PHYS 3313 – Section 001
Lecture #8

Monday, Sept. 24, 2012
Dr. Jaehoon Yu

- Atomic Model of Thomson
- Rutherford Scattering Experiment and Rutherford Atomic Model
- The Classic Atomic Model
- The Bohr Model of the Hydrogen Atom
- Bohr Radius

Announcements

- Reminder: Homework #2
  - CH3 end of the chapter problems: 2, 19, 27, 36, 41, 47 and 57
  - Due this Wednesday, Sept. 26
- Quiz #2 this Wednesday, Sept. 26
  - Beginning of the class
  - Covers CH1.1 – CH4.3 (Rutherford scattering)
- Conference volunteers, please send e-mail to Dr. Jackson (cbjackson@uta.edu) ASAP!
- Not sure if there is a colloquium this week. I will keep you informed in class Wednesday!

Special Project #3

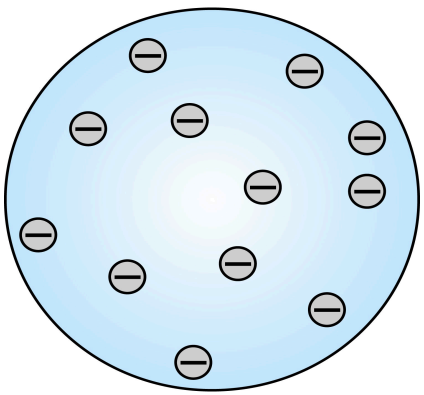
- A total of N_i incident projectile particle of atomic number Z_1 kinetic energy KE scatter on a target of thickness t and atomic number Z_2 and has n atoms per volume. What is the total number of scattered projectile particles at an angle θ? (20 points)
- Please be sure to define all the variables used in your derivation! Points will be deducted for missing variable definitions.
- This derivation must be done on your own. Please do not copy the book or your friends'.
- Due is Monday, Oct. 8.

The Atomic Models of Thomson and Rutherford

- Pieces of evidence that scientists had in 1900 to indicate that the atom was not a fundamental unit
- There are simply too many kinds of atoms (~70 known at that time), belonging to a distinct chemical element
  - Too many to be fundamental!!
- Atoms and electromagnetic phenomena seem to be intimately related
- The issue **valence** → Why certain elements combine with some elements but not with others
  - Is there a characteristic internal atomic structure?
- The discoveries of radioactivity, x rays, and the electron

Thomson's Atomic Model

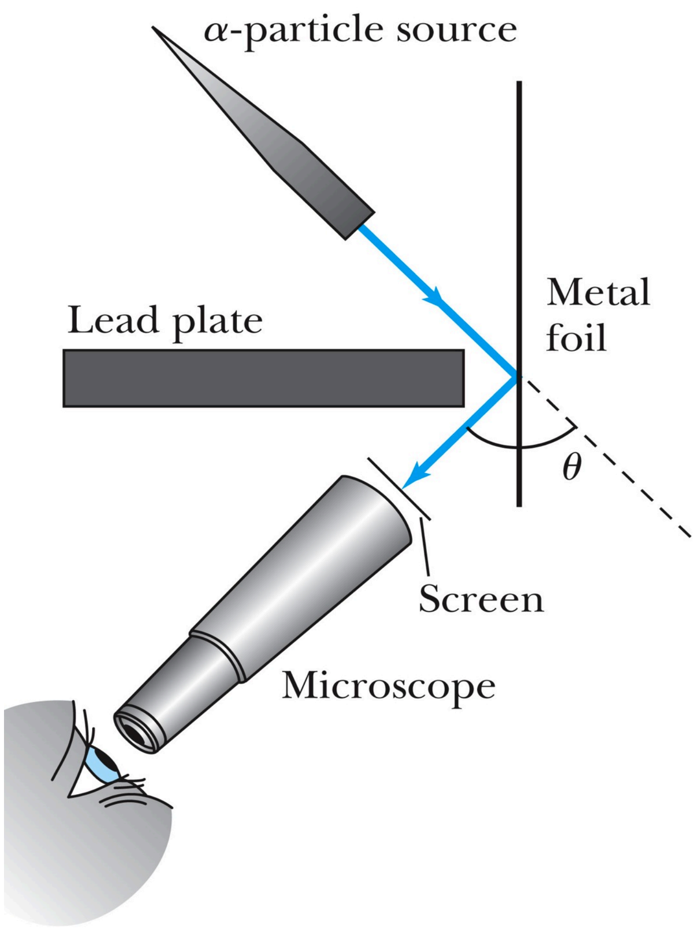
- Thomson's "plum-pudding" model
  - Atoms are electrically neutral and have electrons in them
  - Atoms must have equal amount of positive charges in it to balance electron negative charges
  - So how about positive charges spread uniformly throughout a sphere the size of the atom with, the newly discovered "negative" electrons embedded in the uniform background.



