PHYS 1441 – Section 002
Lecture #21

Monday, Nov. 29, 2010

Dr. Jaehoon Yu

- Rolling Motion of a Rigid Body
- Rotational Dynamics
  - Torque
  - Moment of Inertia
  - Torque and Angular Acceleration
- Rotational Kinetic Energy
- Angular Momentum

Today's homework is homework #12, due 10pm, Thursday, Dec. 9!!

Announcements

- The Final Exam
  - Date and time: 11am, Monday Dec. 13
  - Place: SH103
  - Comprehensive exam
    - Covers from CH1.1 – what we finish Wednesday, Dec. 8
    - Plus appendices A.1 – A.8
    - Combination of multiple choice and free response problems
    - Bring your Planetarium extra credit sheet to the class next Wednesday, Dec. 8, with your name clearly marked on the sheet!
- No more quizzes and no more home works!
- Colloquium this Wednesday

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Undo the Size Effect in Semiconductor Nanostructures

Dr. Shengbai Zhang

Rensselaer Polytechnic Institute

4:00p.m Wednesday December 1, 2010
At SH Rm 101

Abstract:

It is a textbook example that when the size of a semiconductor is reduced, band gap will increase due to the increased kinetic energy of the electron and hole. However, first-principles calculations reveal that there should also be a quantum boundary effect, which can drastically change the band gap to the extent to completely erase the size effect. The boundary effect originates, for instance, from different surface passivations: While a thin silicon film passivated by hydrogen shows a full quantum size effect, the effect diminishes for film size as small as two nanometers when some of the hydrogen atoms are replaced by NH ligands. I will introduce the concept of zero confinement state for semiconductors to elucidate why it is possible to remove the seemingly universal quantum size effect. This finding could of course be highly desirable for certain electronic applications. The quantum boundary effect can also manifest itself as a symmetry effect. Taking the fully hydrogenated zigzag graphene nanoribbon as an example, I will show that due to the underlying, but hidden, triple-period Kekulé symmetry, the band gap will change by a factor of three if one slides one side of the passivation with respect to the other side by one atomic unit, regardless the width of the ribbon. This creates two edge polymorphs of practically identical stability, which could be very challenging for fabricating graphene nanoribbons with well-defined band gap, as well as offering new opportunities for novel electronic applications.

Refreshments will be served in the Physics Lounge at 3:30 pm

Rolling Motion of a Rigid Body

What is a rolling motion?

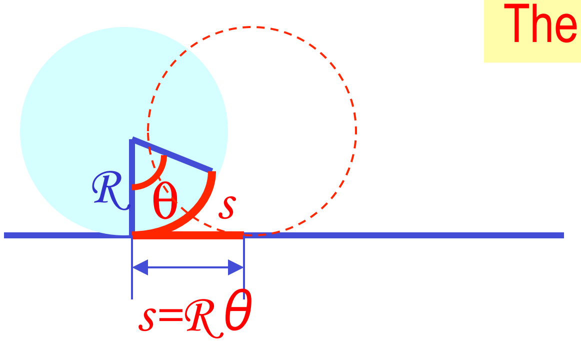
A more generalized case of a motion where the rotational axis moves together with an object

A rotational motion about a moving axis

To simplify the discussion, let's make a few assumptions

1. Limit our discussion on very symmetric objects, such as cylinders, spheres, etc
2. The object rolls on a flat surface

Let's consider a cylinder rolling on a flat surface, without slipping.



Under what condition does this "Pure Rolling" happen?

The total linear distance the CM of the cylinder moved is $s = R\theta$

Thus the linear speed of the CM is

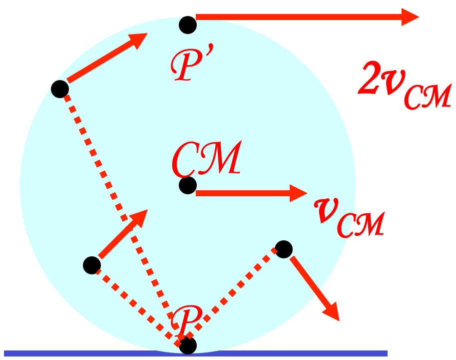
$$\bar{v}_{CM} = \frac{\Delta s}{\Delta t} = R\frac{\Delta \theta}{\Delta t} = R\omega$$

