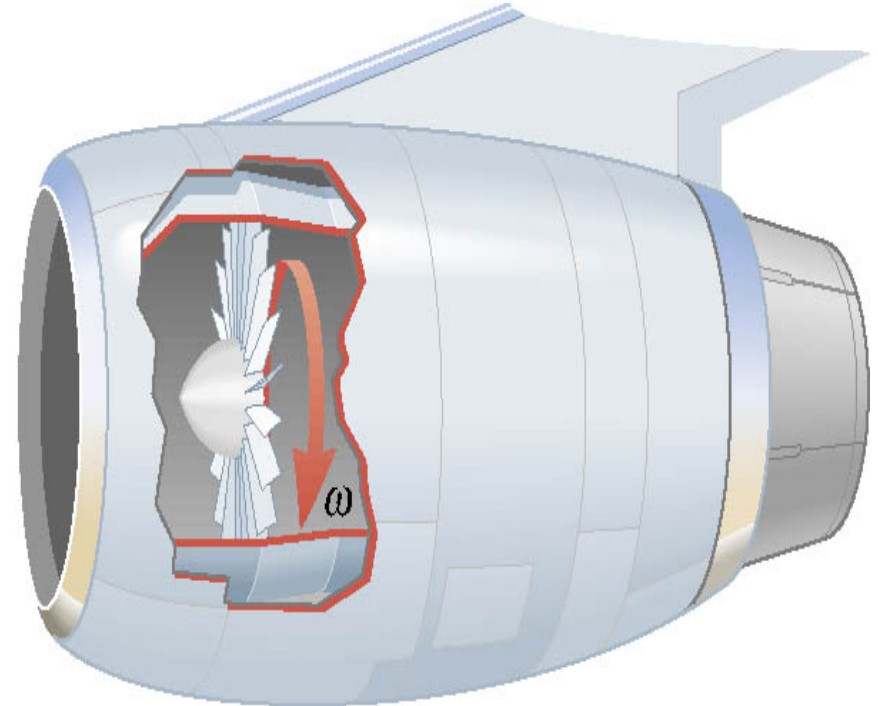
$$= -12.6 \text{ rad}$$

Why negative? Because he is rotating clockwise!!

$$\bar{\omega} = \frac{-12.6 \text{ rad}}{1.90 \text{ s}} = -6.63 \text{ rad/s}$$

Ex. A Jet Revving Its Engines

As seen from the front of the engine, the fan blades are rotating with an angular speed of -110 rad/s. As the plane takes off, the angular velocity of the blades reaches -330 rad/s in a time of 14 s. Find the angular acceleration, assuming it to be constant.

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

$$= \frac{(-330\,\text{rad/s}) - (-110\,\text{rad/s})}{14\,\text{s}} = -16\,\text{rad/s}^2$$

Rotational Kinematics

The first type of motion we have learned in linear kinematics was under the constant acceleration. We will learn about the rotational motion under constant angular acceleration, because these are the simplest motions in both cases.

Just like the case in linear motion, one can obtain

Angular velocity under constant angular acceleration:

$$\omega_f = \omega_0 + \alpha t$$

Linear kinematics $v = v_o + at$

Angular displacement under constant angular acceleration:

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Linear kinematics $x_f = x_0 + v_o t + \frac{1}{2}at^2$

One can also obtain

Linear kinematics $v_f^2 = v_o^2 + 2a\left(x_f - x_i\right)$

$$\omega_f^2 = \omega_0^2 + 2\alpha\left(\theta_f - \theta_0\right)$$

Problem Solving Strategy

- Visualize the problem by drawing a picture.

- Write down the values that are given for any of the five kinematic variables and convert them to SI units.

  - Remember that the unit of the angle must be radians!!

- Verify that the information contains values for at least three of the five kinematic variables. Select the appropriate equation.

- When the motion is divided into segments, remember that the final angular velocity of one segment is the initial velocity for the next.

- Keep in mind that there may be two possible answers to a kinematics problem.

Ex. Blending with a Blender

The blade is whirling with an angular velocity of +375 rad/s when the "puree" button is pushed in. When the "blend" button is pushed, the blade accelerates and reaches a greater angular velocity after the blade has rotated through an angular displacement of +44.0 rad. The angular acceleration has a constant value of +1740 rad/s^2. Find the final angular velocity of the blade.

