

PHYS 1444 – Section 003

Lecture #18

Thursday, Nov. 3, 2011

Dr. Jaehoon Yu

- Torque on a Current Loop
- Magnetic Dipole Moment
- Magnetic Dipole Potential Energy
- Sources of Magnetic Field
- Magnetic Field Due to Straight Wire
- Forces Between Two Parallel Wires
- Ampère's Law and Its Verification

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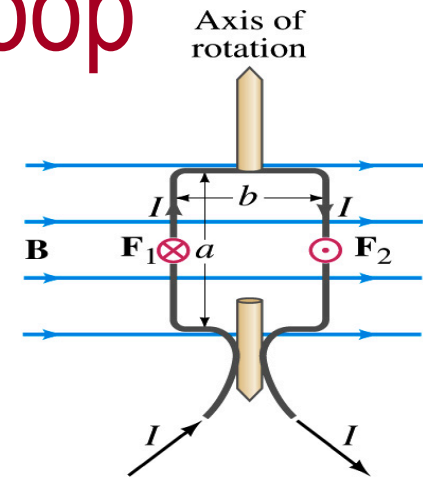
PHYS 1444-003, Fall 2011
Dr. Jaehoon Yu

Announcements

- Quiz #3
 - Beginning of the class coming Tuesday, Nov. 8
 - Covers: CH26.5 through what we finish Today (CH28.4?)!
- Reading Assignments
 - CH 27.6 – 27.9
- Bring your special project at the end of the class!



Torque on a Current Loop



- What do you think will happen to a closed rectangular loop of wire with electric current as shown in the figure?
 - It will rotate! Why?
 - The magnetic field exerts a force on both vertical sections of wire.
 - Where is this principle used in?
 - Ammeters, motors, volt-meters, speedometers, etc
- The two forces on the different sections of the wire exerts net torque to the same direction about the rotational axis along the symmetry axis of the wire.
- What happens when the wire turns 90 degrees?
 - It will not turn unless the direction of the current changes

Torque on a Current Loop

- So what would be the magnitude of this torque?

- What is the magnitude of the force on the section of the wire with length a ?

- $F_a = IaB$
- The moment arm of the coil is $b/2$

- So the total torque is the sum of the torques by each of the forces

$$\tau = IaB \frac{b}{2} + IaB \frac{b}{2} = IabB = IAB$$

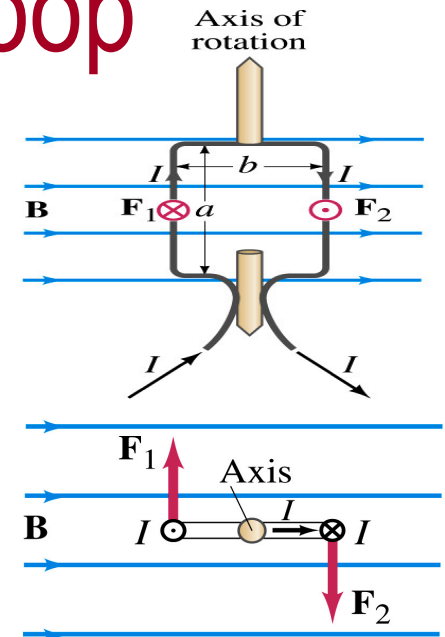
- Where $A = ab$ is the area of the coil loop

- What is the total net torque if the coil consists of N loops of wire?