The condition for a "Pure Rolling motion"

More Rolling Motion of a Rigid Body

The magnitude of the linear acceleration of the CM is

$$a_{CM} = \frac{\Delta v_{CM}}{\Delta t} = R\frac{\Delta\omega}{\Delta t} = R\alpha$$



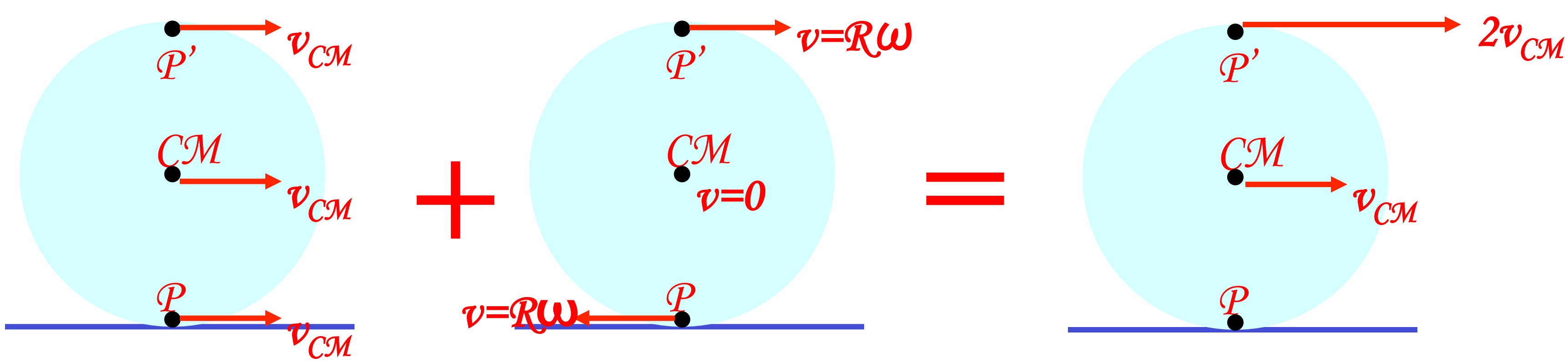
As we learned in rotational motion, all points in a rigid body moves at the same angular speed but at different linear speeds.

CM is moving at the same speed at all times.

At any given time, the point that comes to P has 0 linear speed while the point at P' has twice the speed of CM

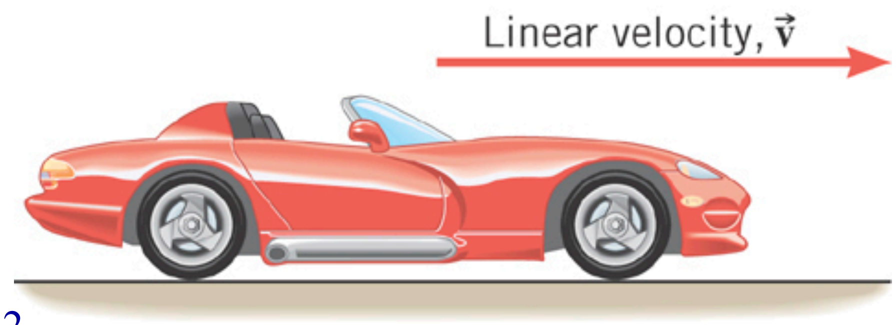
Why??

