

PHYS 1444 – Section 003

Lecture #19

Tuesday, Nov. 8, 2011

Dr. Jaehoon Yu

- Ampère's Law and Its Verification
- Solenoid and Toroidal Magnetic Field
- Biot-Savart Law
- Magnetic Materials
- B in Magnetic Materials
- Hysteresis

Today's homework is #10, due 10pm, Tuesday, Nov. 15!!

Tuesday, Nov. 8, 2011



PHYS 1444-003, Fall 2011
Dr. Jaehoon Yu

Announcements

- Term exam #2
 - Date and time: 12:30 – 2:00pm, Tuesday, Nov. 22
 - Location: SH103
 - Coverage: Ch. 26 – 3 to what we finish Tuesday, Nov. 15
 - A review session on Thursday, Nov. 17, in SH103
 - 150 problem pack will be prepared and distributed coming Thursday, Nov. 10
 - Please do NOT miss the exam!!
- Colloquium this week
 - Dr. R. McCroy from U. of Colorado



**Physics Department
The University of Texas at Arlington
COLLOQUIUM**

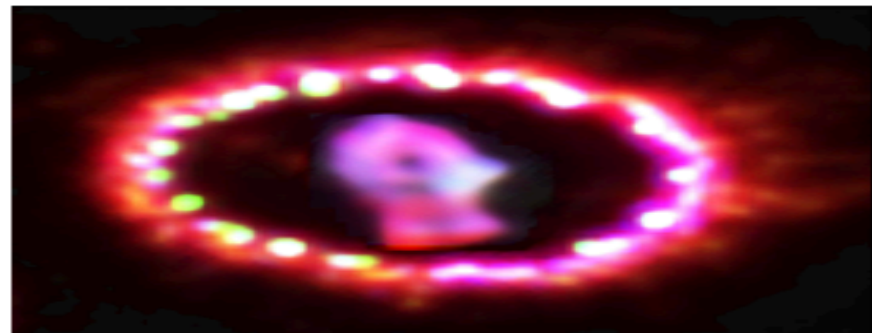
Supernova 1987A at age 24.7 years

**Dr. Richard McCray
University of Colorado**

4:00p.m Wednesday November 9, 2011
Room 101 Science Hall

Abstract:

Supernova 1987A is the brightest and nearest supernova to be observed since ~~Kepler's~~ supernova (1604 AD), and the first to be observed at every wavelength band of the electromagnetic spectrum as well as by neutrino detectors. As a result of intensive observations spanning nearly a quarter century, we have gained an unprecedented understanding of the supernova explosion and the events leading up to it. . In this seminar, I will give a brief summary of what we have learned about this event, the outstanding unsolved mysteries, and what we hope to learn by future observations.



SN 1987A as of 2011 March

Refreshments will be served at 3:30p.m in the physics lounge

Operational Definition of Ampere and Coulomb

- The permeability of free space is defined to be exactly

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

- The unit of current, ampere, is defined using the definition of the force between two wires each carrying 1A of current and separated by 1m

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} = \frac{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}}{2\pi} \frac{1\text{A} \cdot 1\text{A}}{1\text{m}} = 2 \times 10^{-7} \text{ N/m}$$

- So 1A is defined as: the current flowing each of two long parallel conductors 1m apart, which results in a force of exactly $2 \times 10^{-7} \text{ N/m}$.
- Coulomb is then defined as exactly $1\text{C} = 1\text{A} \cdot \text{s}$.
- We do it this way since the electric current is measured more accurately and controlled more easily than the charge.



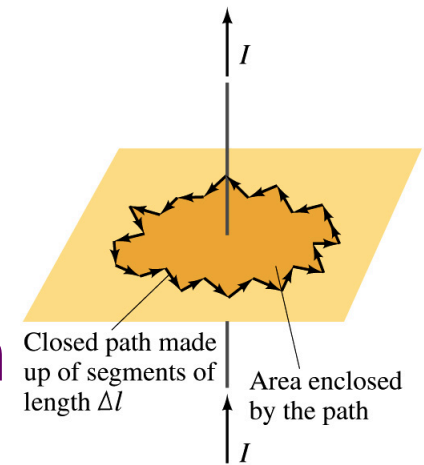
Ampère's Law

- What is the relationship between the magnetic field strength and the current?
$$B = \frac{\mu_0 I}{2\pi r}$$
 - Does this work in all cases?
 - Nope!
 - OK, then when?
 - Only valid for a long straight wire
- Then what would be the more generalized relationship between the current and the magnetic field for any shape of the wire?
 - French scientist André Marie Ampère proposed such a relationship soon after Oersted's discovery



Ampère's Law

- Let's consider an arbitrary closed path around the current as shown in the figure.
 - Let's split this path with small segments each of Δl long.
 - The sum of all the products of the length of each segment and the component of B parallel to that segment is equal to μ_0 times the net current I_{encl} that passes through the surface enclosed by the path



