PHYS 1441 – Section 002
Lecture #22

Wednesday, Dec. 1, 2010

Dr. Jaehoon Yu

- Rotational Kinetic Energy
- Angular Momentum
- Angular Momentum Conservation
- Similarities Between Linear and Rotational Quantities
- Conditions for Equilibrium

Announcements

- The Final Exam
  - Date and time: 11am, Monday Dec. 13
  - Place: SH103
  - Comprehensive exam
    - Covers from CH1.1 – what we finish Wednesday, Dec. 8
    - Plus appendices A.1 – A.8
    - Combination of multiple choice and free response problems
    - Bring your Planetarium extra credit sheet to the class next Wednesday, Dec. 8, with your name clearly marked on the sheet!
- Colloquium this today

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Undo the Size Effect in Semiconductor Nanostructures

Dr. Shengbai Zhang

Rensselaer Polytechnic Institute

4:00p.m Wednesday December 1, 2010

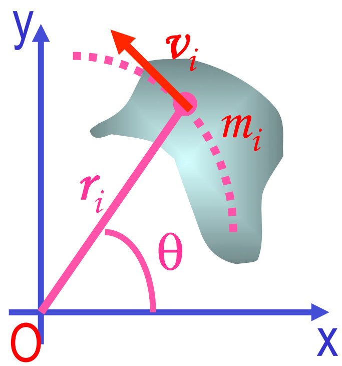
At SH Rm 101

Abstract:

It is a textbook example that when the size of a semiconductor is reduced, band gap will increase due to the increased kinetic energy of the electron and hole. However, first-principles calculations reveal that there should also be a quantum boundary effect, which can drastically change the band gap to the extent to completely erase the size effect. The boundary effect originates, for instance, from different surface passivations: While a thin silicon film passivated by hydrogen shows a full quantum size effect, the effect diminishes for film size as small as two nanometers when some of the hydrogen atoms are replaced by NH ligands. I will introduce the concept of zero confinement state for semiconductors to elucidate why it is possible to remove the seemingly universal quantum size effect. This finding could of course be highly desirable for certain electronic applications. The quantum boundary effect can also manifest itself as a symmetry effect. Taking the fully hydrogenated zigzag graphene nanoribbon as an example, I will show that due to the underlying, but hidden, triple-period Kekulé symmetry, the band gap will change by a factor of three if one slides one side of the passivation with respect to the other side by one atomic unit, regardless the width of the ribbon. This creates two edge polymorphs of practically identical stability, which could be very challenging for fabricating graphene nanoribbons with well-defined band gap, as well as offering new opportunities for novel electronic applications.

Refreshments will be served in the Physics Lounge at 3:30 pm

Rotational Kinetic Energy



What do you think the kinetic energy of a rigid object that is undergoing a circular motion is?

Kinetic energy of a masslet, m_i, moving at a tangential speed, v_i, is $K_i = \frac{1}{2}m_i v_i^2 = \frac{1}{2}m_i r_i^2 \omega^2$

Since a rigid body is a collection of masslets, the total kinetic energy of the rigid object is

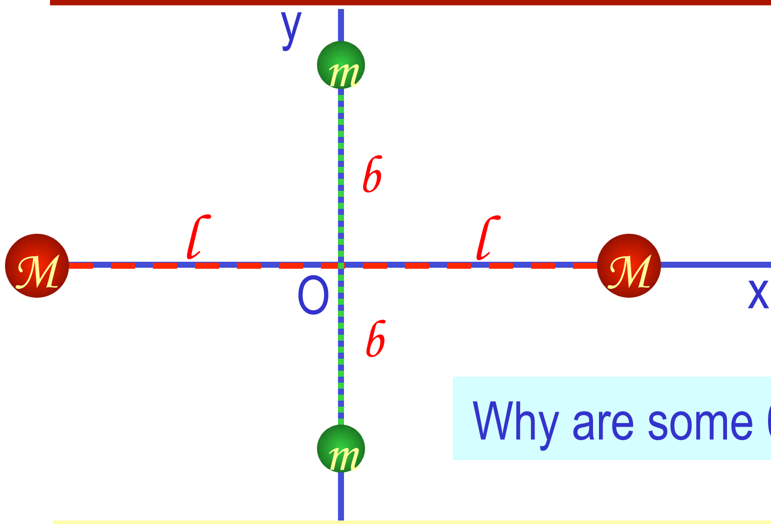
$$K_R = \sum_i K_i = \frac{1}{2}\sum_i m_i r_i^2 \omega^2 = \frac{1}{2}\boxed{\left(\sum_i m_i r_i^2\right)}\omega^2$$