A rolling motion can be interpreted as the sum of Translation and Rotation



Ex. An Accelerating Car

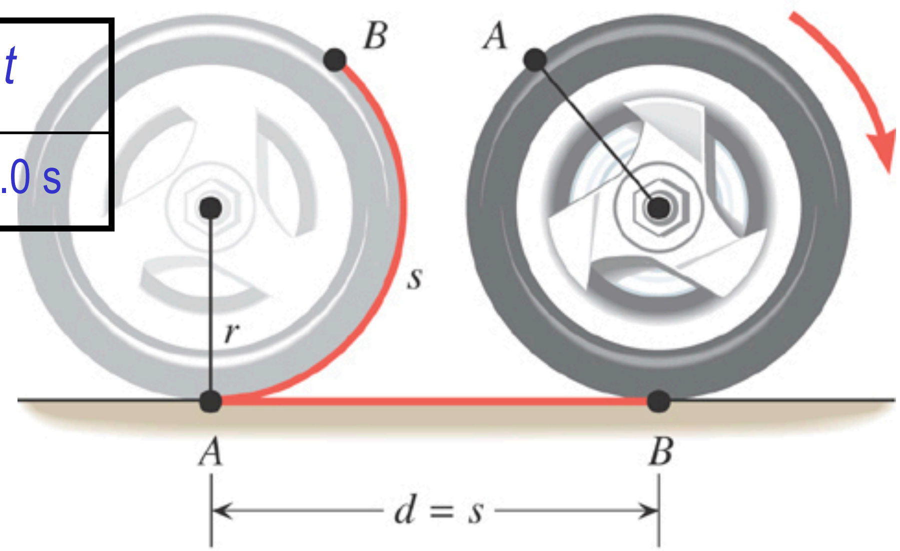
Starting from rest, the car accelerates for 20.0 s with a constant linear acceleration of 0.800 m/s^2. The radius of the tires is 0.330 m. What is the angle through which each wheel has rotated?



(a)

$$\alpha = \frac{a}{r} = \frac{0.800\,\mathrm{m/s^2}}{0.330\ \mathrm{m}} = 2.42\,\mathrm{rad/s^2}$$

θ	α	ω	ω_o	t
?	-2.42 rad/s^2		0 rad/s	20.0 s

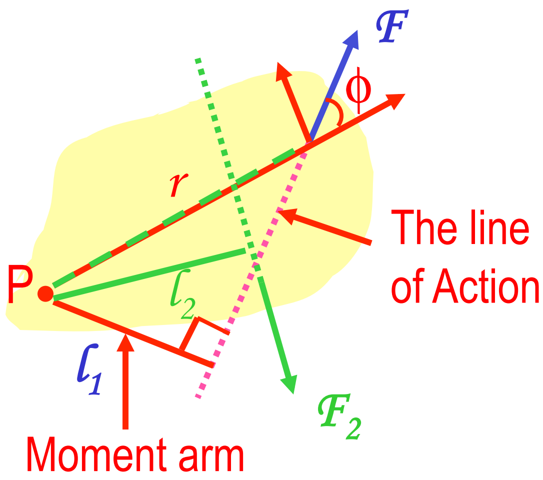


(b)

$$\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$
$$= \tfrac{1}{2}\left(-2.42\,\mathrm{rad/s^2}\right)^2 (20.0\ \mathrm{s})^2$$
$$= -484\ \mathrm{rad}$$

Torque

Torque is the tendency of a force to rotate an object about an axis. Torque, τ, is a vector quantity.



Consider an object pivoting about the point P by the force $\mathcal{F}$ being exerted at a distance r from P.

The line that extends out of the tail of the force vector is called the line of action.

The perpendicular distance from the pivoting point P to the line of action is called the moment arm.

$$\left|\vec{\tau}\right| \equiv (\text{Magnitude of the Force}) \times (\text{Lever Arm})$$
$$= (F)(r\sin\phi) = Fl$$

Magnitude of torque is defined as the product of the force exerted on the object to rotate it and the moment arm.

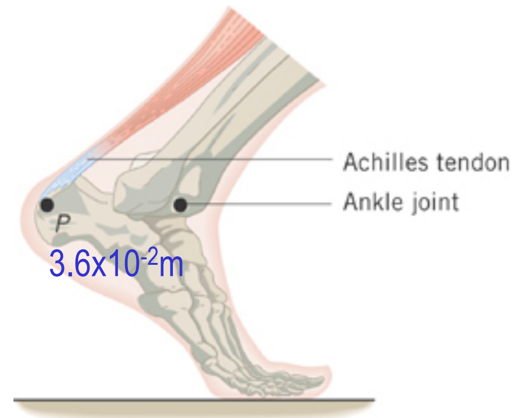
$$\sum \tau = \tau_1 + \tau_2$$
$$= F_1 l_1 - F_2 l_2$$

When there are more than one force being exerted on certain points of the object, one can sum up the torque generated by each force vectorially. The convention for sign of the torque is **positive if rotation is in counter-clockwise** and **negative if clockwise**.

