

PHYS 1444 – Section 003

Lecture #6

Thursday, Sept. 8, 2011

*Dr. **Jaehoon** Yu*

- Chapter 21
 - Electric Dipoles
- Chapter 22 Gauss's Law
 - Electric Flux



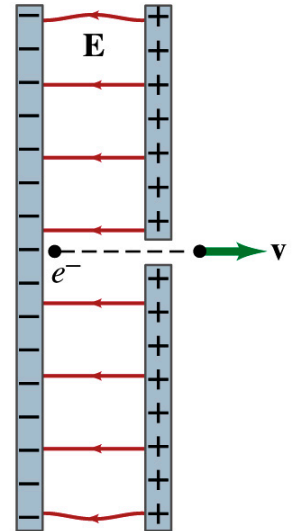
Announcements

- Quiz #2
 - Thursday, Sept. 15
 - Beginning of the class
 - Covers: CH21.1 through what we learn on Tuesday, Sept. 13
- Reading assignments
 - CH21.12 and CH21.13



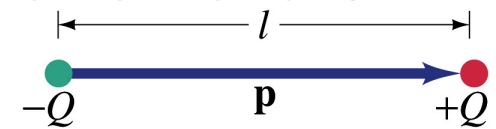
Special Project

- **Particle Accelerator.** A charged particle of mass M with charge $-Q$ is accelerated in the uniform field E between two parallel charged plates whose separation is D as shown in the figure on the right. The charged particle is accelerated from an initial speed v_0 near the negative plate and passes through a tiny hole in the positive plate.
 - Derive the formula for the electric field E to accelerate the charged particle to fraction f of the speed of light c . Express E in terms of M , Q , D , f , c and v_0 .
 - (a) Using the Coulomb force and kinematic equations. (8 points)
 - (b) Using the work-kinetic energy theorem. (8 points)
 - © Using the formula above, evaluate the strength of the electric field E to accelerate an electron from 0.1% of the speed of light to 90% of the speed of light. You need to look up the relevant constants, such as mass of the electron, charge of the electron and the speed of light. (5 points)
- Due beginning of the class Tuesday, Sept. 13

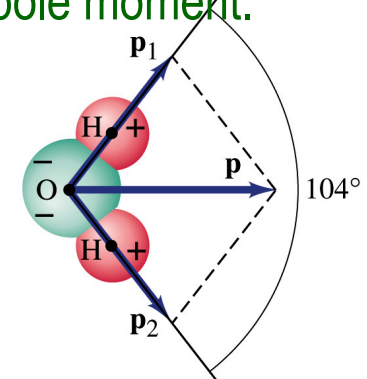


Electric Dipoles

- An electric dipole is the combination of two equal charges of opposite signs, $+Q$ and $-Q$, separated by a distance l , which behaves as one entity.
- The quantity Ql is called the electric dipole moment and is represented by the symbol p .



- The dipole moment is a vector quantity, p
- The magnitude of the dipole moment is Ql Unit? **C-m**
- Its direction is from the negative to the positive charge.
- Many of diatomic molecules like CO have a dipole moment. →
These are referred as polar molecules.
 - Even if the molecule is electrically neutral, their sharing of electron causes separation of charges
 - Symmetric diatomic molecules, such as O_2 , do not have dipole moment.
- The water molecule also has a dipole moment which is the vector sum of two dipole moments between Oxygen and each of Hydrogen atoms.