- Thomson's thought when the atom was heated the electrons could vibrate about their equilibrium positions, thus producing electromagnetic radiation.

Experiments of Geiger and Marsden

- Rutherford, Geiger, and Marsden conceived a new technique for investigating the structure of matter by scattering a particles from atoms.
- Geiger showed that many a particles were scattered from thin gold-leaf targets at backward angles greater than 90°.



Ex 4.1: Maximum Scattering Angle

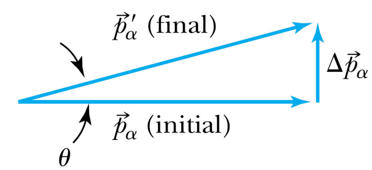
Geiger and Marsden (1909) observed backward-scattered (θ>=90°) α particles when a beam of energetic α particles was directed at a piece of gold foil as thin as 6.0×10^{-7}m. Assuming an α particle scatters from an electron in the foil, what is the maximum scattering angle?

- The maximum scattering angle corresponding to the maximum momentum change
- Using the momentum conservation and the KE conservation for an elastic collision, the maximum momentum change of the α particle is

$$M_\alpha \vec{v}_\alpha = M_\alpha \vec{v}_\alpha^{\,'} + m_e \vec{v}_e^{\,'}$$
$$\frac{1}{2}M_\alpha v_\alpha^2 = \frac{1}{2}M_\alpha v_\alpha^{'2} + \frac{1}{2}m_2 v_e^2$$

$$\Rightarrow \Delta\vec{p}_\alpha = M_\alpha \vec{v}_\alpha - M_\alpha \vec{v}_\alpha^{\,'} = m_e \vec{v}_e^{\,'} \Rightarrow \Delta p_{\alpha-\max} = 2m_e v_\alpha$$

- Determine θ by letting Δp_{max} be perpendicular to the direction of motion.



$$\theta_{\max} = \frac{\Delta p_{\alpha-\max}}{p_\alpha} = \frac{2m_e v_\alpha}{m_\alpha v_\alpha} = \frac{2m_e}{m_\alpha} = 2.7\times10^{-4}\,rad = 0.016^\circ$$

Multiple Scattering from Electrons

- If an α particle were scattered by many electrons and *N* electrons results in $\langle\theta\rangle_{total} \sim \sqrt{N}\theta$
- The number of atoms across the thin gold layer of 6×10^{-7} m:

$$\frac{\text{Number of molecules}}{\text{cm}^3} = [\text{Avogadro's no. (molecules/mol)}] \times \left[\frac{1}{\text{gram} - \text{molecular weight}}\left(\frac{\text{mol}}{\text{g}}\right)\right]\left[\text{density}\left(\frac{\text{g}}{\text{cm}^3}\right)\right]$$

$$= \left(6.02 \times 10^{23}\,\frac{\text{molecules}}{\text{mol}}\right)\left(\frac{1\text{ mol}}{197\text{ g}}\right)\left(19.3\,\frac{\text{g}}{\text{cm}^3}\right)$$

$$= 5.9 \times 10^{22}\,\frac{\text{molecules}}{\text{cm}^3} = 5.9 \times 10^{28}\,\frac{\text{atoms}}{\text{m}^3}$$

- Assume the distance between atoms is $d = (5.9 \times 10^{28})^{-1/3}\text{m} = 2.6 \times 10^{-10}\text{m}$

  and there are $N = \dfrac{6 \times 10^{-7}\text{m}}{2.6 \times 10^{-10}\text{m}} = 2300\text{ atoms}$