Unit? $N \cdot m$

Ex. The Achilles Tendon

The tendon exerts a force of magnitude 790 N on the point P. Determine the torque (magnitude and direction) of this force about the ankle joint which is located 3.6×10^{-2}m away from point P.



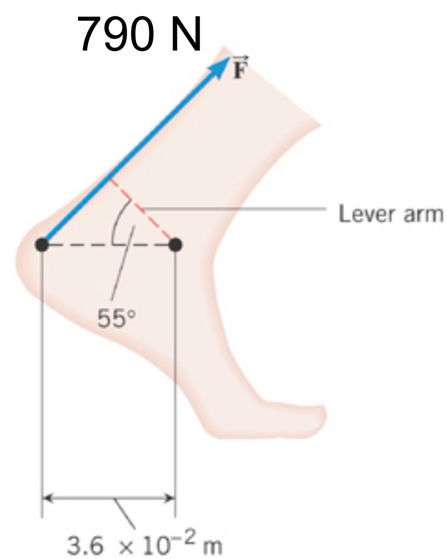
(a)

First, let's find the lever arm length

$$\cos 55^\circ = \frac{\ell}{3.6 \times 10^{-2}\,\mathrm{m}}$$

$$\ell = 3.6 \times 10^{-2} \cos 55^\circ =$$

$$= 3.6 \times 10^{-2} \sin\left(90^\circ - 55^\circ\right) = 2.1 \times 10^{-2}\,(m)$$



(b)

So the torque is

$$\tau = F\ell$$

$$= (720\ \mathrm{N})\left(3.6 \times 10^{-2}\,\mathrm{m}\right)\cos 55^\circ$$

$$= (720\ \mathrm{N})\left(3.6 \times 10^{-2}\,\mathrm{m}\right)\sin 35^\circ = 15\ \mathrm{N}\cdot\mathrm{m}$$

Since the rotation is in clock-wise $\tau = -15 N \cdot m$

Moment of Inertia

Rotational Inertia: Measure of resistance of an object to changes in its rotational motion. Equivalent to mass in linear motion.

For a group of objects $$I \equiv \sum_i m_i r_i^2$$

For a rigid body $$I \equiv \int r^2 dm$$

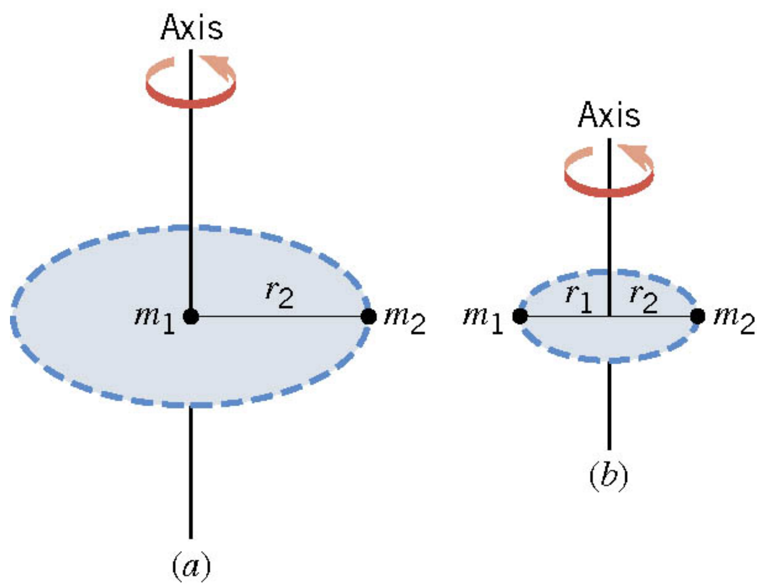
What are the dimension and unit of Moment of Inertia? $$\left[ML^2\right] \quad kg \cdot m^2$$

Determining Moment of Inertia is extremely important for computing equilibrium of a rigid body, such as a building.

Dependent on the axis of rotation!!!

Ex. The Moment of Inertia Depends on Where the Axis Is.

Two particles each have mass m_1 and m_2 and are fixed at the ends of a thin rigid rod. The length of the rod is *L*. Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.