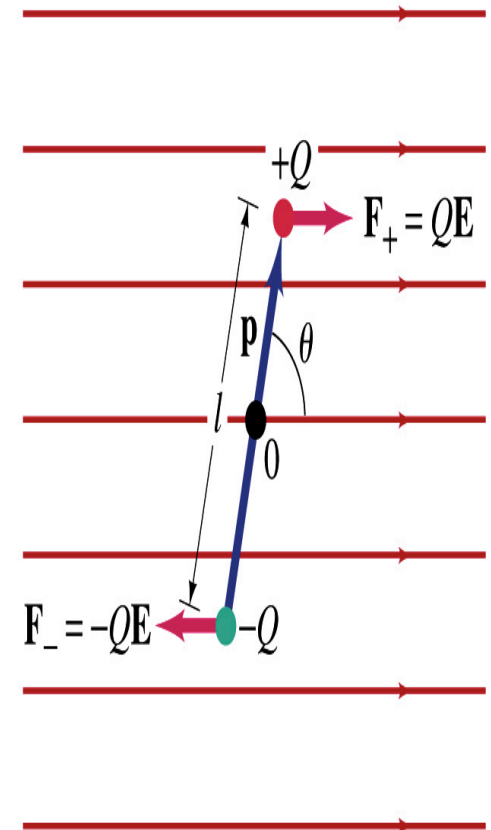
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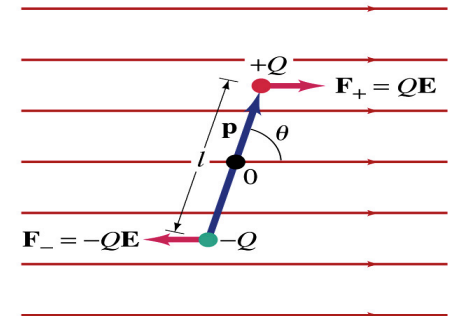
Dipoles in an External Field

- Let's consider a dipole placed in a uniform electric field \mathbf{E} .
- What do you think will happen to the dipole in the figure?
 - Forces will be exerted on the charges.
 - The positive charge will get pushed toward right while the negative charge will get pulled toward left.
 - What is the net force acting on the dipole?
 - Zero
 - So will the dipole not move?
 - Yes, it will.
 - Why?
 - There is torque applied on the dipole.



Dipoles in an External Field, cnt'd

- How much is the torque on the dipole?
 - Do you remember the formula for torque?
 - $\vec{\tau} = \vec{r} \times \vec{F}$
 - The magnitude of the torque exerting on each of the charges with respect to the rotational axis at the center is
 - $\tau_{+Q} = |\vec{r} \times \vec{F}| = rF \sin \theta = \left| \left(\frac{l}{2} \right) (QE) \sin \theta \right| = \frac{l}{2} QE \sin \theta$
 - $\tau_{-Q} = |\vec{r} \times \vec{F}| = rF \sin \theta = \left| \left(-\frac{l}{2} \right) (-QE) \sin \theta \right| = \frac{l}{2} QE \sin \theta$
 - Thus, the total torque is
 - $\tau_{Total} = \tau_{+Q} + \tau_{-Q} = \frac{l}{2} QE \sin \theta + \frac{l}{2} QE \sin \theta = lQE \sin \theta = pE \sin \theta$
 - So the torque on a dipole in vector notation is $\vec{\tau} = \vec{p} \times \vec{E}$
- The effect of the torque is to try to turn the dipole so that the dipole moment is parallel to \vec{E} . Which direction?



Potential Energy of a Dipole in an External Field

- What is the work done on the dipole by the electric field to change the angle from θ_1 to θ_2 ?

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} \vec{\tau} \cdot d\vec{\theta} = \int_{\theta_1}^{\theta_2} -\tau d\theta$$

Why negative?

Because τ and θ are opposite directions to each other.

- The torque is $\tau = pE \sin \theta$.

- Thus the work done on the dipole by the field is

$$W = \int_{\theta_1}^{\theta_2} -pE \sin \theta d\theta = pE [\cos \theta]_{\theta_1}^{\theta_2} = pE (\cos \theta_2 - \cos \theta_1)$$

- What happens to the dipole's potential energy, U , when a positive work is done on it by the field?

– It decreases.

- We choose $U=0$ when $\theta_1=90$ degrees, then the potential energy at $\theta_2=\theta$ becomes $U = -W = -pE \cos \theta = -\vec{p} \cdot \vec{E}$



Electric Field by a Dipole

- Let's consider the case in the picture.
- There are fields by both the charges. So the total electric field by the dipole is $\vec{E}_{Tot} = \vec{E}_{+Q} + \vec{E}_{-Q}$
- The magnitudes of the two fields are equal

$$E_{+Q} = E_{-Q} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\left(\sqrt{r^2 + (l/2)^2}\right)^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2 + (l/2)^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2 + l^2/4}$$

- Now we must work out the x and y components of the total field.

