PHYS 1441 – Section 002
Lecture #23

Monday, Dec. 6, 2010

Dr. Jaehoon Yu

- Similarities Between Linear and Rotational Quantities
- Conditions for Equilibrium
- How to Solve Equilibrium Problems?
- A Few Examples of Mechanical Equilibrium
- Elastic Properties of Solids
- Density and Specific Gravity

Announcements

- The Final Exam
  - Date and time: 11am – 1:30pm, Monday Dec. 13
  - Place: SH103
  - Comprehensive exam
    - Covers from CH1.1 – what we finish Wednesday, Dec. 8
    - Plus appendices A.1 – A.8
    - Combination of multiple choice and free response problems
    - Bring your Planetarium extra credit sheet to the class next Wednesday, Dec. 8, with your name clearly marked on the sheet!
- Reading assignments
  - Ch9.3 – 9.7

Similarity Between Linear and Rotational Motions

All physical quantities in linear and rotational motions show striking similarity.

Quantities	Linear	Rotational
Mass	Mass M	Moment of Inertia I
Length of motion	Distance L	Angle θ (Radian)
Speed	$v = \frac{\Delta r}{\Delta t}$	$\omega = \frac{\Delta \theta}{\Delta t}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta \omega}{\Delta t}$
Force	Force $\vec{F} = m\vec{a}$	Torque $\vec{\tau} = I\vec{\alpha}$
Work	Work $W = \vec{F} \cdot \vec{d}$	Work $W = \tau\theta$
Power	$P = \vec{F} \cdot \vec{v}$	$P = \tau\omega$
Momentum	$\vec{p} = m\vec{v}$	$\vec{L} = I\vec{\omega}$
Kinetic Energy	Kinetic $K = \frac{1}{2}mv^2$	Rotational $K_R = \frac{1}{2}I\omega^2$

Conditions for Equilibrium

What do you think the term "An object is at its equilibrium" means?

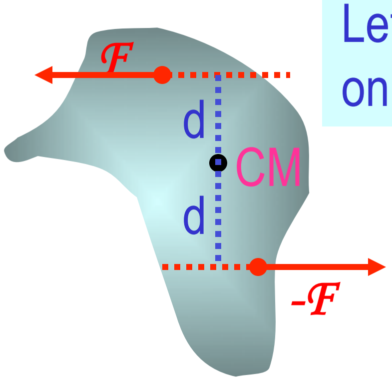
The object is either at rest (<u>Static Equilibrium</u>) or its center of mass is moving at a constant velocity (<u>Dynamic Equilibrium</u>).

When do you think an object is at its equilibrium?

Translational Equilibrium: Equilibrium in linear motion $\sum \vec{F} = 0$

Is this it?

The above condition is sufficient for a point-like object to be at its translational equilibrium. However for an object with size this is not sufficient. One more condition is needed. What is it?



Let's consider two forces equal in magnitude but in opposite direction acting on a rigid object as shown in the figure. What do you think will happen?

The object will rotate about the CM. Thus the net torque acting on the object about any axis must be 0. $\sum \vec{\tau} = 0$

For an object to be at its *static equilibrium*, the object should not have linear or angular speed. $v_{CM} = 0 \quad \omega = 0$

More on Conditions for Equilibrium

To simplify the problem, we will only deal with forces acting on x-y plane, giving torque only along z-axis. What do you think the conditions for equilibrium be in this case?

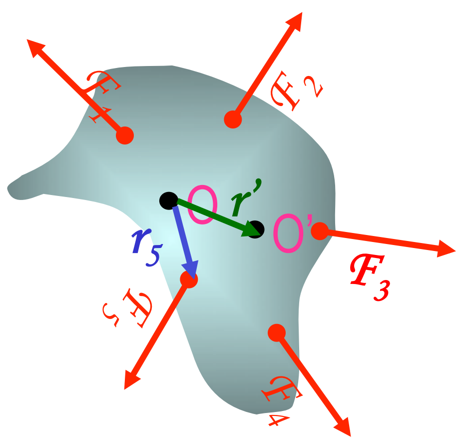
The six possible equations from the two vector equations turns to three equations.