(a) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$$m_1 = m_2 = m \qquad r_1 = 0 \quad r_2 = L$$

$$I = m(0)^2 + m(L)^2 = mL^2$$

(b) $I = \sum\left(mr^2\right) = m_1 r_1^2 + m_2 r_2^2$

$$m_1 = m_2 = m \qquad r_1 = L/2 \quad r_2 = L/2$$

$$I = m(L/2)^2 + m(L/2)^2 = \tfrac{1}{2}mL^2$$

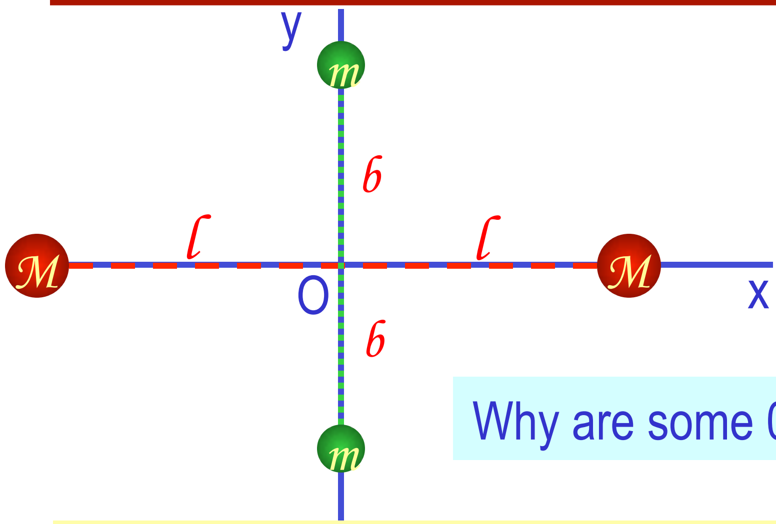
Which case is easier to spin?

Case (b)

Why? Because the moment of inertia is smaller

Example for Moment of Inertia

In a system of four small spheres as shown in the figure, assuming the radii are negligible and the rods connecting the particles are massless, compute the moment of inertia and the rotational kinetic energy when the system rotates about the y-axis at angular speed w.



Since the rotation is about y axis, the moment of inertia about y axis, I_y, is

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + m \cdot 0^2 + m \cdot 0^2 = 2Ml^2$$

Why are some 0s?

This is because the rotation is done about y axis, and the radii of the spheres are negligible.

Thus, the rotational kinetic energy is

$$K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(2Ml^2\right)\omega^2 = Ml^2\omega^2$$

Find the moment of inertia and rotational kinetic energy when the system rotates on the x-y plane about the z-axis that goes through the origin O.

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + mb^2 + mb^2 = 2\left(Ml^2 + mb^2\right) \quad K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(2Ml^2 + 2mb^2\right)\omega^2 = \left(Ml^2 + mb^2\right)\omega^2$$