- $$\sum B_{\parallel} \Delta l = \mu_0 I_{\text{encl}}$$

- In the limit $\Delta l \rightarrow 0$, this relation becomes

- $$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

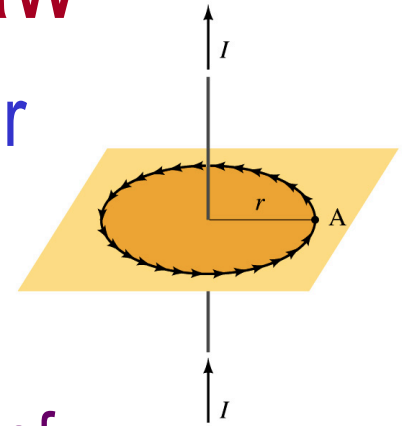
Ampère's Law

Looks very similar to a law in the electricity. Which law is it?

Gauss' Law

Verification of Ampère's Law

- Let's find the magnitude of B at a distance r away from a long straight wire w/ current I
 - This is a verification of Ampere's Law
 - We can apply Ampere's law to a circular path of radius r .



$$\mu_0 I_{encl} = \oint \vec{B} \cdot d\vec{l} = \oint B dl = B \oint dl = 2\pi r B$$

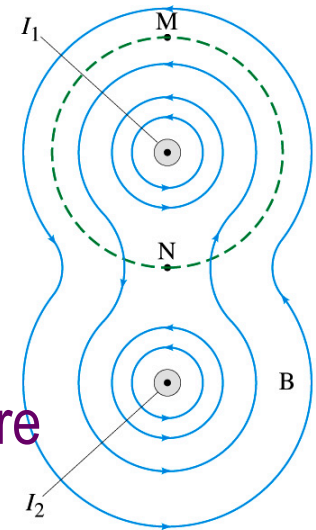
Solving for B

$$B = \frac{\mu_0 I_{encl}}{2\pi r} = \frac{\mu_0}{2\pi} \frac{I}{r}$$

- We just verified that Ampere's law works in a simple case
- Experiments verified that it works for other cases too
- The importance, however, is that it provides means to relate magnetic field to current

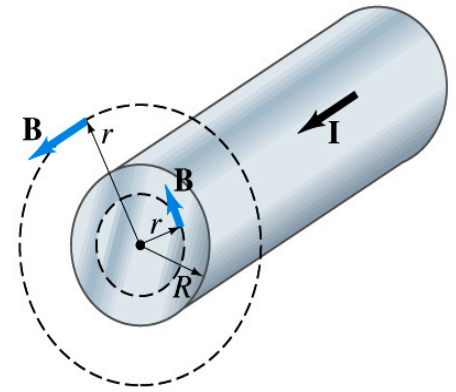
Verification of Ampère's Law

- Since Ampere's law is valid in general, B in Ampere's law is not just due to the current I_{encl} .
- B is the field at each point in space along the chosen path due to all sources
 - Including the current I enclosed by the path but also due to any other sources
 - How do you obtain B in the figure at any point?
 - Vector sum of the field by the two currents
 - The result of the closed path integral in Ampere's law for green dashed path is still $\mu_0 I_1$. Why?
 - While B in each point along the path varies, the integral over the closed path still comes out the same whether there is the second wire or not.



Example 28 – 6

Field inside and outside a wire. A long straight cylindrical wire conductor of radius R carries current I of uniform density in the conductor. Determine the magnetic field at (a) points outside the conductor ($r > R$) and (b) points inside the conductor ($r < R$). Assume that r , the radial distance from the axis, is much less than the length of the wire. (c) If $R = 2.0\text{mm}$ and $I = 60\text{A}$, what is B at $r = 1.0\text{mm}$, $r = 2.0\text{mm}$ and $r = 3.0\text{mm}$?



Since the wire is long, straight and symmetric, the field should be the same at any point the same distance from the center of the wire.

Since B must be tangential to circles around the wire, let's choose a circular path of the closed-path integral outside the wire ($r > R$). What is I_{encl} ? $I_{\text{encl}} = I$

So using Ampere's law

$$\mu_0 I = \oint \vec{B} \cdot d\vec{l} = 2\pi r B$$

Solving for B

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

Example 28 – 6 cont'd

For $r < R$, the current inside the closed path is less than I .
How much is it?