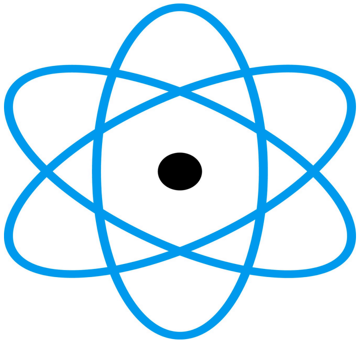
  That gives $\langle\theta\rangle_{\text{total}} = \sqrt{2300}(0.016°) = 0.8°$

Rutherford's Atomic Model

- $<\theta>_{total} \sim 0.8^{o}$ even if the α particle scattered from all 79 electrons in each atom of gold

The experimental results were inconsistent with Thomson's atomic model.

- Rutherford proposed that an atom has a positively charged core (nucleus) surrounded by the negative electrons.



Assumptions of Rutherford Scattering

1. The scatterer is so massive that it does not recoil significantly; therefore the initial and final KE of the α particle are practically equal.
2. The target is so thin that only a single scattering occurs.
3. The bombarding particle and target scatterer are so small that they may be treated as point masses and charges.
4. Only the Coulomb force is effective.

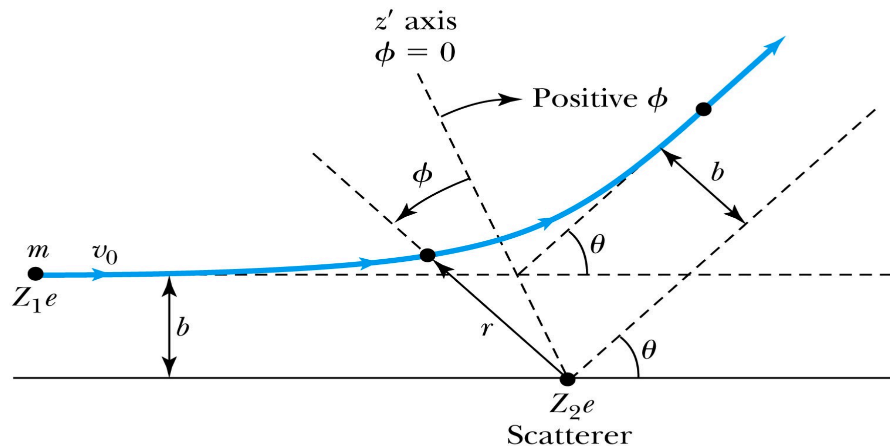
Rutherford Scattering

- Scattering experiments help us study matter too small to be observed directly by measuring the angular distributions of the scattered particles
  - What is the force acting in this scattering?
- There is a relationship between the impact parameter *b* and the scattering angle *θ*.

