PHYS 1444 – Section 003
Lecture #22

Tuesday, Nov. 29, 2011

Dr. Jaehoon Yu

- Electric Inductance
- Energy Stored in the Magnetic Field
- LR circuit
- LC Circuit and EM Oscillation
- LRC circuit
- AC Circuit w/ Resistance only
- AC Circuit w/ Inductance only

Today's homework is #12, due 10pm, Friday, Dec. 9!!

Announcements

- Your planetarium extra credit
  - Please bring your planetarium extra credit sheet by the beginning of the class next Tuesday, Dec. 6
  - Be sure to tape one edge of the ticket stub with the title of the show on top
  - Be sure to write your name onto the sheet
- Quiz #4
  - Coming Tuesday, Dec. 6
  - Covers CH30.1 through what we finish this Thursday
- Reading Assignments
  - CH30.9 – CH30.11
- Colloquium this week
  - Dr. Andy White (just been elected to be an APS fellow!!)

Physics Department
The University of Texas at Arlington

COLLOQUIUM

The International Linear Collider–Accelerator, Detectors, and Physics

Dr. Andrew White

Department of Physics
University of Texas at Arlington

4:00 pm Wednesday November 30, 2011 room 101 SH

Abstract:

This colloquium will describe the context, role, and design of the future International Linear Collider. The physics context, in terms of the latest results from the CERN Large Hadron Collider will be described. The design status of the ILC accelerator will be summarized with a focus on superconducting radio-frequency cavity development. The design and proposed technologies for ILC collider detectors will be presented, including a new type of calorimetry being developed at UTA. The physics potential of the ILC will be described, and, finally, the possible timesale for implementation of the ILC will be discussed.

Refreshments will be s [illegible] Physics Lounge

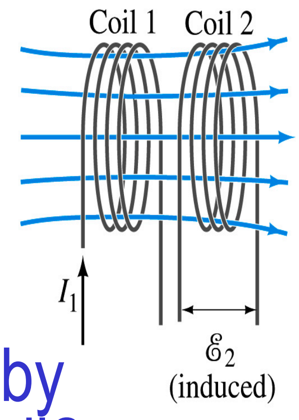
Inductance

- Changing magnetic flux through a circuit induce an emf in that circuit
- An electric current produces a magnetic field
- From these, we can deduce
  - A changing current in one circuit must induce an emf in a nearby circuit ➔ Mutual inductance
  - Or induce an emf in itself ➔ Self inductance

Mutual Inductance

- If two coils of wire are placed near each other, a changing current in one will induce an emf in the other.
- What is the induced emf, ε_2, in coil2 proportional to?
  - Rate of the change of the magnetic flux passing through it
- This flux is due to current I_1 in coil 1
- If Φ_{21} is the magnetic flux in each loop of coil2 created by coil1 and N_2 is the number of closely packed loops in coil2, then $N_2\Phi_{21}$ is the total flux passing through coil2.
- If the two coils are fixed in space, $N_2\Phi_{21}$ is proportional to the current I_1 in coil 1, $N_2\Phi_{21} = M_{21} I_1$.
- The proportionality constant for this is called the Mutual Inductance and defined by $M_{21} = N_2\Phi_{21}/I_1$.
- The emf induced in coil2 due to the changing current in coil1 is

$$\varepsilon_2 = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d\left(N_2\Phi_{21}\right)}{dt} = -M_{21}\frac{dI_1}{dt}$$



Mutual Inductance

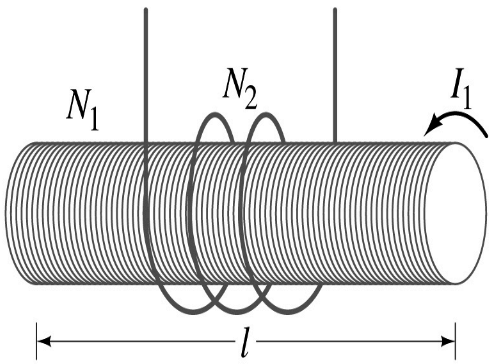
- The mutual induction of coil2 with respect to coil1, M_{21},
  - is a constant and does not depend on I_1.
  - depends only on "geometric" factors such as the size, shape, number of turns and relative position of the two coils, and whether a ferromagnetic material is present

    What? Does this make sense?