$$\tau = NIAB$$

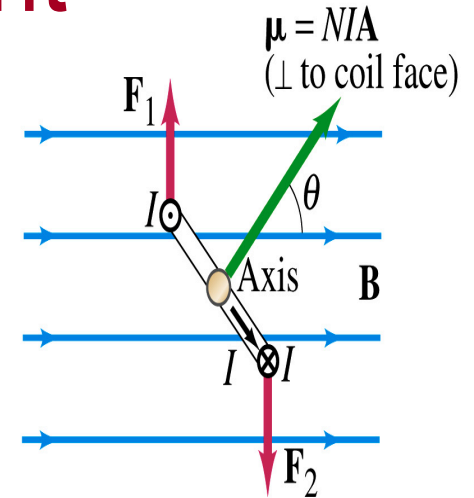
- If the coil makes an angle θ w/ the field

$$\tau = NIAB \sin \theta$$



Magnetic Dipole Moment

- The formula derived in the previous page for a rectangular coil is valid for any shape of the coil
- The quantity $NI\mathcal{A}$ is called the **magnetic dipole moment of the coil**



- It is considered a vector $\vec{\mu} = NI \vec{A}$
 - Its direction is the same as that of the area vector \vec{A} and is perpendicular to the plane of the coil consistent with the right-hand rule
 - Your thumb points to the direction of the magnetic moment when your finger cups around the loop in the direction of the wire
- Using the definition of magnetic moment, the torque can be written in vector form

$$\vec{\tau} = NI \vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}$$

Magnetic Dipole Potential Energy

- Where else did you see the same form of the torque?
 - Remember the torque due to electric field on an electric dipole? $\vec{\tau} = \vec{p} \times \vec{E}$
 - The potential energy of the electric dipole is
 - $U = -\vec{p} \cdot \vec{E}$
- How about the potential energy of a magnetic dipole?
 - The work done by the torque is
 - $U = \int \tau d\theta = \int NIAB \sin \theta d\theta = -\mu B \cos \theta + C$
 - If we chose $U=0$ at $\theta=\pi/2$, then $C=0$
 - Thus the potential energy is $U = -\mu B \cos \theta = -\vec{\mu} \cdot \vec{B}$
 - Very similar to the electric dipole




Example 27 – 12

Magnetic moment of a hydrogen atom. Determine the magnetic dipole moment of the electron orbiting the proton of a hydrogen atom, assuming (in the Bohr model) it is in its ground state with a circular orbit of radius $0.529 \times 10^{-10} \text{ m}$.

What provides the centripetal force? **The Coulomb force**

So we can obtain the speed of the electron from $F = \frac{e^2}{4\pi\epsilon_0 r^2} = \frac{m_e v^2}{r}$

 $v = \sqrt{\frac{e^2}{4\pi\epsilon_0 m_e r}} = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) \cdot (1.6 \times 10^{-19} \text{ C})^2}{(9.1 \times 10^{-31} \text{ kg}) \cdot (0.529 \times 10^{-10} \text{ m})}} = 2.19 \times 10^6 \text{ m/s}$

Since the electric current is the charge that passes through the given point per unit time, we can obtain the current

$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

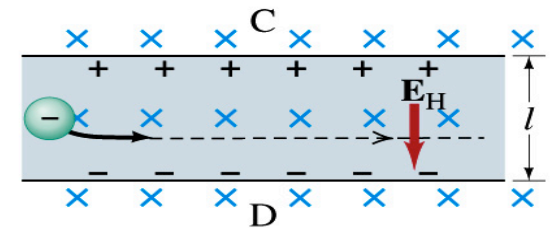
Since the area of the orbit is $A = \pi r^2$, we obtain the hydrogen magnetic moment

$$\mu = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2} = \frac{er}{2} \sqrt{\frac{e^2}{4\pi\epsilon_0 m_e r}} = \frac{e^2}{4} \sqrt{\frac{r}{\pi\epsilon_0 m_e}}$$