$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0, \quad \sum F_y = 0$$

AND

$$\sum \vec{\tau} = 0 \Rightarrow \sum \tau_z = 0$$

What happens if there are many forces exerting on an object?



If an object is at its translational static equilibrium, and if the net torque acting on the object is 0 about one axis, the net torque must be 0 about any arbitrary axis.

Why is this true?

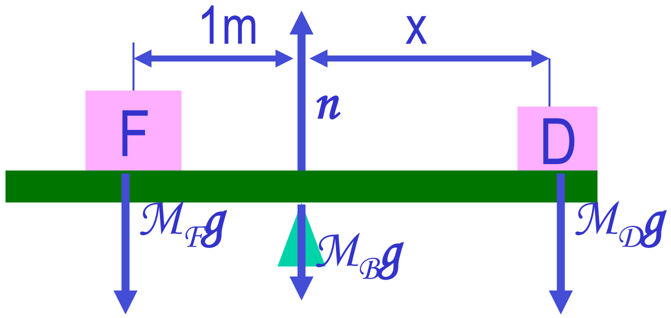
Because the object is *not moving*, no matter what the rotational axis is, there should not be any motion. It is simply a matter of mathematical manipulation.

How do we solve static equilibrium problems?

1. Select the object to which the equations for equilibrium are to be applied.
2. Identify all the forces and draw a free-body diagram with them indicated on it with their directions and locations properly indicated
3. Choose a convenient set of x and y axes and write down force equation for each x and y component with correct signs.
4. Apply the equations that specify the balance of forces at equilibrium. Set the net force in the x and y directions equal to 0.
5. Select the most optimal rotational axis for torque calculations ➔ Selecting the axis such that the torque of one of the unknown forces become 0 makes the problem easier to solve.
6. Write down the torque equation with proper signs.
7. Solve the force and torque equations for the desired unknown quantities.

Example for Mechanical Equilibrium

A uniform 40.0 N board supports the father and the daughter each weighing 800 N and 350 N, respectively, and is not moving. If the support (or fulcrum) is under the center of gravity of the board, and the father is 1.00 m from CoG, what is the magnitude of the normal force n exerted on the board by the support?



Since there is no linear motion, this system is in its translational equilibrium

$$\sum F_x = 0$$

$$\sum F_y = n - M_B g - M_F g - M_D g = 0$$

Therefore the magnitude of the normal force $n = 40.0 + 800 + 350 = 1190\,N$

Determine where the child should sit to balance the system.

The net torque about the fulcrum by the three forces are

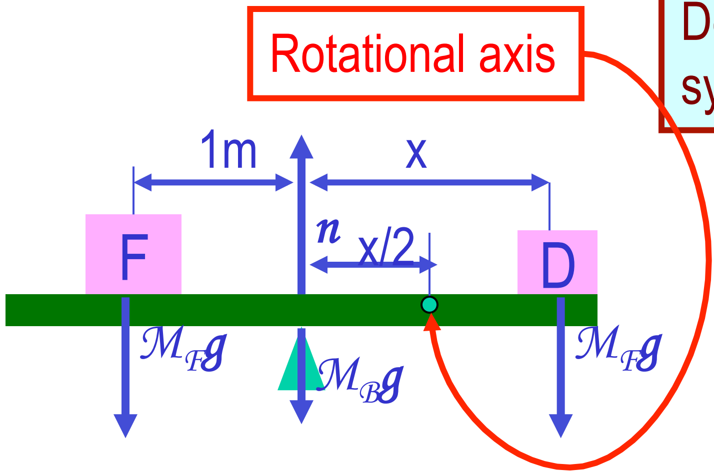
$$\tau = M_B g \cdot 0 + n \cdot 0 + M_F g \cdot 1.00 - M_D g \cdot x = 0$$

Therefore to balance the system the daughter must sit

$$x = \frac{M_F g}{M_D g} \cdot 1.00m = \frac{800}{350} \cdot 1.00m = 2.29m$$

Example for Mech. Equilibrium Cont'd

Determine the position of the child to balance the system for different position of axis of rotation.



