PHYS 1441 – Section 002
Lecture #24

Wednesday, Dec. 8, 2010

Dr. Jaehoon Yu

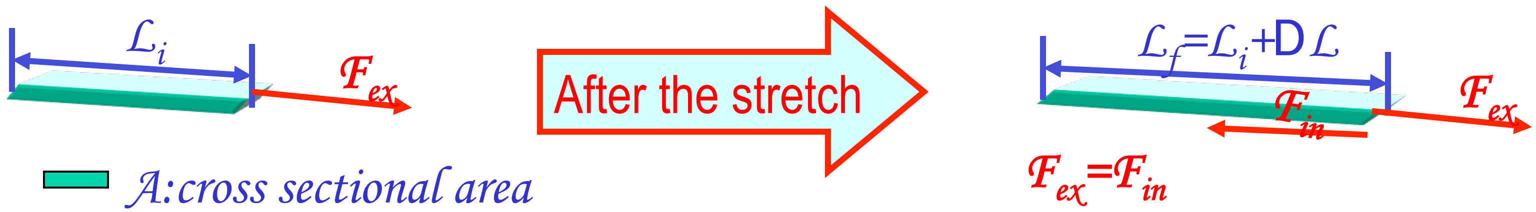
- Elastic Properties of Solids
- Density and Specific Gravity
- Fluid and Pressure
- Depth Dependence of Pressure
- Absolute and Relative Pressure
- Pascal's Principle and Hydraulics
- Buoyant Forces and Archimedes' Principle

Announcements

- The Final Exam
  - Date and time: 11am – 1:30pm, Monday Dec. 13
  - Place: SH103
  - Comprehensive exam
    - Covers from CH1.1 – CH10.7
    - Plus appendices A.1 – A.8
    - Combination of multiple choice and free response problems
    - Submit your Planetarium extra credit sheet

Young's Modulus

Let's consider a long bar with cross sectional area A and initial length L_i.



Tensile stress $\text{Tensile Stress} \equiv \frac{F_{ex}}{A}$ Tensile strain $\text{Tensile Strain} \equiv \frac{\Delta L}{L_i}$

Young's Modulus is defined as

$$Y \equiv \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{F_{ex}/A}{\Delta L/L_i}$$

Used to characterize a rod or wire stressed under tension or compression

What is the unit of Young's Modulus?

Force per unit area

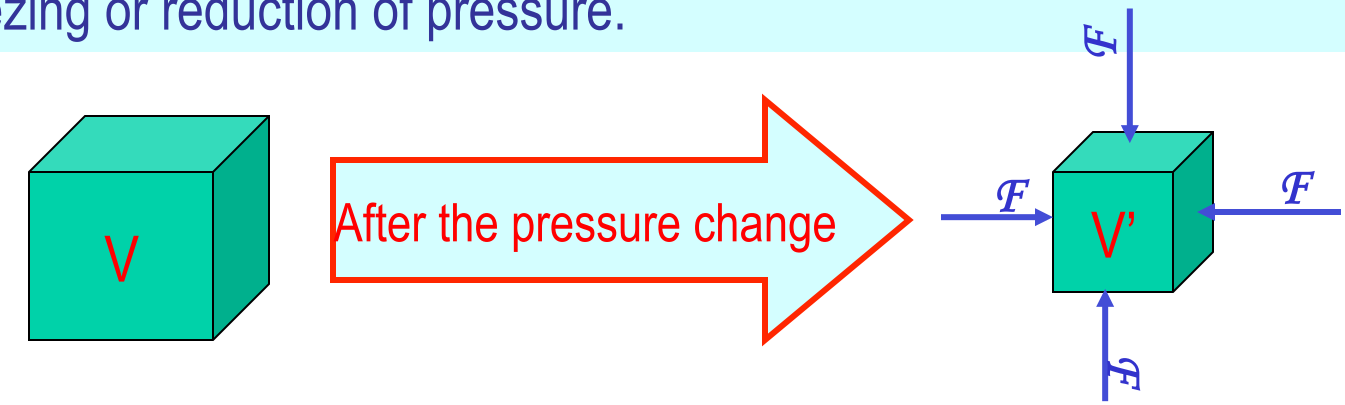
Experimental Observations

1. For a fixed external force, the change in length is proportional to the original length
2. The necessary force to produce the given strain is proportional to the cross sectional area

Elastic limit: Maximum stress that can be applied to the substance before it becomes permanently deformed

Bulk Modulus

Bulk Modulus characterizes the response of a substance to uniform squeezing or reduction of pressure.



Volume stress =pressure

$$\text{Pressure} \equiv \frac{\text{Normal Force}}{\text{Surface Area the force applies}} = \frac{F}{A}$$

If the pressure on an object changes by ΔP=ΔF/A, the object will undergo a volume change ΔV.

Bulk Modulus is defined as

$$B \equiv \frac{\text{Volume Stress}}{\text{Volume Strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V_i}$$

Because the change of volume is reverse to change of pressure.