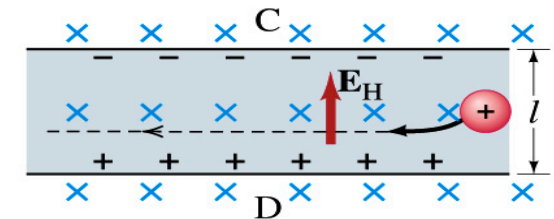


The Hall Effect

- What do you think will happen to the electrons flowing through a conductor immersed in a magnetic field?
 - Magnetic force will push the electrons toward one side of the conductor. Then what happens?
 - $\vec{F}_B = -e\vec{v}_d \times \vec{B}$
 - A potential difference will be created due to continued accumulation of electrons on one side. Till when? Forever?
 - Nope. Till the electric force inside the conductor is equal and opposite to the magnetic force
- This is called the **Hall Effect**
 - The potential difference produced is called
 - The Hall emf
 - The electric field due to the separation of charge is called the Hall field, \mathbf{E}_H , and it points to the direction opposite to the magnetic force



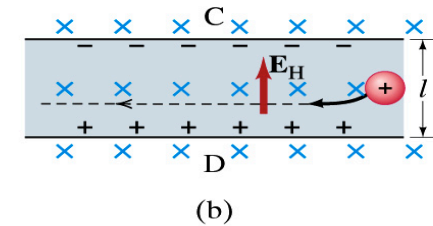
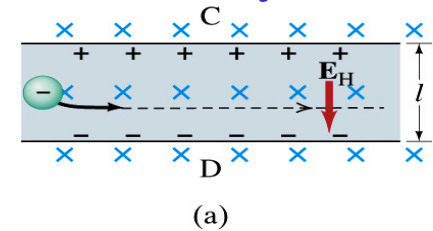
(a)



(b)

The Hall Effect

- In equilibrium, the force due to Hall field is balanced by the magnetic force $e v_d \mathcal{B}$, so we obtain
- $e E_H = e v_d B$ and $E_H = v_d B$
- The Hall emf is then $\mathcal{E}_H = E_H l = v_d B l$
 - Where l is the width of the conductor
- What do we use the Hall effect for?
 - The current of negative charge moving to right is equivalent to the positive charge moving to the left
 - The Hall effect can distinguish these since the direction of the Hall field or direction of the Hall emf is opposite
 - Since the magnitude of the Hall emf is proportional to the magnetic field strength → can measure the B-field strength
 - Hall probe



Sources of Magnetic Field

- We have learned so far about the effects of magnetic field on electric currents and moving charge
- We will now learn about the dynamics of magnetism
 - How do we determine magnetic field strengths in certain situations?
 - How do two wires with electric current interact?
 - What is the general approach to finding the connection between current and magnetic field?



Magnetic Field due to a Straight Wire

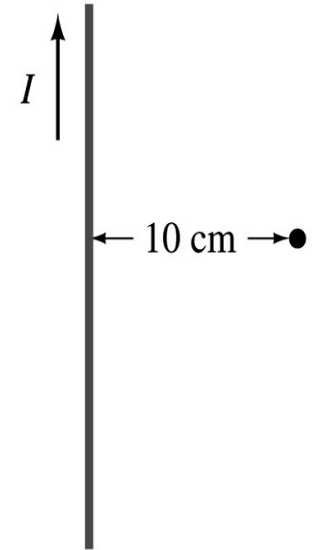
- The magnetic field due to the current flowing through a straight wire forms a circular pattern around the wire
 - What do you imagine the strength of the field is as a function of the distance from the wire?
 - It must be weaker as the distance increases
 - How about as a function of current?
 - Directly proportional to the current
 - Indeed, the above are experimentally verified $B \propto \frac{I}{r}$
 - This is valid as long as $r \ll$ the length of the wire
 - The proportionality constant is $\mu_0/2\pi$, thus the field strength becomes

$$B = \frac{\mu_0 I}{2\pi r}$$
 - μ_0 is the permeability of free space $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m}/\text{A}$



Example 28 – 1

Calculation of B near wire. A vertical electric wire in the wall of a building carries a dc current of 25A upward. What is the magnetic field at a point 10cm due north of this wire?



