# PHYS 1444 – Section 003
# Lecture #15

*Tuesday, Oct. 25, 2011*
*Dr. Jaehoon Yu*

- Kirchhoff's Rules
- EMFs in Series and Parallel
- RC Circuits
  - Analysis of RC Circuits
  - Discharging of RC Circuits
  - Application of RC Circuits
- Magnetism and Magnetic Field

Today's homework is #8, due 10pm, Tuesday, Nov. 1!!

# Announcements

- Midterm grade discussions will occur from later this week through early next week
  - Please do not miss this discussion session
  - Will announce Thursday the sequence
- Colloquium tomorrow
  - 4pm, SH101

# Physics Department
# The University of Texas at Arlington
# COLLOQUIUM

## Anisotropic Scaling and Horava's Theory of Quantum Gravity

***Dr. Anzhong Wang / Baylor University***

4:00p.m Wednesday October 26, 2011
Room 101 Science Hall

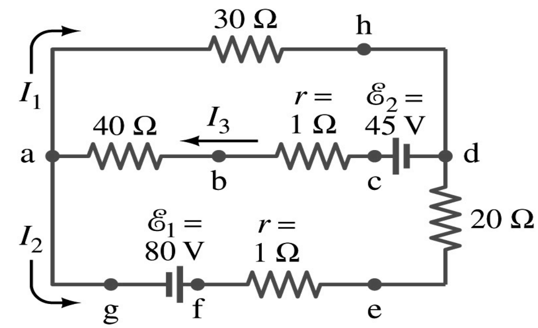
**Abstract:**

I shall first give a brief and basic introduction to anisotropic scalings of space and time introduced by Lifshitz in 1940's in order to make the scalar field renormalizable in condensed matter physics, and then apply them to gravity, as done recently by Horava, the so-called Horava-Lifshitz theory of quantum gravity. I will explain in some detail its major challenges found so far, and show some very promising approaches to resolve these problems. Finally, I shall point out some future work along the lines.

**Refreshments will be served at 3:30p.m in the physics lounge**

# Special Project #5

- In the circuit on the right, find out what the currents $I_1$, $I_2$ and $I_3$ are using Kirchhoff's rules in the following two cases:
  - All the directions of the current flows are as shown in the figure. (3points)
  - When the directions of the flow of the current $I_1$ and $I_3$ are opposite than what is drawn in the figure but the direction of $I_2$ is the same. (5 points)
  - When the directions of the flow of the current $I_2$ and $I_3$ are opposite than what is drawn in the figure but the direction of $I_1$ is the same. (5 points)



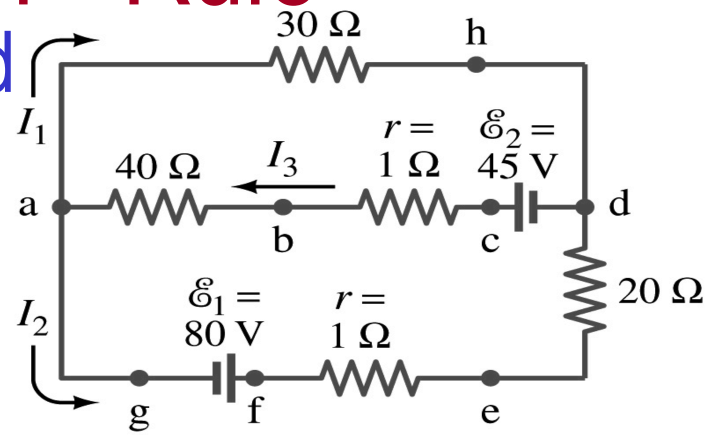

- Show the details of your OWN work to obtain credit.
- Due is at the beginning of the class Thursday, Nov. 3.