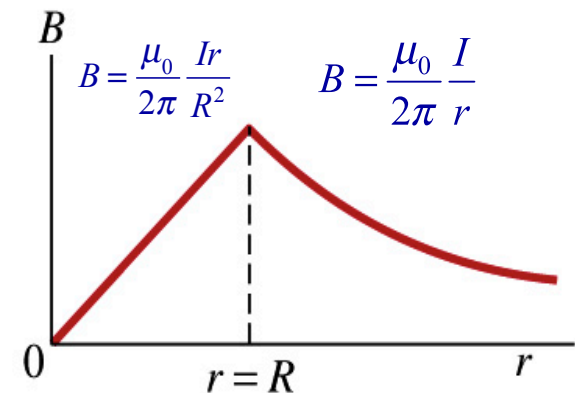
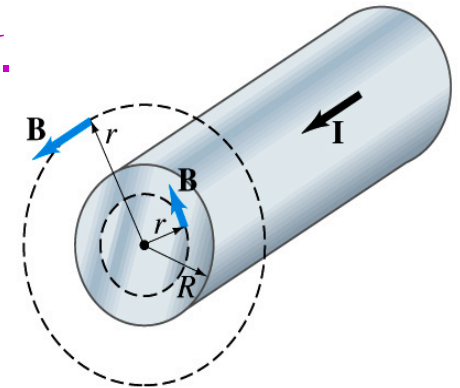
$$I_{\text{encl}} = I \frac{\pi r^2}{\pi R^2} = I \left(\frac{r}{R} \right)^2$$

So using Ampere's law

$$\mu_0 I \left(\frac{r}{R} \right)^2 = \oint \vec{B} \cdot d\vec{l} = 2\pi r B \quad \xrightarrow{\text{Solving for B}} \quad B = \frac{\mu_0}{2\pi} \frac{I}{r} \left(\frac{r}{R} \right)^2 = \frac{\mu_0}{2\pi} \frac{I r}{R^2}$$

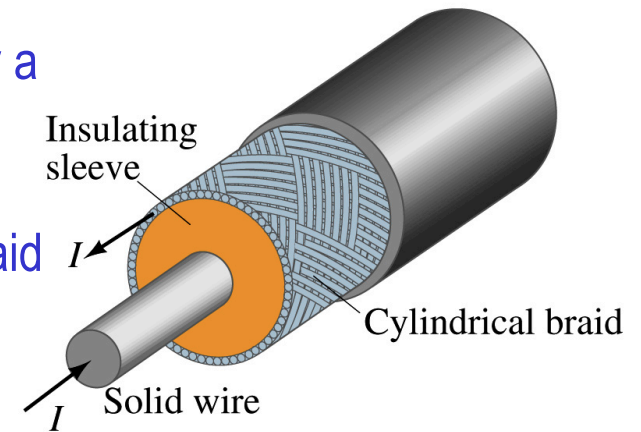
What does this mean?

The field is 0 at $r=0$ and increases linearly as a function of the distance from the center of the wire up to $r=R$ then decreases as $1/r$ beyond the radius of the conductor.



Example 28 – 7

Coaxial cable. A coaxial cable is a single wire surrounded by a cylindrical metallic braid, as shown in the figure. The two conductors are separated by an insulator. The central wire carries current to the other end of the cable, and the outer braid carries the return current and is usually considered ground. Describe the magnetic field (a) in the space between the conductors and (b) outside the cable.



(a) The magnetic field between the conductors is the same as the long, straight wire case since the current in the outer conductor does not impact the enclosed current.

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

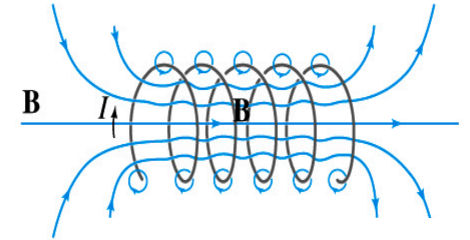
(b) Outside the cable, we can draw a similar circular path, since we expect the field to have a circular symmetry. What is the sum of the total current inside the closed path? $I_{encl} = I - I = 0$.

So there is no magnetic field outside a coaxial cable. In other words, the coaxial cable self-shields. The outer conductor also shields against an external electric field. Cleaner signal and less noise.

Solenoid and Its Magnetic Field

- What is a solenoid?

- A long coil of wire consisting of many loops
- If the space between loops are wide

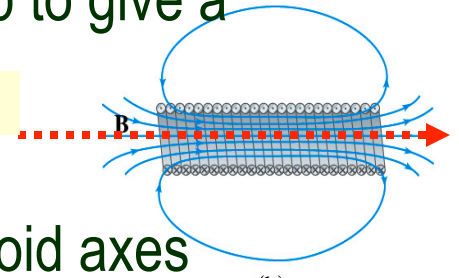


- The field near the wires are nearly circular
- Between any two wires, the fields due to each loop cancel
- Toward the center of the solenoid, the fields add up to give a field that can be fairly large and uniform

- For a long, densely packed loops

- The field is nearly uniform and parallel to the solenoid axes within the entire cross section
- The field outside the solenoid is very small compared to the field inside, except the ends