θ	α	ω	ω_o	t
+44.0rad	+1740rad/s^2	?	+375rad/s	

Which kinematic eq? $\quad \omega^2 = \omega_o^2 + 2\alpha\theta$

$$\omega = \pm\sqrt{\omega_o^2 + 2\alpha\theta}$$

$$= \pm\sqrt{\left(375\,\text{rad/s}\right)^2 + 2\left(1740\,\text{rad/s}^2\right)\left(44.0\text{rad}\right)} = \pm542\,\text{rad/s}$$

Which sign? $\quad \omega = +542\,\text{rad/s} \quad$ *Why?* *Because the blade is accelerating in counter-clockwise!*
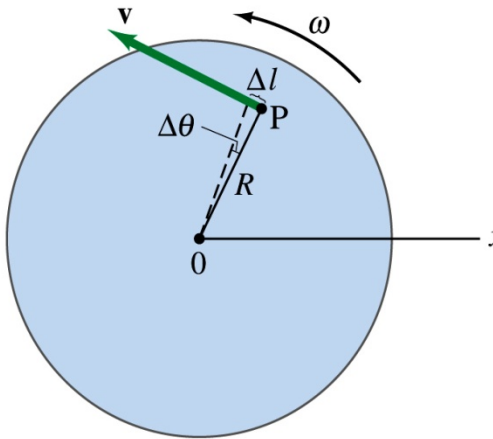
Relationship Between Angular and Linear Quantities

What do we know about a rigid object that rotates about a fixed axis of rotation?

Every particle (or masslet) in the object moves in a circle centered at the same axis of rotation with the same angular velocity.

When a point rotates, it has both the linear and angular components in its motion.

What is the linear component of the motion you see?

The direction of ω follows the right-hand rule.

Linear velocity along the tangential direction.

How do we related this linear component of the motion with angular component?

The arc-length is $\boxed{l = r\theta}$ So the tangential speed v is

$$v = \frac{\Delta l}{\Delta t} = \frac{\Delta(r\theta)}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega$$

What does this relationship tell you about the tangential speed of the points in the object and their angular speed?

Although every particle in the object has the same angular speed, its tangential speed differs and is proportional to its distance from the axis of rotation.

The farther away the particle is from the center of rotation, the higher the tangential speed.

Is the lion faster than the horse?

A rotating carousel has one child sitting on a horse near the outer edge and another child on a lion halfway out from the center. (a) Which child has the greater linear speed? (b) Which child has the greater angular speed?
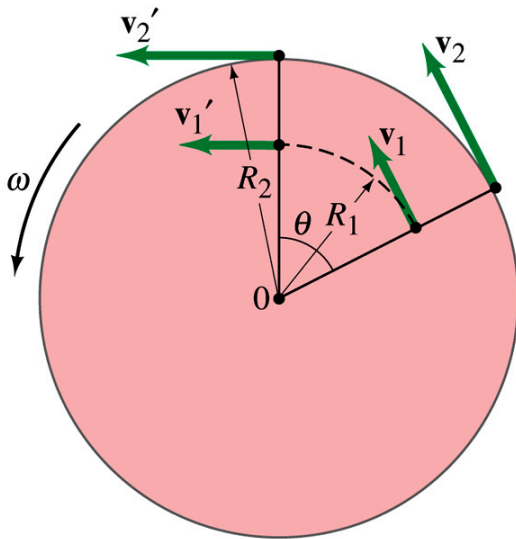
(a) Linear speed is the distance traveled divided by the time interval. So the child sitting at the outer edge travels more distance within the given time than the child sitting closer to the center. Thus, the horse is faster than the lion.

(b) Angular speed is the angle traveled divided by the time interval. The angle both the children travel in the given time interval is the same. Thus, both the horse and the lion have the same angular speed.

How about the acceleration?
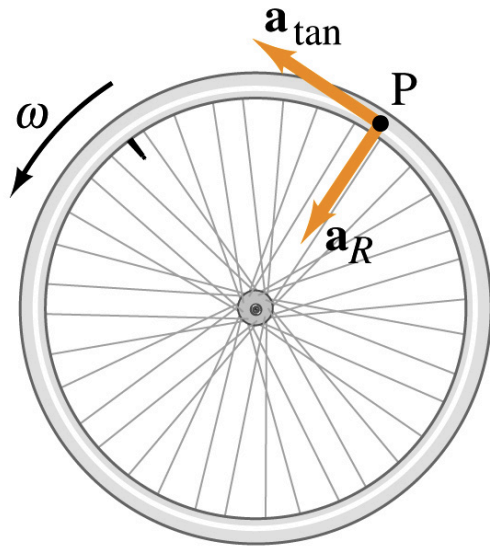
$\mathbf{a}_{tan}$
ω
P
$\mathbf{a}_R$

How many different linear acceleration components do you see in a circular motion and what are they? **Two**

Tangential, a_t, and the radial acceleration, a_r.

Since the tangential speed v is $\quad v_t = r\omega$

The magnitude of tangential acceleration a_t is

$$a_t = \frac{v_{tf} - v_{t0}}{\Delta t} = \frac{r\omega_f - r\omega_0}{\Delta t} = r\frac{\omega_f - \omega_0}{\Delta t} = r\alpha$$

What does this relationship tell you?