Since moment of Inertia, I, is defined as $I = \sum_i m_i r_i^2$

The above expression is simplified as $K_R = \frac{1}{2}I\omega^2$

Example for Moment of Inertia

In a system of four small spheres as shown in the figure, assuming the radii are negligible and the rods connecting the particles are massless, compute the moment of inertia and the rotational kinetic energy when the system rotates about the y-axis at angular speed w.



Since the rotation is about y axis, the moment of inertia about y axis, I_y, is

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + m \cdot 0^2 + m \cdot 0^2 = 2Ml^2$$

Why are some 0s?

This is because the rotation is done about y axis, and the radii of the spheres are negligible.

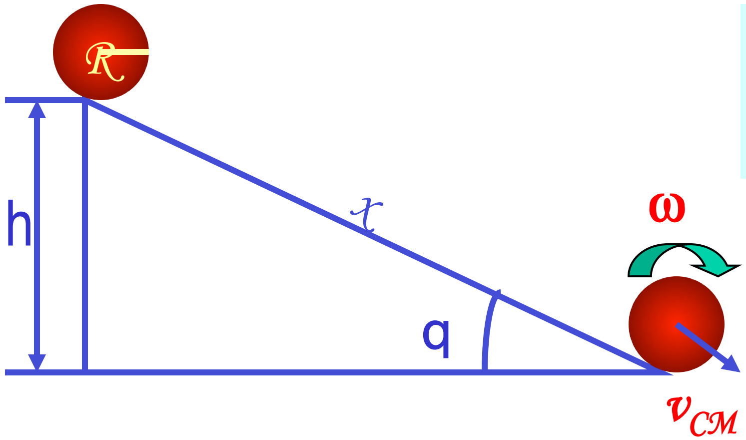
Thus, the rotational kinetic energy is

$$K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(2Ml^2\right)\omega^2 = Ml^2\omega^2$$

Find the moment of inertia and rotational kinetic energy when the system rotates on the x-y plane about the z-axis that goes through the origin O.

$$I = \sum_i m_i r_i^2 = Ml^2 + Ml^2 + mb^2 + mb^2 = 2\left(Ml^2 + mb^2\right) \quad K_R = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(2Ml^2 + 2mb^2\right)\omega^2 = \left(Ml^2 + mb^2\right)\omega^2$$

Kinetic Energy of a Rolling Sphere



Let's consider a sphere with radius R rolling down the hill without slipping.

Since $v_{CM}=R\omega$

$$K = \frac{1}{2}I_{CM}\omega^2 + \frac{1}{2}MR^2\omega^2$$

$$= \frac{1}{2}I_{CM}\left(\frac{v_{CM}}{R}\right)^2 + \frac{1}{2}Mv_{CM}^2$$

$$= \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right)v_{CM}^2$$

What is the speed of the CM in terms of known quantities and how do you find this out?