    - The farther apart the two coils are the less flux can pass through coil, 2, so M_{21} will be less.
  - Most cases the mutual inductance is determined experimentally
- Conversely, the changing current in coil2 will induce an emf in coil1
- $$\varepsilon_1 = -M_{12}\frac{dI_2}{dt}$$
  - M_{12} is the mutual inductance of coil1 with respect to coil2 and $M_{12} = M_{21}$
  - We can put $M=M_{12}=M_{21}$ and obtain $\varepsilon_1 = -M\frac{dI_2}{dt}$ and $\varepsilon_2 = -M\frac{dI_1}{dt}$
  - SI unit for mutual inductance is henry (H) $1H = 1V\cdot s/A = 1\Omega\cdot s$

Example 30 – 1

Solenoid and coil. A long thin solenoid of length ℓ and cross-sectional area A contains N_1 closely packed turns of wire. Wrapped around it is an insulated coil of N_2 turns. Assume all the flux from coil 1 (the solenoid) passes through coil 2, and calculate the mutual inductance.



First we need to determine the flux produced by the solenoid.

What is the magnetic field inside the solenoid? $B = \dfrac{\mu_0 N_1 I_1}{l}$

Since the solenoid is closely packed, we can assume that the field lines are perpendicular to the surface area of the coils. Thus the flux through coil 2 is

$$\Phi_{21} = BA = \frac{\mu_0 N_1 I_1}{l} A$$

Thus the mutual inductance of coil 2 is

$$M_{21} = \frac{N_2 \Phi_{21}}{I_1} = \frac{N_2}{I_1}\frac{\mu_0 N_1 I_1}{l} A = \frac{\mu_0 N_1 N_2}{l} A$$

Note that M_{21} only depends on geometric factors!

Self Inductance

- The concept of inductance applies to a single isolated coil of N turns. How does this happen?
  - When a changing current passes through a coil
  - A changing magnetic flux is produced inside the coil
  - The changing magnetic flux in turn induces an emf in the same coil
  - This emf opposes the change in flux. Whose law is this?
    - Lenz's law
- What would this do?
  - When the current through the coil is increasing?
    - The increasing magnetic flux induces an emf that opposes the original current
    - This tends to impedes its increase, trying to maintain the original current
  - When the current through the coil is decreasing?
    - The decreasing flux induces an emf in the same direction as the current
    - This tends to increase the flux, trying to maintain the original current

Self Inductance

- Since the magnetic flux Φ_B passing through N turn coil is proportional to current I in the coil, $N\Phi_B = L I$
- We define self-inductance, $\mathcal{L}$: $L = \frac{N\Phi_B}{I}$ **Self Inductance**
- The induced emf in a coil of self-inductance $\mathcal{L}$ is
  - $\varepsilon = -N\frac{d\Phi_B}{dt} = -L\frac{dI}{dt}$
  - What is the unit for self-inductance? $1H = 1V \cdot s/A = 1\Omega \cdot s$
- What does magnitude of $\mathcal{L}$ depend on?
  - Geometry and the presence of a ferromagnetic material
- Self inductance can be defined for any circuit or part of a circuit

So what in the world is the Inductance?

- It is an impediment onto the electrical current due to the existence of changing flux
- So what?
- In other words, it behaves like a resistance to the varying current, such as AC, that causes the constant change of flux
- But it also provides means to store energy, just like the capacitance

Inductor

- An electrical circuit always contains some inductance but is normally negligibly small
  - If a circuit contains a coil of many turns, it could have large inductance
- A coil that has significant inductance, $\mathcal{L}$, is called an inductor and is express with the symbol 
  - Precision resisters are normally wire wound
    - Would have both resistance and inductance
    - The inductance can be minimized by winding the wire back on itself in opposite direction to cancel magnetic flux
    - This is called a "non-inductive winding"
- If an inductor has negligible resistance, inductance controls a changing current
- For an AC current, the greater the inductance the less the AC current
  - An inductor thus acts like a resistor to impede the flow of alternating current (not to DC, though. Why?)
  - The quality of an inductor is indicated by the term **reactance** or **impedance**

Example 30 – 3

Solenoid inductance. (a) Determine a formula for the self inductance $\mathcal{L}$ of a tightly wrapped solenoid (a long coil) containing N turns of wire in its length ℓ and whose cross-sectional area is A. (b) Calculate the value of $\mathcal{L}$ if N=100, ℓ=5.0cm, A=0.30cm^2 and the solenoid is air filled. (c) calculate $\mathcal{L}$ if the solenoid has an iron core with $\mu=4000\mu_0$.