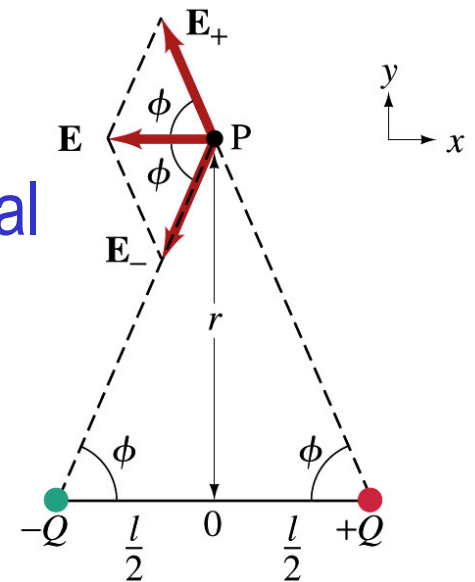
- Sum of the two y components is
 - Zero since they are the same but in opposite direction
- So the magnitude of the total field is the same as the sum of the two x-components:

$$E = 2E_+ \cos \phi = \frac{1}{2\pi\epsilon_0} \frac{Q}{r^2 + l^2/4} \frac{l}{2\sqrt{r^2 + l^2/4}} = \frac{1}{4\pi\epsilon_0} \frac{p}{\left(r^2 + l^2/4\right)^{3/2}}$$

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Dipole Electric Field from Afar

- What happens when $r \gg l$?

•

$$E_D = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2 + l^2/4)^{3/2}} \approx \frac{1}{4\pi\epsilon_0} \frac{p}{r^3} \quad (\text{when } r \gg l)$$

- Why does this make sense?
 - Since from a far distance, the two charges are very close so that the overall charge gets close to 0!!
 - This dependence works for the point not on bisector as well



Example 21 – 17

- **Dipole in a field.** The dipole moment of a water molecule is $6.1 \times 10^{-30} \text{C}\cdot\text{m}$. A water molecule is placed in a uniform electric field with magnitude $2.0 \times 10^5 \text{N/C}$. (a) What is the magnitude of the maximum torque that the field can exert on the molecule? (b) What is the potential energy when the torque is at its maximum? (c) In what position will the potential energy take on its greatest value? Why is this different than the position where the torque is maximized?

(a) The torque is maximized when $\theta = 90$ degrees. Thus the magnitude of the maximum torque is

$$\begin{aligned}\tau &= pE \sin \theta = pE = \\ &= (6.1 \times 10^{-30} \text{C} \cdot \text{m})(2.5 \times 10^5 \text{N/C}) = 1.2 \times 10^{-24} \text{N} \cdot \text{m}\end{aligned}$$



Example 21 – 17

(b) What is the potential energy when the torque is at its maximum?

Since the dipole potential energy is $U = -\vec{p} \cdot \vec{E} = -pE \cos \theta$

And τ is at its maximum at $\theta=90$ degrees, the potential energy, U , is

$$U = -pE \cos \theta = -pE \cos (90^\circ) = 0$$

Is the potential energy at its minimum at $\theta=90$ degrees? **No**

Why not? **Because U will become negative as θ increases.**

(c) In what position will the potential energy take on its greatest value?

The potential energy is maximum when $\cos \theta = -1$, $\theta = 180$ degrees.

Why is this different than the position where the torque is maximized?

The potential energy is maximized when the dipole is oriented so that it has to rotate through the largest angle against the direction of the field, to reach the equilibrium position at $\theta=0$.

Torque is maximized when the field is perpendicular to the dipole, $\theta=90$.