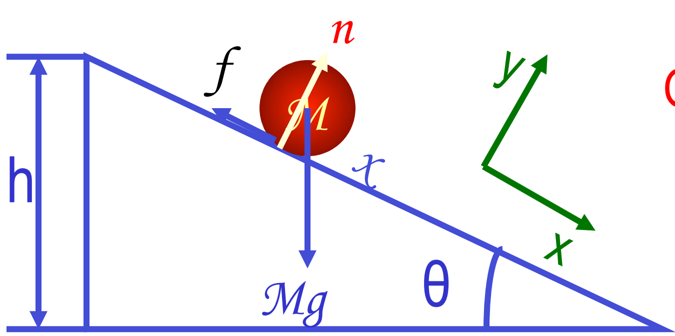
Since the kinetic energy at the bottom of the hill must be equal to the potential energy at the top of the hill

$$K = \frac{1}{2}\left(\frac{I_{CM}}{R^2} + M\right)v_{CM}^2 = Mgh$$

$$v_{CM} = \sqrt{\frac{2gh}{1 + I_{CM}/MR^2}}$$

Example for Rolling Kinetic Energy

For solid sphere as shown in the figure, calculate the linear speed of the CM at the bottom of the hill and the magnitude of linear acceleration of the CM. Solve this problem using Newton's second law, the dynamic method.



What are the forces involved in this motion?

Gravitational Force, Frictional Force, Normal Force

Newton's second law applied to the CM gives

$$\sum F_x = Mg\sin\theta - f = Ma_{CM}$$

$$\sum F_y = n - Mg\cos\theta = 0$$

Since the forces $\mathcal{M}g$ and **n** go through the CM, their moment arm is 0 and do not contribute to torque, while the static friction f causes torque

$$\tau_{CM} = fR = I_{CM}\alpha$$

We know that

$$I_{CM} = \frac{2}{5}MR^2$$

$$a_{CM} = R\alpha$$

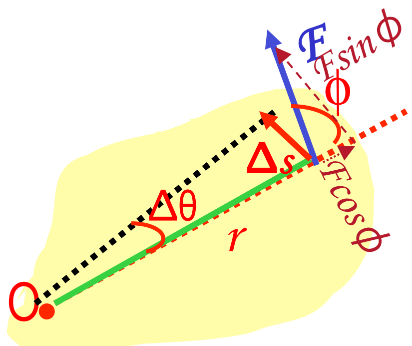
We obtain

$$f = \frac{I_{CM}\alpha}{R} = \frac{\frac{2}{5}MR^2}{R}\left(\frac{a_{CM}}{R}\right) = \frac{2}{5}Ma_{CM}$$

Substituting f in dynamic equations

$$Mg\sin\theta = \frac{7}{5}Ma_{CM} \qquad a_{CM} = \frac{5}{7}g\sin\theta$$

Work, Power, and Energy in Rotation



Let's consider the motion of a rigid body with a single external force $\mathcal{F}$ exerting on the point P, moving the object by Δs. The work done by the force $\mathcal{F}$ as the object rotates through the infinitesimal distance $\Delta s = r\Delta\theta$ is

$$\Delta W = \vec{F} \cdot \Delta\vec{s} = (F\sin\phi)r\Delta\theta$$

What is $\mathcal{F}\sin\phi$? The tangential component of the force $\mathcal{F}$.

What is the work done by radial component $\mathcal{F}\cos\phi$? Zero, because it is perpendicular to the displacement.

Since the magnitude of torque is $r\mathcal{F}\sin\phi$,

$$\Delta W = (rF\sin\phi)\Delta\theta = \tau\Delta\theta$$

The rate of work, or power, becomes

$$P = \frac{\Delta W}{\Delta t} = \frac{\tau\Delta\theta}{\Delta t} = \tau\omega$$

How was the power defined in linear motion?

The rotational work done by an external force equals the change in rotational Kinetic energy.

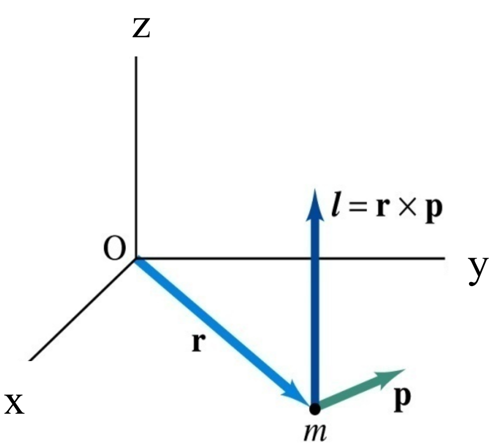
$$\sum\tau = I\alpha = I\left(\frac{\Delta\omega}{\Delta t}\right) \Rightarrow \sum\tau\Delta\theta = I\omega\Delta\omega$$

The work put in by the external force then

$$\Delta W = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

Angular Momentum of a Particle

If you grab onto a pole while running, your body will rotate about the pole, gaining angular momentum. We've used the linear momentum to solve physical problems with linear motions, the angular momentum will do the same for rotational motions.