# Special Project Spread Sheet

**PHYS1444-003, Fall11, Special Project #4**

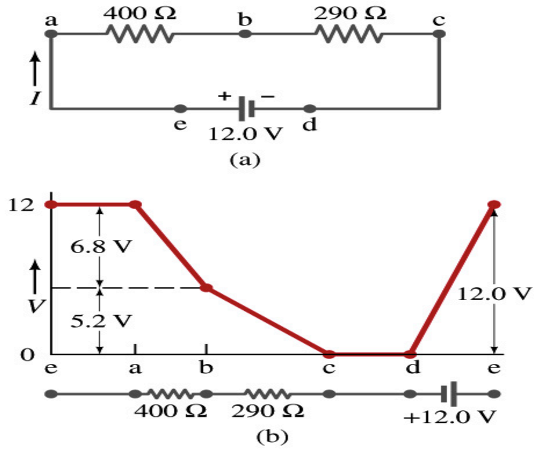
| Item Name | Rated power (W) | Number of devices | Number of Hours per day | Daily Power Consumption (kWh) | Energy Cost per kWh (cents) | Daily Energy Cost ($) | Monthly Energy Cost ($) | Yearly Energy Cost ($) | Note |
|---|---|---|---|---|---|---|---|---|---|
| Light Bulbs | 30, 40, 60, 100, etc | 40 | | | | | | | |
| Heaters | | | | | | | | | |
| Fans | | | | | | | | | |
| Air Conditioner | | | | | | | | | |
| Fridgers, Freezers | | | | | | | | | |
| Computers | | | | | | | | | |
| Game consoles | | | | | | | | | |
| Total | | | | 0 | | 0 | 0 | 0 | |

# Kirchhoff's Rules – 1st Rule




- Some circuits are very complicated to do the analysis using the simple combinations of resisters
  - G. R. Kirchhoff devised two rules to deal with complicated circuits.
- Kirchhoff's rules are based on **conservation of charge and energy**
  - Kirchhoff's 1st rule: The junction rule, charge conservation.
    - At any junction point, the sum of all currents entering the junction must equal to the sum of all currents leaving the junction.
    - In other words, what goes in must come out.
    - At junction *a* in the figure, $I_3$ comes into the junction while $I_1$ and $I_2$ leaves: $I_3 = I_1 + I_2$

# Kirchhoff's Rules – 2nd Rule

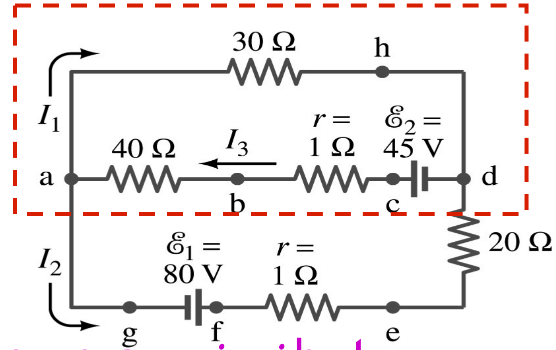



- Kirchoff's 2nd rule: The loop rule, uses conservation of energy.
  - The sum of the changes in potential around any closed path of a circuit must be zero.
- The current in the circuit in the figure is *I*=12/690=0.017A.
  - Point *e* is the high potential point while point *d* is the lowest potential.
  - When the test charge starts at *e* and returns to *e*, the total potential change is 0.
  - Between point *e* and *a*, no potential change since there is no source of potential nor any resistance.
  - Between *a* and *b*, there is a 400Ω resistance, causing IR=0.017*400 =6.8V drop.
  - Between *b* and *c*, there is a 290Ω resistance, causing IR=0.017*290 =5.2V drop.
  - Since these are voltage drops, we use negative sign for these, -6.8V and -5.2V.
  - No change between *c* and *d* while from *d* to *e* there is +12V change.
  - Thus the total change of the voltage through the loop is: -6.8V-5.2V+12V=0V.

# Using Kirchhoff's Rules

1. Determine the flow of currents at the junctions and label each and everyone of the currents.
   - It does not matter which direction, you decide.
   - If the value of the current after completing the calculations are negative, you just need to flip the direction of the current flow.
2. Write down the current equation based on Kirchhoff's 1$^{st}$ rule at various junctions.
   - Be sure to see if any of them are the same.
3. Choose closed loops in the circuit
4. Write down the potential in each interval of the junctions, keeping the sign properly.
5. Write down the potential equations for each loop.
6. Solve the equations for unknowns.

# Example 26 – 9




**Use Kirchhoff's rules.** Calculate the currents $I_1$, $I_2$ and $I_3$ in each of the branches of the circuit in the figure.

The directions of the current through the circuit is not known a *priori* but since the current tends to move away from the positive terminal of a battery, we arbitrarily choose the direction of the currents as shown.

We have three unknowns so we need three equations.

Using Kirchhoff's junction rule at point *a*, we obtain $I_3 = I_1 + I_2$

This is the same for junction *d* as well, so no additional information.