The net torque about the axis of rotation by all the forces are

$$\tau = M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2) - n \cdot x/2 - M_D g \cdot x/2 = 0$$

Since the normal force is $n = M_B g + M_F g + M_D g$

The net torque can be rewritten

$$\begin{aligned}\tau &= M_B g \cdot x/2 + M_F g \cdot (1.00 + x/2) \\ &\quad - (M_B g + M_F g + M_D g) \cdot x/2 - M_D g \cdot x/2 \\ &= M_F g \cdot 1.00 - M_D g \cdot x = 0\end{aligned}$$

Therefore

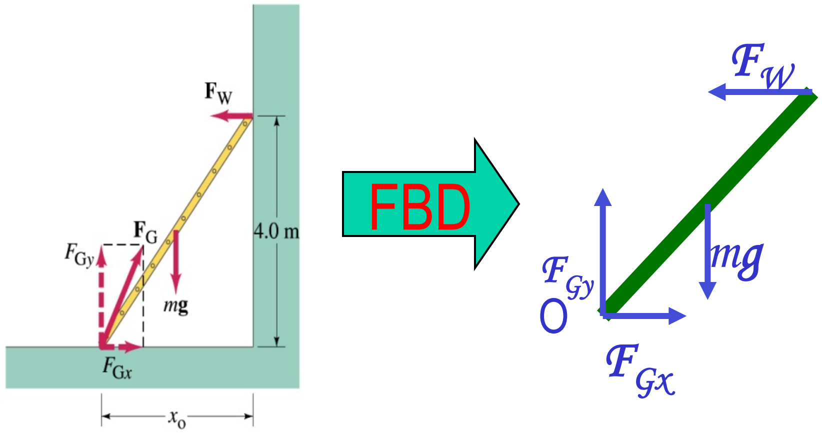
$$x = \frac{M_F g}{M_D g} \cdot 1.00m = \frac{800}{350} \cdot 1.00m = 2.29m$$

What do we learn?

No matter where the rotation axis is, net effect of the torque is identical.

Example 9 – 7

A 5.0 m long ladder leans against a wall at a point 4.0m above the ground. The ladder is uniform and has mass 12.0kg. Assuming the wall is frictionless (but ground is not), determine the forces exerted on the ladder by the ground and the wall.



First the translational equilibrium, using components

$$\sum F_x = F_{Gx} - F_W = 0$$

$$\sum F_y = -mg + F_{Gy} = 0$$

Thus, the y component of the force by the ground is

$$F_{Gy} = mg = 12.0 \times 9.8N = 118N$$

The length x_0 is, from Pythagorian theorem

$$x_0 = \sqrt{5.0^2 - 4.0^2} = 3.0m$$

Example 9 – 7 cont'd

From the rotational equilibrium $\sum \tau_O = -mg\, x_0/2 + F_W 4.0 \quad = 0$

Thus the force exerted on the ladder by the wall is

$$F_W = \frac{mg\, x_0/2}{4.0} = \frac{118 \cdot 1.5}{4.0} = 44N$$

The x component of the force by the ground is

$$\sum F_x = F_{Gx} - F_W = 0$$

Solve for F_{Gx}

$$F_{Gx} = F_W = 44N$$

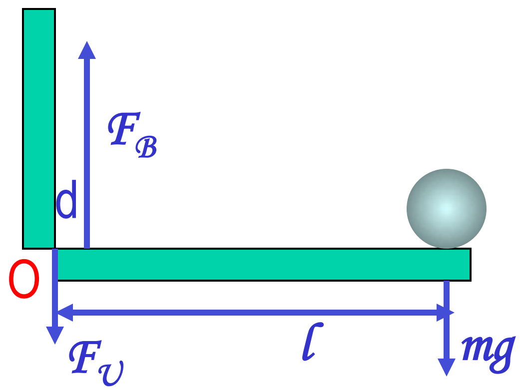
Thus the force exerted on the ladder by the ground is