Let's consider a point-like object (particle) with mass m located at the vector location r and moving with linear velocity v

The angular momentum L of this particle relative to the origin O is

$$\boxed{\vec{L} \equiv \vec{r} \times \vec{p}}$$

What is the unit and dimension of angular momentum? $kg \cdot m^2/s$ $[ML^2T^{-1}]$

Note that L depends on origin O. Why? Because r changes

What else do you learn? The direction of L is +z.

Since p is mv, the magnitude of L becomes $L = mvr = mr^2\varpi = I\varpi$

What do you learn from this? If the direction of linear velocity points to the origin of rotation, the particle does not have any angular momentum.

If the linear velocity is perpendicular to position vector, the particle moves exactly the same way as a point on a rim.

Angular Momentum of a System of Particles

The total angular momentum of a system of particles about some point is the vector sum of the angular momenta of the individual particles

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \ldots\ldots + \vec{L}_n = \sum \vec{L}_i$$

Since the individual angular momentum can change, the total angular momentum of the system can change.

Both internal and external forces can provide torque to individual particles. However, the internal forces do not generate net torque due to Newton's third law.

Let's consider a two particle system where the two exert forces on each other.

Since these forces are the action and reaction forces with directions lie on the line connecting the two particles, the vector sum of the torque from these two becomes 0.

Thus the time rate change of the angular momentum of a system of particles is equal to <u>only the net external torque</u> acting on the system

$$\boxed{\sum \vec{\tau}_{ext} = \frac{\Delta \vec{L}}{\Delta t}}$$

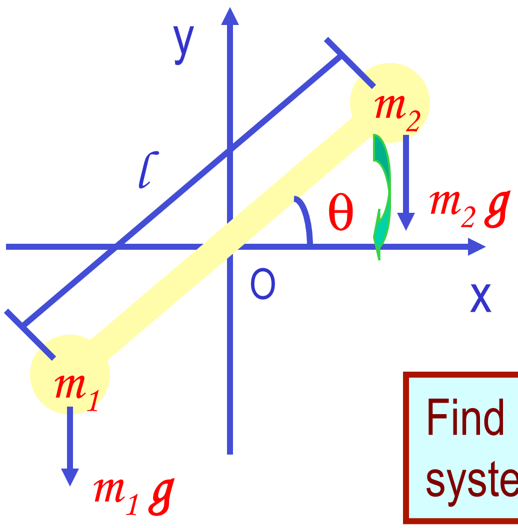
Just like $\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$

For a rigid body, the external torque is written

$$\boxed{\sum \tau_{ext} = \frac{\Delta L_z}{\Delta t} = \frac{\Delta(I\varpi)}{\Delta t} = \frac{I\Delta\varpi}{\Delta t} = I\alpha}$$

Example for Rigid Body Angular Momentum

A rigid rod of mass M and length l is pivoted without friction at its center. Two particles of mass m_1 and m_2 are attached to either end of the rod. The combination rotates on a vertical plane with an angular speed of w. Find an expression for the magnitude of the angular momentum.



The moment of inertia of this system is

$$I = I_{rod} + I_{m_1} + I_{m_2} = \frac{1}{12}Ml^2 + \frac{1}{4}m_1 l^2 + \frac{1}{4}m_2 l^2$$

$$= \frac{l^2}{4}\left(\frac{1}{3}M + m_1 + m_2\right) \qquad \boxed{L = I\omega = \frac{\omega l^2}{4}\left(\frac{1}{3}M + m_1 + m_2\right)}$$

Find an expression for the magnitude of the angular acceleration of the system when the rod makes an angle θ with the horizon.