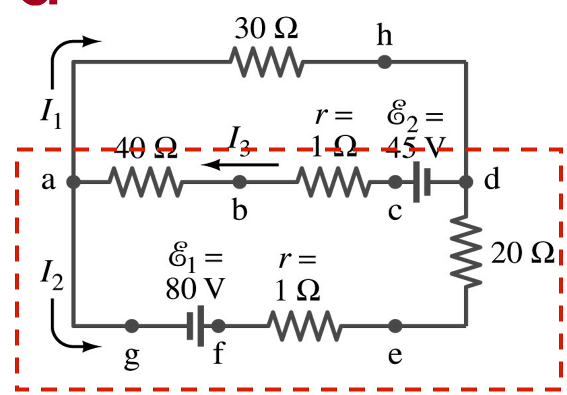
Now the second rule on the loop *ahdcba*.

$V_{ah} = -I_1 30 \qquad V_{hd} = 0 \qquad V_{dc} = +45 \qquad V_{cb} = -I_3 \qquad V_{ba} = -40 I_3$

The total voltage change in the loop *ahdcba* is.

$$V_{ahdcba} = -30I_1 + 45 - 1 \cdot I_3 - 40I_3 = 45 - 30I_1 - 41I_3 = 0$$

# Example 26 – 9, cnt'd




Now the second rule on the other loop *agfedcba*.

$V_{ag} = 0 \quad V_{gf} = +80 \quad V_{fe} = -I_2 \cdot 1 \quad V_{ed} = -I_2 \cdot 20$

$V_{dc} = +45 \quad V_{cb} = -I_3 \cdot 1 \quad V_{ba} = -40 \cdot I_3$

The total voltage change in loop *agfedcba* is. $V_{agfedcba} = -21I_2 + 125 - 41I_3 = 0$

So the three equations become

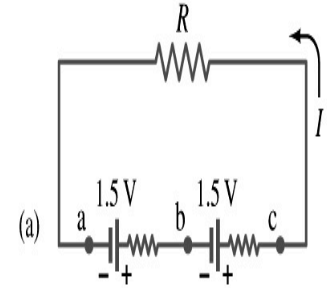
$$I_3 = I_1 + I_2$$
$$45 - 30I_1 - 41I_3 = 0$$
$$125 - 21I_2 - 41I_3 = 0$$

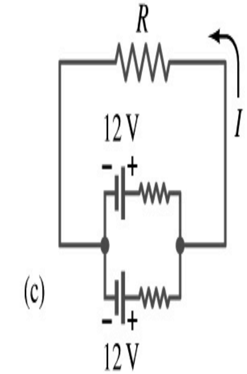
We can obtain the three current by solving these equations for $I_1$, $I_2$ and $I_3$.

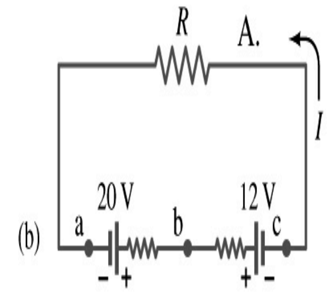
Do this yourselves!!

# EMFs in Series and Parallel: Charging a Battery

- When two or more sources of emfs, such as batteries, are connected in series
  - The total voltage is the algebraic sum of their voltages, if their direction is the same
    - $V_{ab}$=1.5 + 1.5=3.0V in figure (a).
  - If the batteries are arranged in an opposite direction, the total voltage is the difference between them
    - $V_{ac}$=20 – 12=8.0V in figure (b)
    - Connecting batteries in opposite direction is wasteful.
    - This, however, is the way a battery charger works.
    - Since the 20V battery is at a higher voltage, it forces charges into 12V battery
    - Some battery are rechargeable since their chemical reactions are reversible but most the batteries do not reverse their chemical reactions

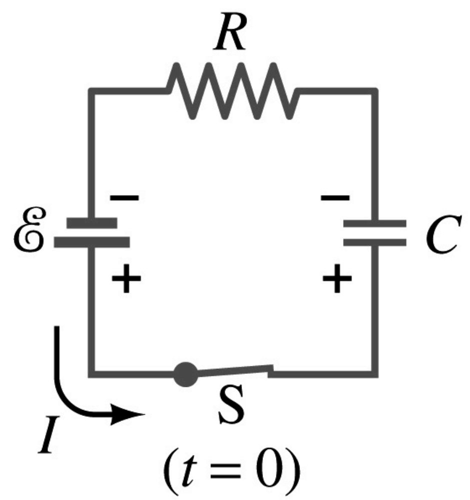



Parallel arrangements (c) are used only to increase currents. An example?