Check out Figure 8 – 21 for moment of inertia for various shaped objects

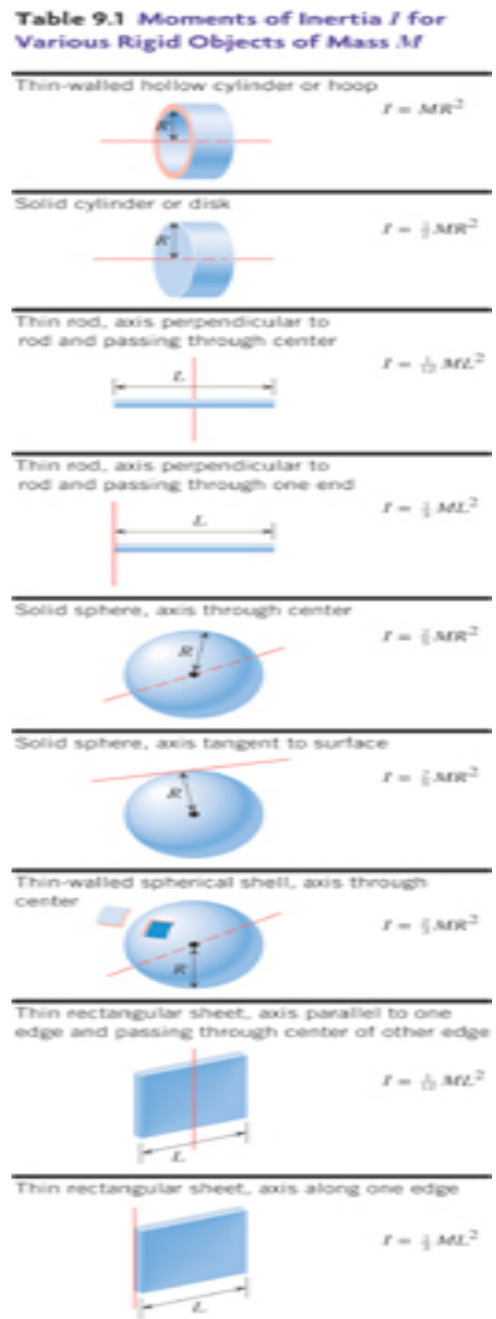
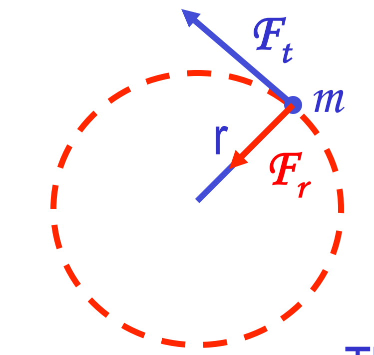


Table 9.1 **Moments of Inertia I for Various Rigid Objects of Mass M**

Thin-walled hollow cylinder or hoop	$I = MR^2$
Solid cylinder or disk	$I = \frac{1}{2}MR^2$
Thin rod, axis perpendicular to rod and passing through center	$I = \frac{1}{12}ML^2$
Thin rod, axis perpendicular to rod and passing through one end	$I = \frac{1}{3}ML^2$
Solid sphere, axis through center	$I = \frac{2}{5}MR^2$
Solid sphere, axis tangent to surface	$I = \frac{7}{5}MR^2$
Thin-walled spherical shell, axis through center	$I = \frac{2}{3}MR^2$
Thin rectangular sheet, axis parallel to one edge and passing through center of other edge	$I = \frac{1}{12}ML^2$
Thin rectangular sheet, axis along one edge	$I = \frac{1}{3}ML^2$

Torque & Angular Acceleration



Let's consider a point object with mass m rotating on a circle.

What forces do you see in this motion?

The tangential force $\mathcal{F}_t$ and the radial force $\mathcal{F}_r$

The tangential force $\mathcal{F}_t$ is $F_t = ma_t = mr\alpha$

The torque due to tangential force $\mathcal{F}_t$ is $\tau = F_t r = ma_t r = mr^2\alpha = I\alpha$

What do you see from the above relationship? $\tau = I\alpha$

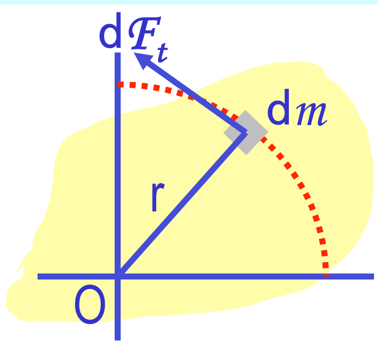
What does this mean?

Torque acting on a particle is proportional to the angular acceleration.

What law do you see from this relationship?

Analogs to Newton's 2nd law of motion in rotation.

How about a rigid object?



The external tangential force $\delta\mathcal{F}_t$ is $\delta F_t = \delta m a_t = \delta m r\alpha$

The torque due to tangential force $\mathcal{F}_t$ is $\delta\tau = \delta F_t r = \left(r^2\delta m\right)\alpha$