If $m_1 = m_2$, no angular momentum because the net torque is 0.

If θ=+/−π/2, at equilibrium so no angular momentum.

First compute the net external torque

$$\tau_1 = m_1 g \frac{l}{2}\cos\theta \qquad \tau_2 = -m_2 g \frac{l}{2}\cos\theta$$

$$\tau_{ext} = \tau_1 + \tau_2 = \frac{gl\cos\theta(m_1 - m_2)}{2}$$

Thus α becomes

$$\alpha = \frac{\sum \tau_{ext}}{I} = \frac{\frac{1}{2}(m_1 - m_2)gl\cos\theta}{\frac{l^2}{4}\left(\frac{1}{3}M + m_1 + m_2\right)} = \frac{2(m_1 - m_2)\cos\theta}{\left(\frac{1}{3}M + m_1 + m_2\right)}\frac{g}{l}$$

Conservation of Angular Momentum

Remember under what condition the linear momentum is conserved?

Linear momentum is conserved when the net external force is 0.

$$\sum \vec{F} = 0 = \frac{\Delta \vec{p}}{\Delta t}$$

$$\vec{p} = const$$

By the same token, the angular momentum of a system is constant in both magnitude and direction, if the resultant external torque acting on the system is 0.

$$\sum \vec{\tau}_{ext} = \frac{\Delta \vec{L}}{\Delta t} = 0$$

$$\boxed{\vec{L} = const}$$

What does this mean?

Angular momentum of the system before and after a certain change is the same.

$$\vec{L}_i = \vec{L}_f = \text{constant}$$

Three important conservation laws for isolated system that does not get affected by external forces

$$K_i + U_i = K_f + U_f$$ **Mechanical Energy**

$$\vec{p}_i = \vec{p}_f$$ **Linear Momentum**

$$\vec{L}_i = \vec{L}_f$$ **Angular Momentum**

Example for Angular Momentum Conservation

A star rotates with a period of 30 days about an axis through its center. After the star undergoes a supernova explosion, the stellar core, which had a radius of 1.0×10^4km, collapses into a neutron star of radius 3.0km. Determine the period of rotation of the neutron star.

What is your guess about the answer?

The period will be significantly shorter, because its radius got smaller.

Let's make some assumptions:

1. There is no external torque acting on it
2. The shape remains spherical
3. Its mass remains constant

Using angular momentum conservation

$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

The angular speed of the star with the period T is

$$\omega = \frac{2\pi}{T}$$

Thus

$$\omega_f = \frac{I_i \omega_i}{I_f} = \frac{m r_i^2}{m r_f^2} \frac{2\pi}{T_i}$$

$$T_f = \frac{2\pi}{\omega_f} = \left(\frac{r_f^2}{r_i^2} \right) T_i = \left(\frac{3.0}{1.0 \times 10^4} \right)^2 \times 30 days = 2.7 \times 10^{-6} days = 0.23 s$$

Similarity Between Linear and Rotational Motions

All physical quantities in linear and rotational motions show striking similarity.

Quantities	Linear	Rotational
Mass	Mass M	Moment of Inertia $I = mr^2$
Length of motion	Distance L	Angle θ (Radian)
Speed	$v = \frac{\Delta r}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$
Force	Force $\overrightarrow{F} = m\overrightarrow{a}$	Torque $\overrightarrow{\tau} = I\overrightarrow{\alpha}$
Work	Work $W = \vec{F} \cdot \vec{d}$	Work $W = \tau\theta$
Power	$P = \overrightarrow{F} \cdot \vec{v}$	$P = \tau\omega$
Momentum	$\overrightarrow{p} = m\overrightarrow{v}$	$\overrightarrow{L} = I\overrightarrow{\omega}$
Kinetic Energy	Kinetic $K = \frac{1}{2}mv^2$	Rotational $K_R = \frac{1}{2}I\omega^2$