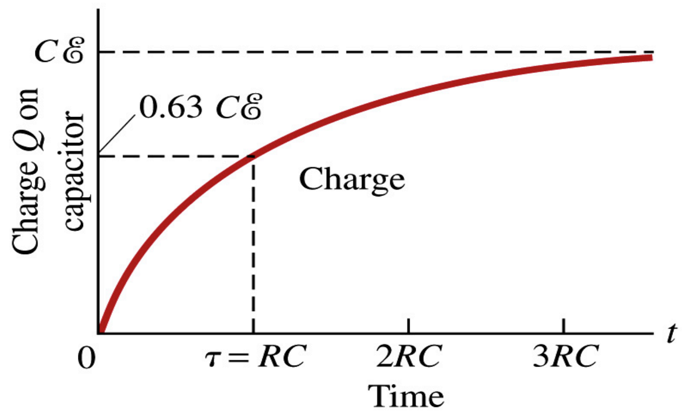
# RC Circuits

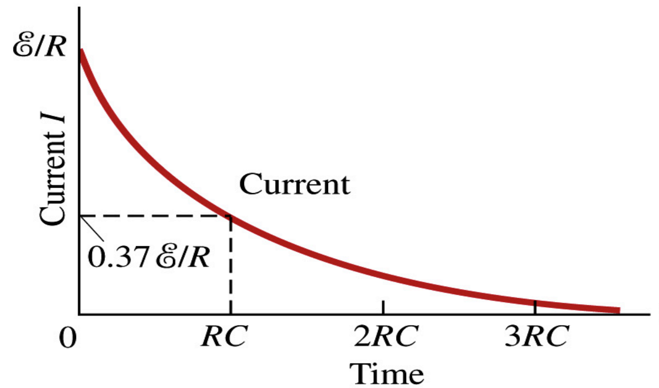



- Circuits containing both resisters and capacitors
  - RC circuits are used commonly in everyday life
    - Control windshield wiper
    - Timing of traffic light from red to green
    - Camera flashes and heart pacemakers
- How does an RC circuit look?
  - There should be a source of emf, capacitors and resisters
- What happens when the switch S is closed?
  - Current immediately starts flowing through the circuit.
  - Electrons flow out of negative terminal of the emf source, through the resister R and accumulates on the upper plate of the capacitor.
  - The electrons from the bottom plate of the capacitor will flow into the positive terminal of the battery, leaving only positive charge on the bottom plate.
  - As the charge accumulates on the capacitor, the potential difference across it increases
  - The current reduces gradually to 0 till the voltage across the capacitor is the same as emf.
  - The charge on the capacitor increases till it reaches to its maximum $C\mathcal{E}$.

# RC Circuits

- How does all this look like in graphs?
  - The charge and the current on the capacitor as a function of time




  - From energy conservation (Kirchhoff's 2nd rule), the emf $\mathcal{E}$ must be equal to the voltage drop across the capacitor and the resister
    - $\mathcal{E}=IR+Q/C$
    - R includes all resistance in the circuit, including the internal resistance of the battery, $I$ is the current in the circuit at any instance, and Q is the charge of the capacitor at that same instance.

# Analysis of RC Circuits

- From the energy conservation, we obtain $\mathcal{E}=I\mathcal{R}+Q/C$
- Which ones are constant in the above equation?
  - $\mathcal{E}$, $\mathcal{R}$ and $C$ are constant
  - $Q$ and $I$ are functions of time
- How do we write the rate at which the charge is accumulated on the capacitor?
  - We can rewrite the above equation as $\varepsilon = R\frac{dQ}{dt}+\frac{1}{C}Q$
  - This equation can be solved by rearranging the terms as $\frac{dQ}{C\varepsilon - Q}=\frac{dt}{RC}$