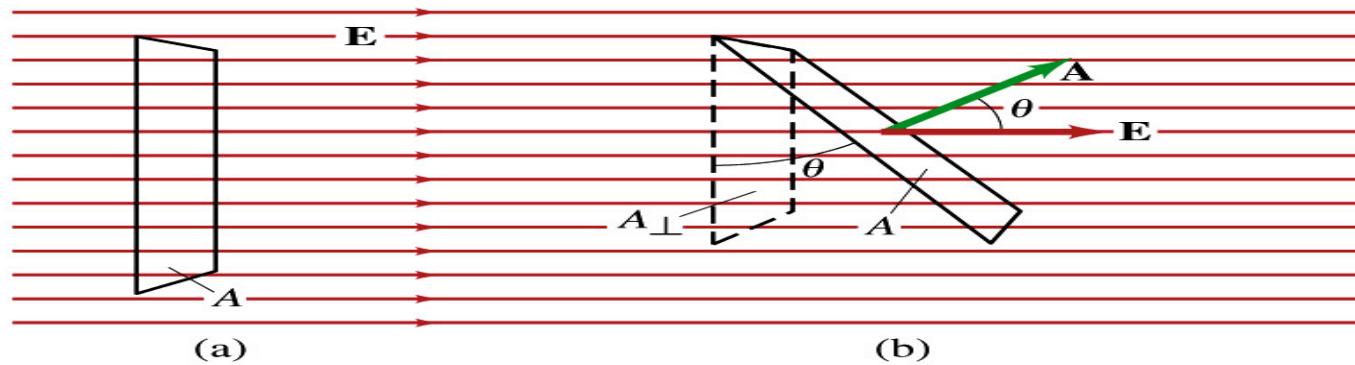


Gauss' Law

- Gauss' law states the relationship between electric charge and electric field.
 - More generalized and elegant form of Coulomb's law.
- The electric field by the distribution of charges can be obtained using Coulomb's law by summing (or integrating) over the charge distributions.
- Gauss' law, however, gives an additional insight into the nature of electrostatic field and a more general relationship between the charge and the field



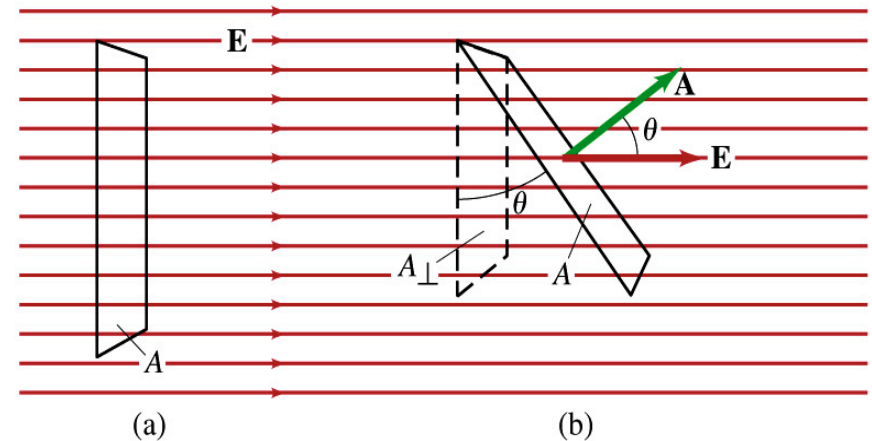
Electric Flux



- Let's imagine a surface of area A through which a uniform electric field E passes
- The electric flux Φ_E is defined as
 - $\Phi_E = EA$, if the field is perpendicular to the surface
 - $\Phi_E = EA \cos \theta$, if the field makes an angle θ to the surface
- So the electric flux is defined as $\Phi_E = \vec{E} \cdot \vec{A}$.
- How would you define the electric flux in words?
 - The total number of field lines passing through the unit area perpendicular to the field. $N_E \propto EA_{\perp} = \Phi_E$

Example 22 – 1

- Electric flux.** (a) Calculate the electric flux through the rectangle in the figure (a). The rectangle is 10cm by 20cm and the electric field is uniform with magnitude 200N/C. (b) What is the flux in figure if the angle is 30 degrees?