Compressibility is the reciprocal of Bulk Modulus

Example for Solid's Elastic Property

A solid brass sphere is initially under normal atmospheric pressure of $1.0 \times 10^5 N/m^2$. The sphere is lowered into the ocean to a depth at which the pressures is $2.0 \times 10^7 N/m^2$. The volume of the sphere in air is $0.5m^3$. By how much its volume change once the sphere is submerged? The bulk modulus of brass is 6.1×10^{10} N/m^2

Since bulk modulus is $$\mathrm{B} = -\frac{\Delta P}{\Delta V / V_i}$$

The amount of volume change is $$\Delta V = -\frac{\Delta P V_i}{\mathrm{B}}$$

The pressure change ΔP is $$\Delta P = P_f - P_i = 2.0 \times 10^7 - 1.0 \times 10^5 \approx 2.0 \times 10^7$$

Therefore the resulting volume change ΔV is $$\Delta V = V_f - V_i = -\frac{2.0 \times 10^7 \times 0.5}{6.1 \times 10^{10}} = -1.6 \times 10^{-4} m^3$$

The volume has decreased.

Density and Specific Gravity

Density, ρ(rho), of an object is defined as mass per unit volume

$$\rho \equiv \frac{M}{V}$$

Unit? kg/m^3

Dimension? $[ML^{-3}]$

Specific Gravity of a substance is defined as the ratio of the density of the substance to that of water at 4.0 ^{o}C (ρ_{H2O}=1.00g/cm^3).

$$SG \equiv \frac{\rho_{\text{substance}}}{\rho_{H_2O}}$$

Unit? None

Dimension? None

What do you think would happen of a substance in the water dependent on SG?

$SG > 1$ Sink in the water

$SG < 1$ Float on the surface

Fluid and Pressure

What are the three states of matter?

Solid, Liquid and Gas

How do you distinguish them?

Using the time it takes for the particular substance to change its shape in reaction to external forces.

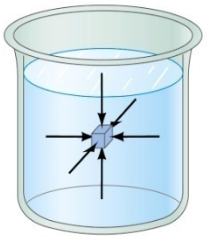
What is a fluid?

A collection of molecules that are **randomly arranged** and **loosely bound** by forces between them or by an external container.

We will first learn about mechanics of fluid at rest, *fluid statics.*

In what ways do you think fluid exerts stress on the object submerged in it?

Fluid cannot exert shearing or tensile stress. Thus, the only force the fluid exerts on an object immersed in it is the force perpendicular to the surface of the object.



This force by the fluid on an object usually is expressed in the form of the force per unit area at the given depth, the pressure, defined as

$$P \equiv \frac{F}{A}$$

Expression of pressure for an infinitesimal area δA by the force δF is

$$P = \frac{\delta F}{\delta A}$$

Note that pressure is a scalar quantity because it's the magnitude of the force on a surface area A.

What is the unit and the dimension of pressure?

Unit: N/m^2

Dim.: $[M][L^{-1}][T^{-2}]$

Special SI unit for pressure is Pascal

$$1Pa \equiv 1N/m^2$$

Example for Pressure

The mattress of a water bed is 2.00m long by 2.00m wide and 30.0cm deep. a) Find the weight of the water in the mattress.

The volume density of water at the normal condition (0^oC and 1 atm) is $1000kg/m^3$. So the total mass of the water in the mattress is

$$m = \rho_W V_M = 1000\times2.00\times2.00\times0.300 = 1.20\times10^3\,kg$$

Therefore the weight of the water in the mattress is

$$W = mg = 1.20\times10^3\times9.8 = 1.18\times10^4\,N$$

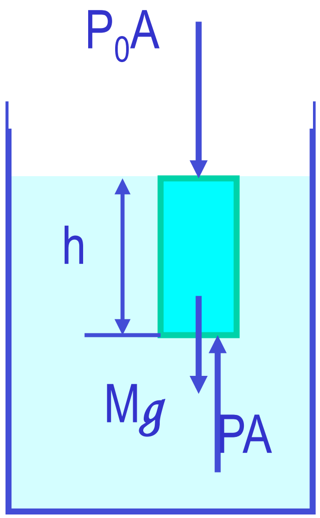
b) Find the pressure exerted by the water on the floor when the bed rests in its normal position, assuming the entire lower surface of the mattress makes contact with the floor.

Since the surface area of the mattress is 4.00 m^2, the pressure exerted on the floor is

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{1.18\times10^4}{4.00} = 2.95\times10^3$$

Variation of Pressure and Depth

Water pressure increases as a function of depth, and the air pressure decreases as a function of altitude. Why?



It seems that the pressure has a lot to do with the total mass of the fluid above the object that puts weight on the object.

Let's imagine the liquid contained in a cylinder with height *h* and the cross sectional area *A* immersed in a fluid of density ρ at rest, as shown in the figure, and the system is in its equilibrium.