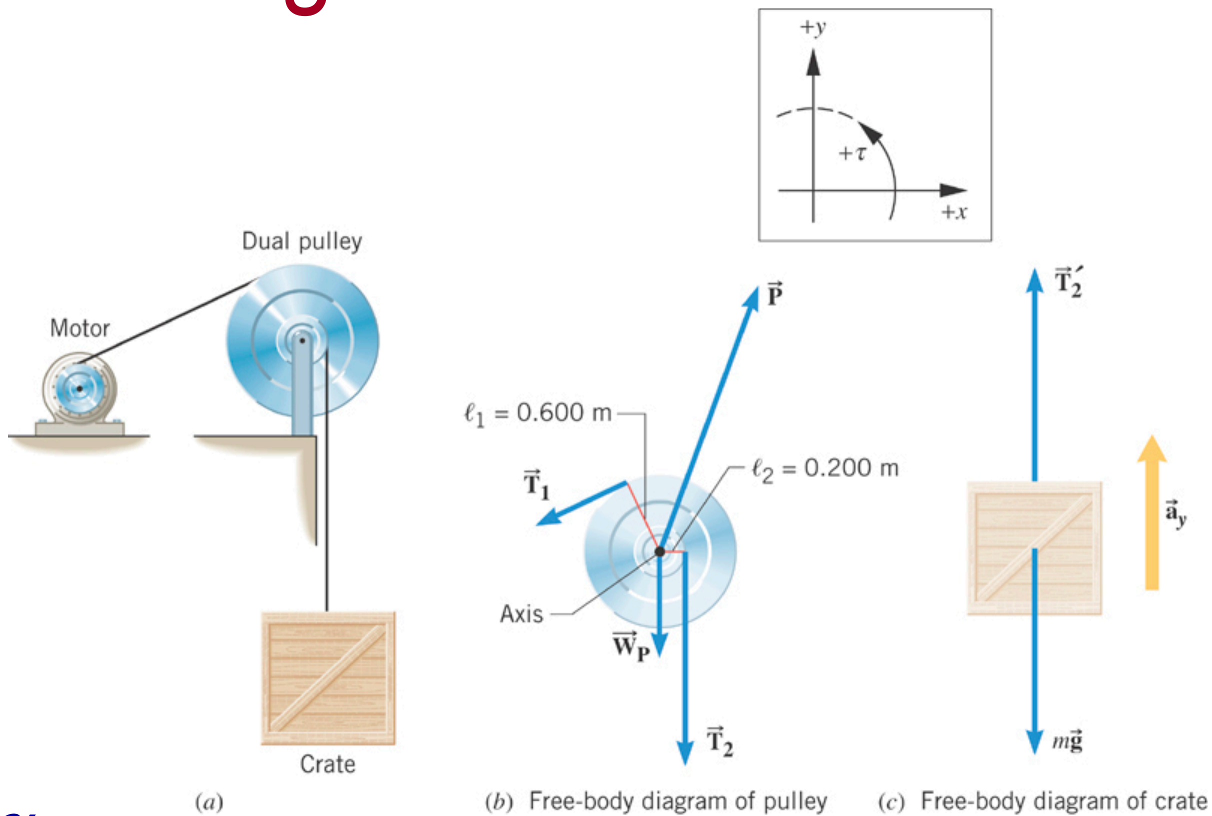
The total torque is $\sum\delta\tau = \alpha\sum r^2\delta m = I\alpha$

What is the contribution due to radial force and why?

Contribution from radial force is 0, because its line of action passes through the pivoting point, making the moment arm 0.

Ex. Hoisting a Crate

The combined moment of inertia of the dual pulley is 50.0 kg·m^2. The crate weighs 4420 N. A tension of 2150 N is maintained in the cable attached to the motor. Find the angular acceleration of the dual pulley.



$$\sum F_y = T_2' - mg = ma_y$$

$$T_2' = mg + ma_y$$

$$\sum \tau = T_1\ell_1 - T_2'\ell_2 = I\alpha$$

$$T_1\ell_1 - \left(mg + ma_y\right)\ell_2 = I\alpha$$

$$a_y = \ell_2\alpha \qquad T_1\ell_1 - \left(mg + m\ell_2\alpha\right)\ell_2 = I\alpha$$

$$\alpha = \frac{T_1\ell_1 - mg\ell_2}{I + m\ell_2^2} = \frac{(2150\ \mathrm{N})(0.600\ \mathrm{m}) - (451\ \mathrm{kg})\left(9.80\,\mathrm{m/s^2}\right)(0.200\ \mathrm{m})}{46.0\ \mathrm{kg\cdot m^2} + (451\ \mathrm{kg})(0.200\ \mathrm{m})^2} = 6.3\,\mathrm{rad/s^2}$$