The electric flux is defined as

$$\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \theta$$

So when (a) $\theta=0$, we obtain

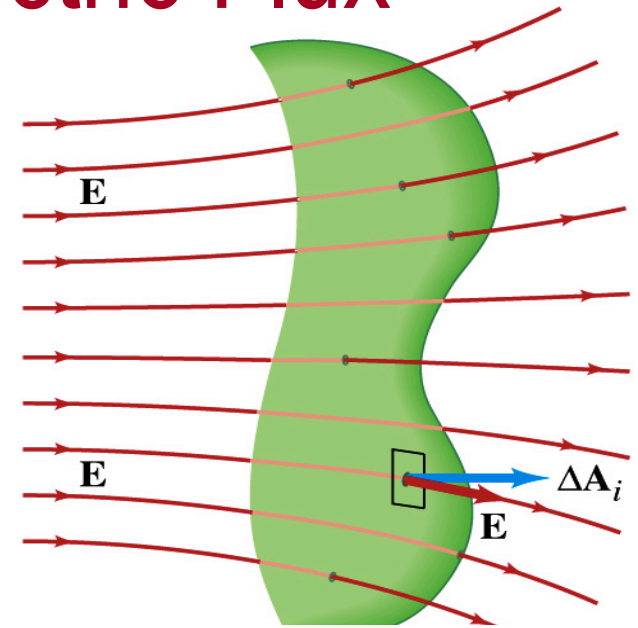
$$\Phi_E = EA \cos \theta = EA = (200 \text{ N/C}) \cdot (0.1 \times 0.2 \text{ m}^2) = 4.0 \text{ N} \cdot \text{m}^2/\text{C}$$

And when (b) $\theta=30$ degrees, we obtain

$$\Phi_E = EA \cos 30^\circ = (200 \text{ N/C}) \cdot (0.1 \times 0.2 \text{ m}^2) \cos 30^\circ = 3.5 \text{ N} \cdot \text{m}^2/\text{C}$$

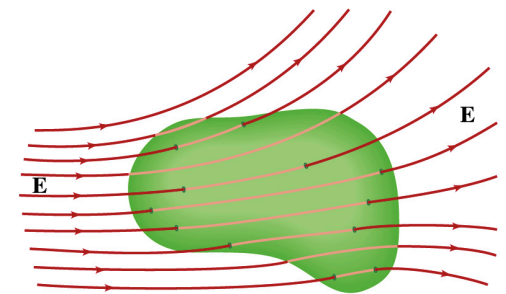
Generalization of the Electric Flux

- Let's consider a surface of area A that is not a square or flat but in some random shape, and that the field is not uniform.
- The surface can be divided up into infinitesimally small areas of $\Delta\mathbf{A}_i$ that can be considered flat.
- And the electric field through this area can be considered uniform since the area is very small.



- Then the electric flux through the entire surface is approximately

$$\Phi_E \approx \sum_{i=1}^n \vec{E}_i \cdot \Delta\vec{A}_i$$



- In the limit where $\Delta\mathbf{A}_i \rightarrow 0$, the discrete summation becomes an integral.

$$\Phi_E = \int \vec{E}_i \cdot d\vec{A} \quad \text{open surface}$$

$$\Phi_E = \oint \vec{E}_i \cdot d\vec{A} \quad \text{enclosed surface}$$

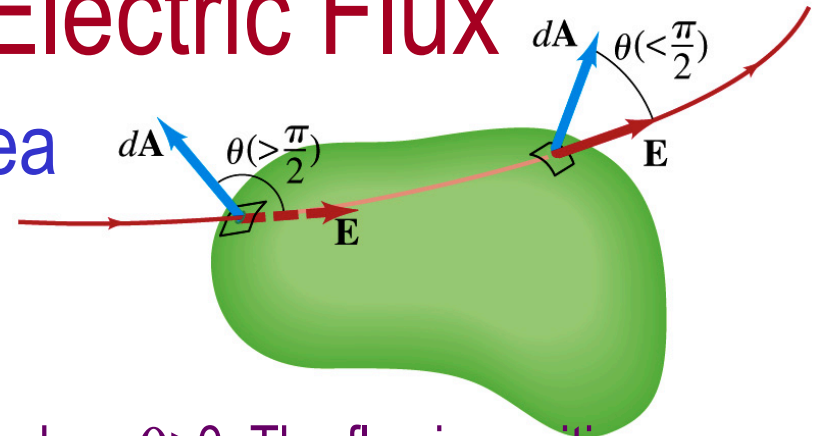
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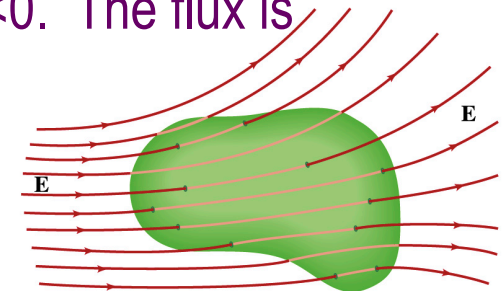
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Generalization of the Electric Flux

- We arbitrarily define that the area vector points outward from the enclosed volume.