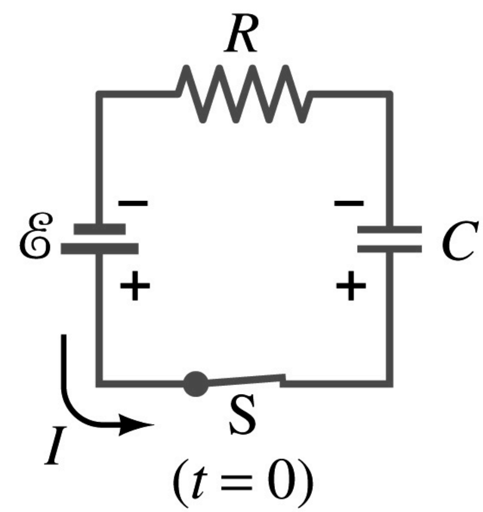
# Analysis of RC Circuits

- Now integrating from $t=0$ when there was no charge on the capacitor to $t$ when the capacitor is fully charged, we obtain
- $$\int_0^Q \frac{dQ}{C\varepsilon - Q} = \frac{1}{RC}\int_0^t dt \quad \rightarrow$$
- $$-\ln\left(C\varepsilon - Q\right)\Big|_0^Q = -\ln\left(C\varepsilon - Q\right) - \left(-\ln C\varepsilon\right) = \frac{t}{RC}\Bigg|_0^t = \frac{t}{RC}$$
- So, we obtain $\ln\left(1 - \frac{Q}{C\varepsilon}\right) = -\frac{t}{RC}$ $\rightarrow$ $1 - \frac{Q}{C\varepsilon} = e^{-t/RC}$
- Or $Q = C\varepsilon\left(1 - e^{-t/RC}\right)$
- The potential difference across the capacitor is V=Q/C, so $V_C = \varepsilon\left(1 - e^{-t/RC}\right)$

# Analysis of RC Circuits

- Since $Q = C\varepsilon\left(1 - e^{-t/RC}\right)$ and $V_C = \varepsilon\left(1 - e^{-t/RC}\right)$
- What can we see from the above equations?
  - Q and $V_C$ increase from 0 at t=0 to maximum value $Q_{max}=C\mathscr{E}$ and $V_C=\mathscr{E}$.
- In how much time?
  - The quantity RC is called the time constant of the circuit, $\tau$
    - $\tau=RC$, What is the unit? Sec.
  - What is the physical meaning?
    - The time required for the capacitor to reach $(1-e^{-1})=0.63$ or 63% of the full charge
- The current is $I = \frac{dQ}{dt} = \frac{\varepsilon}{R}e^{-t/RC}$

# Example 26 – 12




**RC circuit, with emf.** The capacitance in the circuit of the figure is C=0.30μF, the total resistance is 20kΩ, and the battery emf is 12V. Determine (a) the time constant, (b) the maximum charge the capacitor could acquire, (c) the time it takes for the charge to reach 99% of this value, (d) the current *I* when the charge Q is half its maximum value, (e) the maximum current, and (f) the charge Q when, the current *I* is 0.20 its maximum value.

(a) Since $\tau = RC$ We obtain $\tau = 20\times10^{3}\cdot0.30\times10^{-6} = 6.0\times10^{-3}\ \mathrm{sec}$

(b) Maximum charge is $Q_{\max} = C\varepsilon = 0.30\times10^{-6}\cdot12 = 3.6\times10^{-6}\ C$

(c) Since $Q = C\varepsilon\left(1-e^{-t/RC}\right)$ For 99% we obtain $0.99C\varepsilon = C\varepsilon\left(1-e^{-t/RC}\right)$

$$e^{-t/RC} = 0.01;\quad -t/RC = -2\ln10;\quad t = RC\cdot2\ln10 = 4.6RC = 28\times10^{-3}\ \mathrm{sec}$$

(d) Since $\varepsilon = IR + Q/C$ We obtain $I = \left(\varepsilon - Q/C\right)/R$

The current when Q is $0.5Q_{max}$ $I = \left(12 - 1.8\times10^{-6}/0.30\times10^{-6}\right)/20\times10^{3} = 3\times10^{-4}\ A$

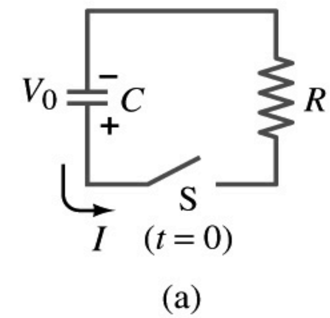
(e) When is I maximum? when Q=0: $I = 12/20\times10^{3} = 6\times10^{-4}\ A$