When *b* is small,

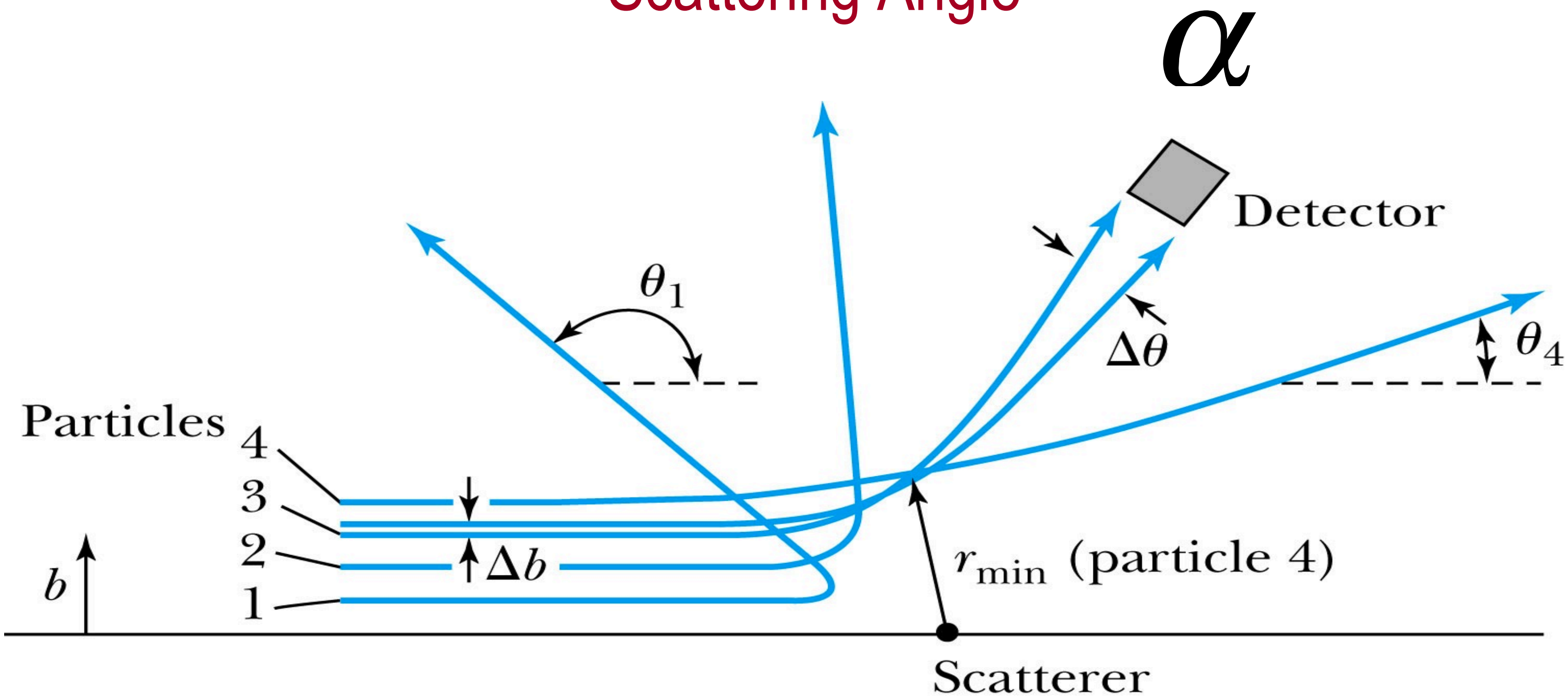
r gets small.

Coulomb force gets large.



θ can be large and the particle can be repelled backward.

The Relationship Between the Impact Parameter b and the Scattering Angle



The relationship between the impact parameter b and scattering angle $\Delta\theta$. Particles with small impact parameters approach the nucleus most closely (r_{min}) and scatter to the largest angles. Particles within the range of impact parameters b will be scattered within $\Delta\theta$.

Rutherford Scattering

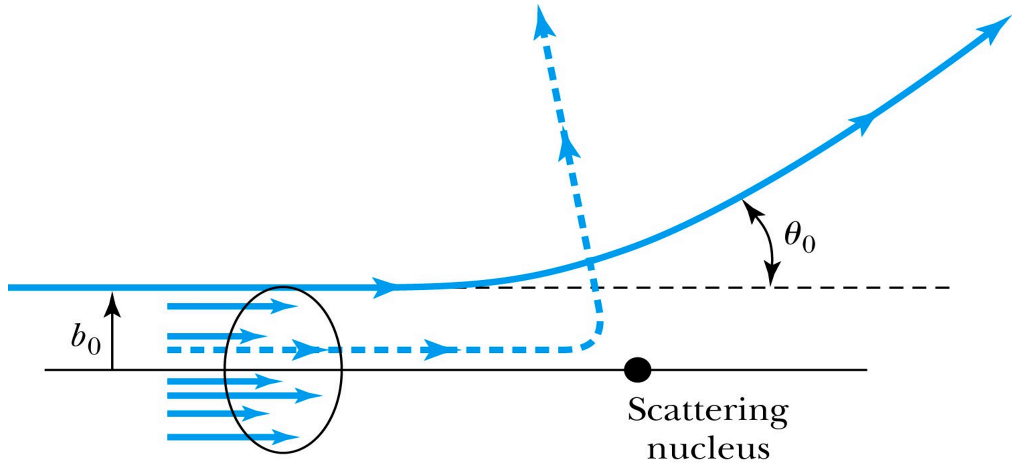
- What are the quantities that can affect the scattering?
  - What was the force again?
    - The Coulomb force