Although every particle in the object has the same angular acceleration, its tangential acceleration differs proportional to its distance from the axis of rotation.

The radial or centripetal acceleration a_r is $\quad a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$

What does this tell you?

The father away the particle is from the rotation axis, the more radial acceleration it receives. In other words, it receives more centripetal force.

Total linear acceleration is $\quad a = \sqrt{a_t^2 + a_r^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$

Ex. A Helicopter Blade

A helicopter blade has an angular speed of 6.50 rev/s and an angular acceleration of 1.30 rev/s². For point 1 on the blade, find the magnitude of (a) the tangential speed and (b) the tangential acceleration.
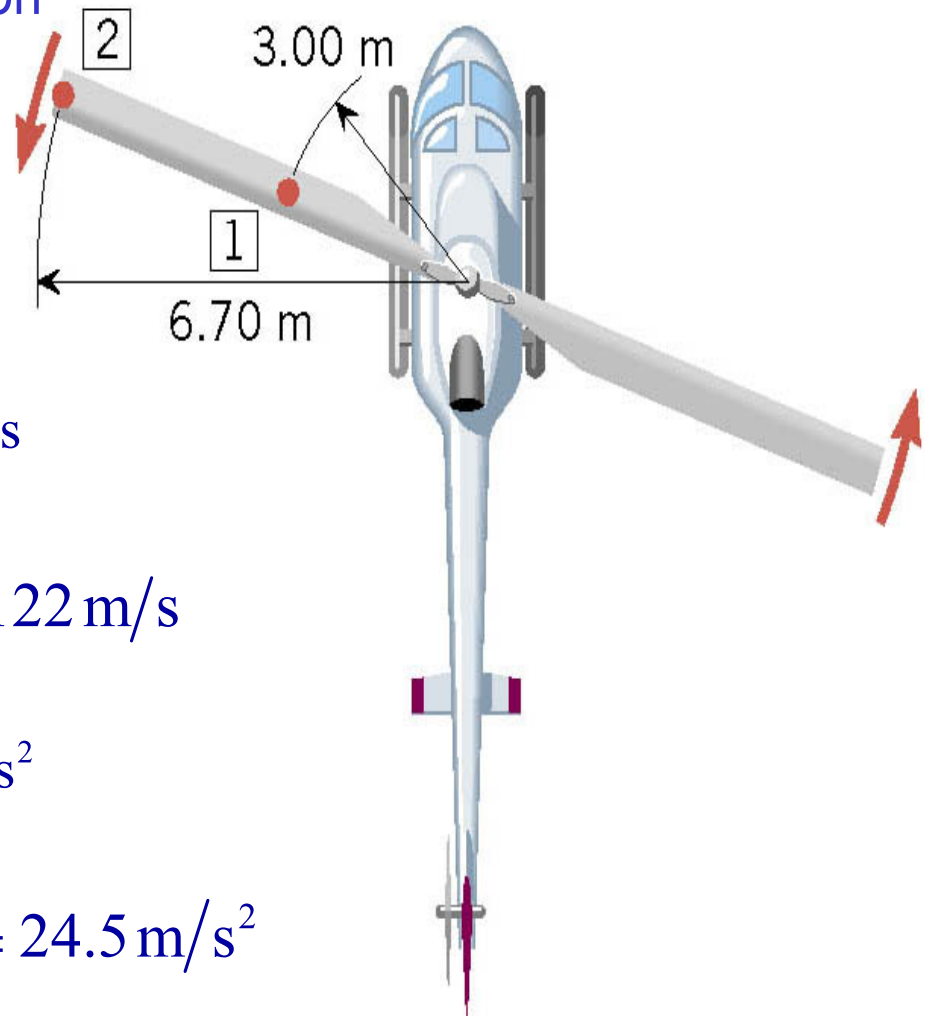
$$\omega = \left(6.50 \ \frac{\text{rev}}{\text{s}} \right)\left(\frac{2\pi \ \text{rad}}{1 \ \text{rev}} \right) = 40.8 \ \text{rad/s}$$

$$v_T = r\overline{\omega} = (3.00 \ \text{m})(40.8 \ \text{rad/s}) = 122 \ \text{m/s}$$

$$\alpha = \left(1.30 \ \frac{\text{rev}}{\text{s}^2} \right)\left(\frac{2\pi \ \text{rad}}{1 \ \text{rev}} \right) = 8.17 \ \text{rad/s}^2$$

$$a_T = r\alpha = (3.00 \ \text{m})(8.17 \ \text{rad/s}^2) = 24.5 \ \text{m/s}^2$$

2   3.00 m

1

6.70 m