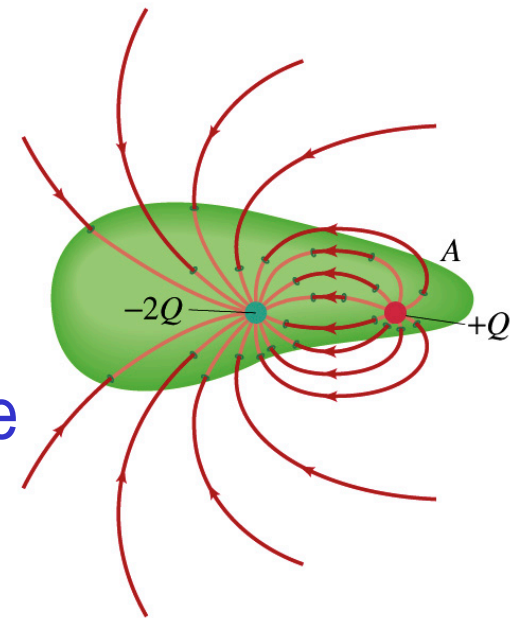
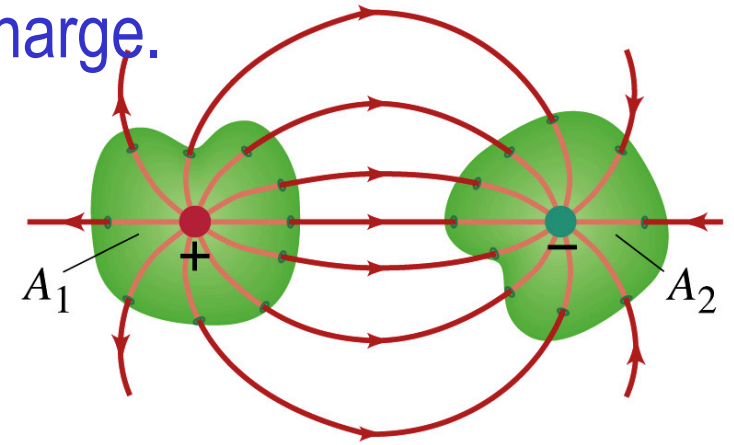
- For the line leaving the volume, $\theta < \pi/2$ and $\cos\theta > 0$. The flux is positive.
- For the line coming into the volume, $\theta > \pi/2$ and $\cos\theta < 0$. The flux is negative.
- If $\Phi_E > 0$, there is net flux out of the volume.
- If $\Phi_E < 0$, there is flux into the volume.



- In the above figures, each field that enters the volume also leaves the volume, so $\Phi_E = \oint \vec{E} \cdot d\vec{A} = 0$.
- The flux is non-zero only if one or more lines start or end inside the surface.

Generalization of the Electric Flux

- The field line starts or ends only on a charge.
- Sign of the net flux on the surface A_1 ?
 - The net outward flux (positive flux)
- How about A_2 ?
 - Net inward flux (negative flux)
- What is the flux in the bottom figure?
 - There should be a net inward flux (negative flux) since the total charge inside the volume is negative.
- The net flux that crosses an enclosed surface is proportional to the total charge inside the surface. ➔ This is the crux of Gauss' law.



A Brain Teaser of Electric Flux

- What would change the electric flux through a circle lying in the xz plane where the electric field is $(10\text{N/C})\mathbf{j}$?
 1. Changing the magnitude of the electric field
 2. Changing the surface area of the circle
 3. Tipping the circle so that it is lying in the xy plane
 4. All of the above
 5. None of the above