If the liquid in the cylinder is the same substance as the fluid, the mass of the liquid in the cylinder is $M = \rho V = \rho A h$

Since the system is in its equilibrium $PA - P_0A - Mg = PA - P_0A - \rho Ahg = 0$

Therefore, we obtain $P = P_0 + \rho g h$

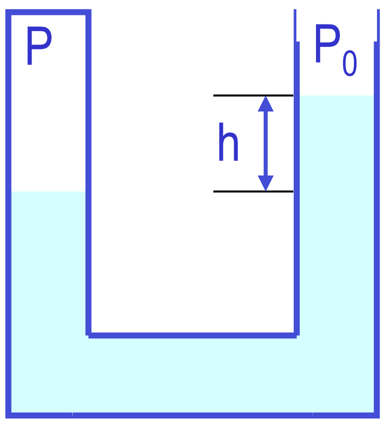
The pressure at the depth *h* below the surface of the fluid open to the atmosphere is greater than the atmospheric pressure by ρ*gh*.

Atmospheric pressure P_0 is

$$1.00 atm = 1.013 \times 10^5 Pa$$

Absolute and Relative Pressure

How can one measure pressure?



One can measure the pressure using an open-tube manometer, where one end is connected to the system with unknown pressure P and the other open to air with pressure P_0.

The measured pressure of the system is $$P = P_0 + \rho g h$$

This is called the **absolute pressure**, because it is the actual value of the system's pressure.

In many cases we measure the pressure difference with respect to the atmospheric pressure to avoid the effect of the changes in P_0 that depends on the environment. This is called **gauge or relative pressure**.

$$P_G = P - P_0 = \rho g h$$

The common barometer which consists of a mercury column with one end closed at vacuum and the other open to the atmosphere was invented by Evangelista Torricelli.

Since the closed end is at vacuum, it does not exert any force. 1 atm of air pressure pushes mercury up 76cm. So 1 atm is

$$P_0 = \rho g h = (13.595\times10^3 kg/m^3)(9.80665 m/s^2)(0.7600 m)$$
$$= 1.013\times10^5 Pa = 1 atm$$

If one measures the tire pressure with a gauge at 220kPa the actual pressure is 101kPa+220kPa=303kPa.

Pascal's Principle and Hydraulics

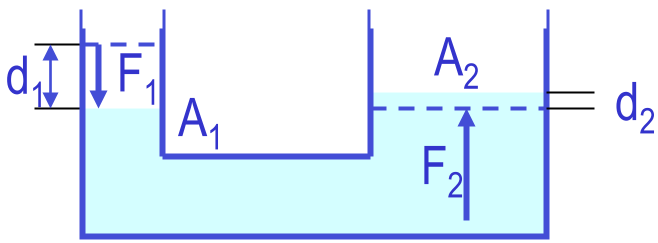
A change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.

$$P = P_0 + \rho g h$$

What happens if P_0 is changed?

The resultant pressure P at any given depth h increases as much as the change in P_0.

This is the principle behind hydraulic pressure. How?



Since the pressure change caused by the the force F_1 applied onto the area A_1 is transmitted to the F_2 on an area A_2.

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Therefore, the resultant force F_2 is

$$F_2 = \frac{A_2}{A_1} F_1$$

In other words, the force gets multiplied by the ratio of the areas A_2/A_1 and is transmitted to the force F_2 on the surface.

This seems to violate some kind of conservation law, doesn't it?

No, the actual displaced volume of the fluid is the same. And the work done by the forces are still the same.

$$F_2 = \frac{d_1}{d_2} F_1$$

Example for Pascal's Principle

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of 5.00cm. This pressure is transmitted by a liquid to a piston that has a radius of 15.0cm. What force must the compressed air exert to lift a car weighing 13,300N? What air pressure produces this force?

Using the Pascal's principle, one can deduce the relationship between the forces, the force exerted by the compressed air is

$$F_1 = \frac{A_1}{A_2} F_2 \quad = \frac{\pi (0.05)^2}{\pi (0.15)^2} \times 1.33 \times 10^4 = 1.48 \times 10^3\, N$$

Therefore the necessary pressure of the compressed air is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3}{\pi (0.05)^2} = 1.88 \times 10^5\, Pa$$

Example for Pascal's Principle

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of the pool with a depth 5.0 m.

We first need to find out the pressure difference that is being exerted on the eardrum. Then estimate the area of the eardrum to find out the force exerted on the eardrum.

Since the outward pressure in the middle of the eardrum is the same as normal air pressure

$$P - P_0 = \rho_W g h = 1000 \times 9.8 \times 5.0 = 4.9 \times 10^4 \, Pa$$

Estimating the surface area of the eardrum at $1.0cm^2 = 1.0x10^{-4}$ m^2, we obtain

$$F = (P - P_0)A \approx 4.9 \times 10^4 \times 1.0 \times 10^{-4} \approx 4.9N$$