(f) What is Q when I=120mA? $Q = C\left(\varepsilon - IR\right) =$

$$= 0.30\times10^{-6}\left(12 - 1.2\times10^{-4}\cdot2\times10^{4}\right) = 2.9\times10^{-6}\ C$$

# Discharging RC Circuits

- When a capacitor is already charged, it is allowed to discharge through a resistance R.
  - When the switch S is closed, the voltage across the resistor at any instant equals that across the capacitor. Thus IR=Q/C.
  - The rate at which the charge leaves the capacitor equals the negative the current flows through the resistor
    - $I$= - dQ/dt. Why negative?
    - Since the current is leaving the capacitor
  - Thus the voltage equation becomes a differential equation




$$-\frac{dQ}{dt}R=\frac{Q}{C}$$

**Rearrange terms**

$$\frac{dQ}{Q}=-\frac{dt}{RC}$$

# Discharging RC Circuits

– Now, let's integrate from t=0 when the charge is $Q_0$ to t when the charge is Q $\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{RC}$

– The result is $\ln Q\Big|_{Q_0}^{Q} = \ln \frac{Q}{Q_0} = -\frac{t}{RC}$

– Thus, we obtain

$$Q(t) = Q_0 e^{-t/RC}$$

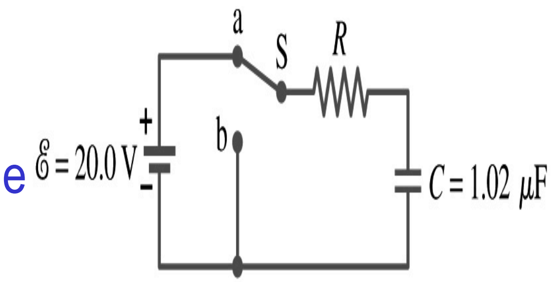
– What does this tell you about the charge on the capacitor?
  - It decreases exponentially w/ time and w/ the time constant RC
  - Just like the case of charging

What is this?

– The current is: $I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC}$ $\quad I(t) = I_0 e^{-t/RC}$

  - The current also decreases exponentially w/ time w/ the constant RC

# Example 26 – 13




**Discharging RC circuit.** In the RC circuit shown in the figure the battery has fully charged the capacitor, so $Q_0 = C\mathcal{E}$. Then at t=0, the switch is thrown from position a to b. The battery emf is 20.0V, and the capacitance C=1.02μF. The current $I$ is observed to decrease to 0.50 of its initial value in 40μs. (a) what is the value of R? (b) What is the value of Q, the charge on the capacitor, at t=0? (c) What is Q at t=60μs?

(a) Since the current reaches to 0.5 of its initial value in 40μs, we can obtain

$I(t) = I_0 e^{-t/RC}$ **For 0.5$I_0$** → $0.5I_0 = I_0 e^{-t/RC}$ **Rearrange terms** → $-t/RC = \ln 0.5 = -\ln 2$

**Solve for R** → $R = t/(C\ln 2) = 40\times10^{-6}/\left(1.02\times10^{-6}\cdot\ln 2\right) = 56.6\Omega$

(b) The value of Q at t=0 is

$$Q_0 = Q_{\max} = C\varepsilon = 1.02\times10^{-6}\cdot 20.0 = 20.4\mu C$$

(c) What do we need to know first for the value of Q at t=60μs?

The RC time $\tau = RC = 56.6\cdot 1.02\times10^{-6} = 57.7\mu s$

Thus $Q(t = 60\mu s) = Q_0 e^{-t/RC} = 20.4\times10^{-6}\cdot e^{-60\mu s/57.7\mu s} = 7.2\mu C$

# Application of RC Circuits

- What do you think the charging and discharging characteristics of RC circuits can be used for?
  - To produce voltage pulses at a regular frequency
  - How?
    - The capacitor charges up to a particular voltage and discharges
    - A simple way of doing this is to use breakdown of voltage in a gas filled tube
      - The discharge occurs when the voltage breaks down at $V_0$
      - After the completion of discharge, the tube no longer conducts
      - Then the voltage is at $V_0$' and it starts charging up
      - How do you think the voltage as a function of time look?
        - » A sawtooth shape
    - Pace maker, intermittent windshield wiper, etc