What is the magnetic field inside a solenoid? $B = \mu_0 nI = \mu_0 NI/l$

The flux is, therefore, $\Phi_B = BA = \mu_0 NIA/l$

Using the formula for self inductance: $L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 A}{l}$

(b) Using the formula above

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\left(4\pi \times 10^{-7}\, T\cdot m/A\right)100^2\left(0.30\times 10^{-4}\, m^2\right)}{5.0\times 10^{-2}\, m} = 7.5\mu H$$

(c) The magnetic field with an iron core solenoid is $B = \mu NI/l$

$$L = \frac{\mu N^2 A}{l} = \frac{4000\left(4\pi \times 10^{-7}\, T\cdot m/A\right)100^2\left(0.30\times 10^{-4}\, m^2\right)}{5.0\times 10^{-2}\, m} = 0.030H = 30mH$$

Energy Stored in a Magnetic Field

- When an inductor of inductance $\mathcal{L}$ is carrying current I which is changing at a rate dI/dt, energy is supplied to the inductor at a rate
  - $P = I\varepsilon = IL\frac{dI}{dt}$
- What is the work needed to increase the current in an inductor from 0 to I?
  - The work, dW, done in time dt is $dW = Pdt = LI\,dI$
  - Thus the total work needed to bring the current from 0 to I in an inductor is

$$W = \int dW = \int_0^I LI\,dI = L\left[\frac{1}{2}I^2\right]_0^I = \frac{1}{2}LI^2$$

Energy Stored in a Magnetic Field

- The work done to the system is the same as the energy stored in the inductor when it is carrying current I
  - $$U = \frac{1}{2}LI^2$$

    Energy Stored in a magnetic field inside an inductor
  - This is compared to the energy stored in a capacitor, C, when the potential difference across it is V: $U = \frac{1}{2}CV^2$
  - Just like the energy stored in a capacitor is considered to reside in the electric field between its plates
  - The energy in an inductor can be considered to be stored in its magnetic field

Stored Energy in terms of B

- So how is the stored energy written in terms of magnetic field B?
  - Inductance of an ideal solenoid without a fringe effect
    $$L = \mu_0 N^2 A / l$$
  - The magnetic field in a solenoid is $B = \mu_0 NI / l$
  - Thus the energy stored in an inductor is
    $$U = \frac{1}{2}LI^2 = \frac{1}{2}\frac{\mu_0 N^2 A}{l}\left(\frac{Bl}{\mu_0 N}\right)^2 = \frac{1}{2}\frac{B^2}{\mu_0}Al$$
  - Thus the energy density is
    $$u = \frac{U}{V} = \frac{U}{Al} = \frac{1}{2}\frac{B^2}{\mu_0}$$
  - This formula is valid in any region of space
  - If a ferromagnetic material is present, μ_0 becomes μ.

What is this?

Volume V

$$U = \frac{1}{2}\frac{B^2}{\mu_0}Al$$ E

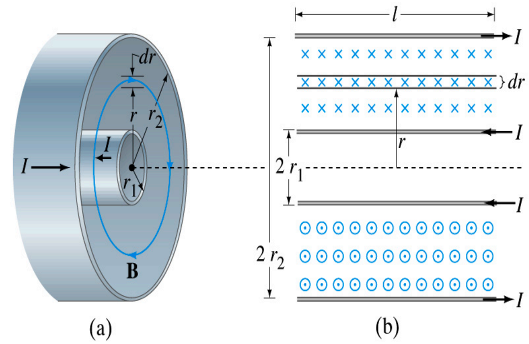
$$u = \frac{1}{2}\frac{B^2}{\mu_0}$$ E density

What volume does Al represent?

The volume inside a solenoid!!

Example 30 – 5

Energy stored in a coaxial cable. (a) How much energy is being stored per unit length in a coaxial cable whose conductors have radii r_1 and r_2 and which carry a current I? (b) Where is the energy density highest?



(a) The total flux through l of the cable is $\Phi_B = \int Bl\,dr = \frac{\mu_0 Il}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{\mu_0 Il}{2\pi}\ln\frac{r_2}{r_1}$

Thus inductance per unit length for a coaxial cable is $\frac{L}{l} = \frac{\mu_0}{2\pi}\ln\frac{r_2}{r_1}$

Thus the energy stored per unit length is $\frac{U}{l} = \frac{1}{2}\frac{LI^2}{l} = \frac{\mu_0 I^2}{4\pi}\ln\frac{r_2}{r_1}$

(b) Since the magnetic field is $B = \frac{\mu_0 I}{2\pi r}$

And the energy density is $u = \frac{1}{2}\frac{B^2}{\mu_0}$

The energy density is highest where B is highest. Since B is highest close to $r=r_1$, near the surface of the inner conductor.