Using the formula for the magnetic field near a straight wire

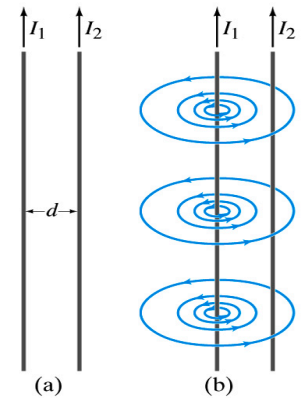
$$B = \frac{\mu_0 I}{2\pi r}$$

So we can obtain the magnetic field at 10cm away as

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \cdot (25 \text{ A})}{(2\pi) \cdot (0.01 \text{ m})} = 5.0 \times 10^{-5} \text{ T}$$

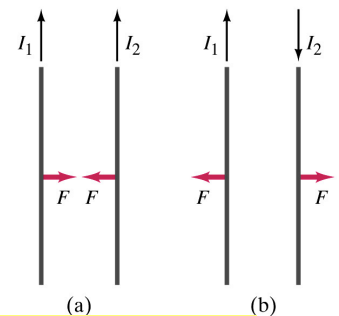
Force Between Two Parallel Wires

- We have learned that a wire carrying the electric current produces magnetic field
- Now what do you think will happen if we place two current carrying wires next to each other?
 - They will exert force onto each other. Repel or attract?
 - Depending on the direction of the currents
- This was first pointed out by Ampère.
- Let's consider two long parallel conductors separated by a distance d , carrying currents I_1 and I_2 .
- At the location of the second conductor, the magnitude of the magnetic field produced by I_1 is
$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$



Force Between Two Parallel Wires

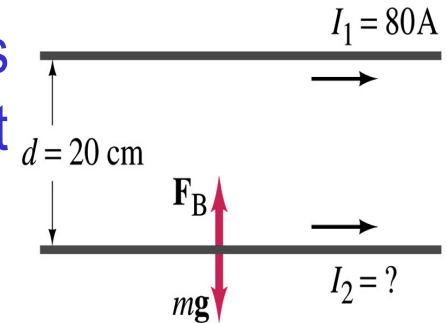
- The force F by a magnetic field B_1 on a wire of length l , carrying the current I_2 when the field and the current are perpendicular to each other is: $F = I_2 B_1 l$
 - So the force per unit length is $\frac{F}{l} = I_2 B_1 = I_2 \frac{\mu_0}{2\pi} \frac{I_1}{d}$
 - This force is only due to the magnetic field generated by the wire carrying the current I_1
 - There is the force exerted on the wire carrying the current I_1 by the wire carrying current I_2 of the same magnitude but in opposite direction
- So the force per unit length is $\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$
- How about the direction of the force?



If the currents are in the same direction, the attractive force. If opposite, repulsive.

Example 28 – 5

Suspending a wire with current. A horizontal wire carries a current $I_1=80\text{A}$ DC. A second parallel wire 20cm below it must carry how much current I_2 so that it doesn't fall due to the gravity? The lower has a mass of 0.12g per meter of length.



Which direction is the gravitational force? **Down to the center of the Earth**

This force must be balanced by the magnetic force exerted on the wire by the first wire.

$$\frac{F_g}{l} = \frac{mg}{l} = \frac{F_M}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$$

Solving for I_2

$$I_2 = \frac{mg}{l} \frac{2\pi d}{\mu_0 I_1} =$$

$$\frac{2\pi (9.8 \text{ m/s}^2) \cdot (0.12 \times 10^{-3} \text{ kg}) \cdot (0.20 \text{ m})}{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) \cdot (80 \text{ A})} = 15 \text{ A}$$

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

- The unit of current, ampere, is defined using the definition of the force between two wires each carrying 1A of current and separated by 1m

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{2\pi} \frac{1\text{A} \cdot 1\text{A}}{1\text{m}} = 2 \times 10^{-7} \text{ N/m}$$

- So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly $2 \times 10^{-7} \text{ N/m}$.
- Coulomb is then defined as exactly $1\text{C} = 1\text{A} \cdot \text{s}$.
- We do it this way since the electric current is measured more accurately and controlled more easily than charge.