$$F_G = \sqrt{F_{Gx}^2 + F_{Gy}^2} = \sqrt{44^2 + 118^2} \approx 130N$$

The angle between the ground force to the floor $\theta = \tan^{-1}\left(\frac{F_{Gy}}{F_{Gx}}\right) = \tan^{-1}\left(\frac{118}{44}\right) = 70^\circ$

Ex. 9.8 for Mechanical Equilibrium

A person holds a 50.0N sphere in his hand. The forearm is horizontal. The biceps muscle is attached 3.00 cm from the joint, and the sphere is 35.0cm from the joint. Find the upward force exerted by the biceps on the forearm and the downward force exerted by the upper arm on the forearm and acting at the joint. Neglect the weight of forearm.



Since the system is in equilibrium, from the translational equilibrium condition

$$\sum F_x = 0$$

$$\sum F_y = F_B - F_U - mg = 0$$

From the rotational equilibrium condition $\sum \tau = F_U \cdot 0 + F_B \cdot d - mg \cdot l = 0$

Thus, the force exerted by the biceps muscle is

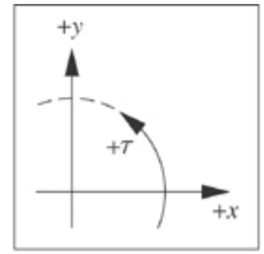
$$F_B \cdot d = mg \cdot l$$

$$F_B = \frac{mg \cdot l}{d} = \frac{50.0 \times 35.0}{3.00} = 583N$$

Force exerted by the upper arm is $F_U = F_B - mg = 583 - 50.0 = 533N$

Ex. A Diving Board

A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m. The board has negligible weight and is supported by a fulcrum 1.40 m away from the left end. Find the forces that the bolt and the fulcrum exert on the board.



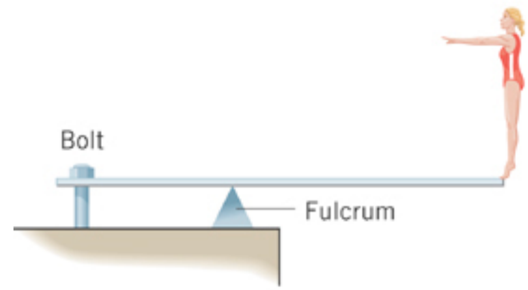
First the torque eq. $\sum \tau = F_2 \ell_2 - W \ell_W = 0$

How large is the torque by the bolt? None Why?

Because the lever arm is 0.

So the force by the fulcrum is $F_2 = \dfrac{W \ell_W}{\ell_2}$

$$F_2 = \frac{(530\text{ N})(3.90\text{ m})}{1.40\text{ m}} = 1480\text{ N}$$

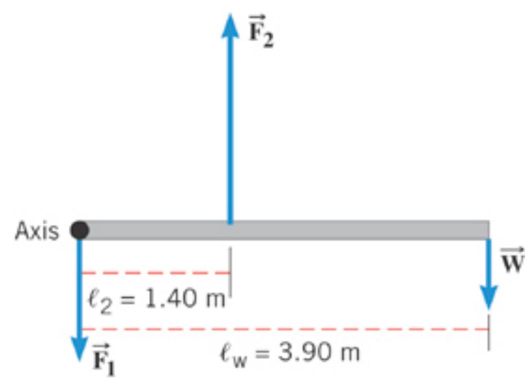


(a)

Now the force eq. $\sum F_y = -F_1 + F_2 - W = 0$

$$-F_1 + 1480\text{ N} - 530\text{ N} = 0$$

So the force by the bolt is $F_1 = 950\text{ N}$



(b) Free-body diagram of the diving board