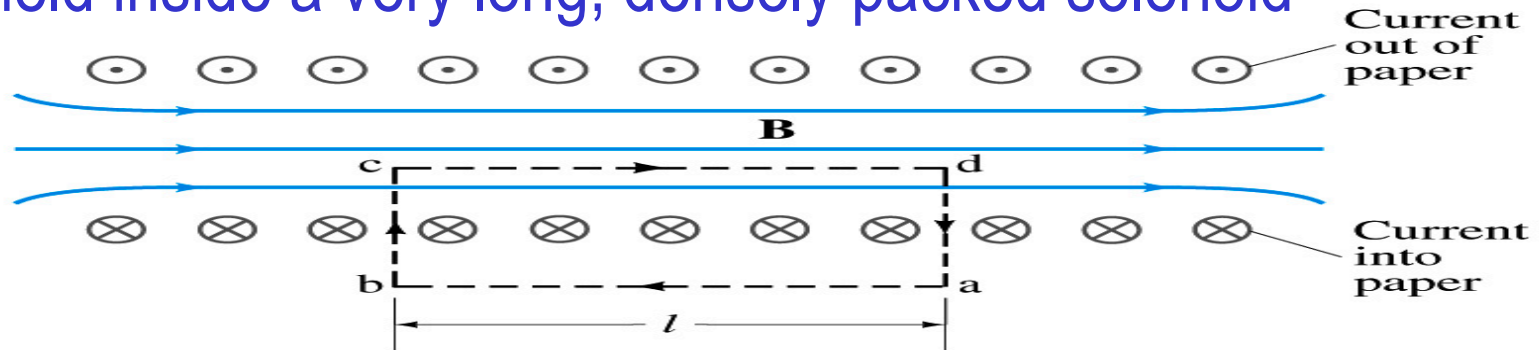
Solenoid Axis



- The same number of field lines spread out to an open space

Solenoid Magnetic Field

- Now let's use Ampere's law to determine the magnetic field inside a very long, densely packed solenoid



- Let's choose the path $abcd$, far away from the ends
 - We can consider four segments of the loop for integral
 - $$\oint \vec{B} \cdot d\vec{l} = \int_a^b \vec{B} \cdot d\vec{l} + \int_b^c \vec{B} \cdot d\vec{l} + \int_c^d \vec{B} \cdot d\vec{l} + \int_d^a \vec{B} \cdot d\vec{l}$$
 - The field outside the solenoid is negligible. So the integral on $a \rightarrow b$ is 0.
 - Now the field B is perpendicular to the bc and da segments. So these integrals become 0, also.

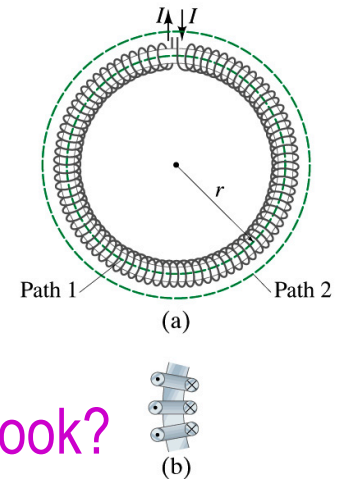
Solenoid Magnetic Field

- So the sum becomes: $\oint \vec{B} \cdot d\vec{l} = \int_c^d \vec{B} \cdot d\vec{l} = Bl$
- If the current I flows in the wire of the solenoid, the total current enclosed by the closed path is $\mathcal{N}I$
 - Where \mathcal{N} is the number of loops (or turns of the coil) enclosed
- Thus Ampere's law gives us $Bl = \mu_0 \mathcal{N}I$
- If we let $n = \mathcal{N}/l$ be the number of loops per unit length, the magnitude of the magnetic field within the solenoid becomes
- $B = \mu_0 nI$

 - B depends on the number of loops per unit length, n , and the current I
 - Does not depend on the position within the solenoid but uniform inside it, like a bar magnet

Example 28 – 10

Toroid. Use Ampere's law to determine the magnetic field (a) inside and (b) outside a toroid, which is like a solenoid bent into the shape of a circle.



(a) How do you think the magnetic field lines inside the toroid look?

Since it is a bent solenoid, it should be a circle concentric with the toroid. If we choose path of integration one of these field lines of radius r inside the toroid, path 1, to use the symmetry of the situation, making B the same at all points on the path, we obtain from Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 NI$$

Solving for B

$$B = \frac{\mu_0 NI}{2\pi r}$$

So the magnetic field inside a toroid is not uniform. It is larger on the inner edge. However, the field will be uniform if the radius is large and the toroid is thin. The field in this case is $B = \mu_0 n I$.

(b) Outside the solenoid, the field is 0 since the net enclosed current is 0.