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r^2}\hat{r}_e$$

  - The charge of the incoming particle (Z_1e)
  - The charge of the target particle (Z_2e)
  - The minimum distance the projectile approaches the target (r)
- Using the fact that this is a totally elastic scattering under a central force, we know
  - Linear momentum is conserved $\vec{p}_i^{\,\alpha} = \vec{p}_f^{\,\alpha} + \vec{p}^{\,N}$
  - KE is conserved $\frac{1}{2}mv_{\alpha i}^2 = \frac{1}{2}mv_{\alpha f}^2 + \frac{1}{2}mv_n^2$
  - Angular momentum is conserved $mr^2\varpi = mv_{\alpha i}b$
- From this, impact parameter $b = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 mv_{\alpha i}^2}\cot\frac{\theta}{2} = \frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 KE_i}\cot\frac{\theta}{2}$

Rutherford Scattering - probability

- Any particle inside the circle of area πb_0^2 will be similarly scattered.



- The **cross section** $\sigma = \pi b^2$ is related to the **probability** for a particle being scattered by a nucleus.

$$nt\pi b^2 = \pi nt \left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 KE_i} \cot\frac{\theta}{2} \right)^2$$

t: target thickness
n: atomic number density

- The fraction of incident particles scattered is

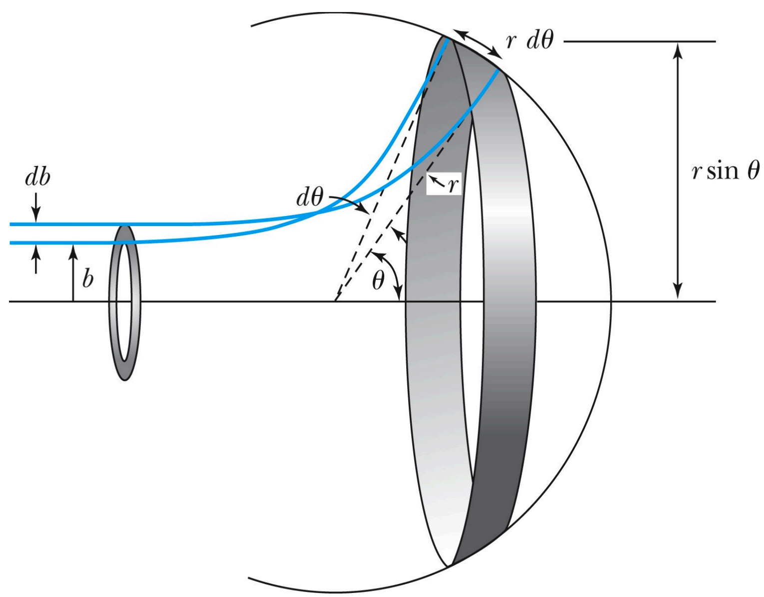
$$f = \frac{\text{target area exposed by scatterers}}{\text{total target area}}$$

- The number of scattering nuclei per unit area

$$nt = \frac{\rho N_A N_M t}{M_g} \frac{\text{atoms}}{\text{cm}^2}$$

Rutherford Scattering Equation

- In actual experiment a detector is positioned from θ to $\theta + d\theta$ that corresponds to incident particles between *b* and *b* + *db*.



- The number of particles scattered into the the angular coverage per unit area is

$$N(\theta) = \frac{N_i nt}{16}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{Z_1^{\ 2} Z_2^{\ 2}}{r^2 K^2 \sin^4(\theta/2)}$$

The Important Points

1. The scattering is proportional to the square of the atomic number of *both* the incident particle (Z_1) and the target scatterer (Z_2).
2. The number of scattered particles is inversely proportional to the square of the kinetic energy of the incident particle.
3. For the scattering angle θ, the scattering is proportional to 4^{th} power of $\sin(\theta/2)$.
4. The Scattering is proportional to the target thickness for thin targets.

The Classical Atomic Model

As suggested by the Rutherford Model the atom consisted of a small, massive, positively charged nucleus surrounded by moving electrons. This then suggested consideration of a planetary model of the atom.

Let's consider atoms as a planetary model.

- The force of attraction on the electron by the nucleus and Newton's 2nd law give

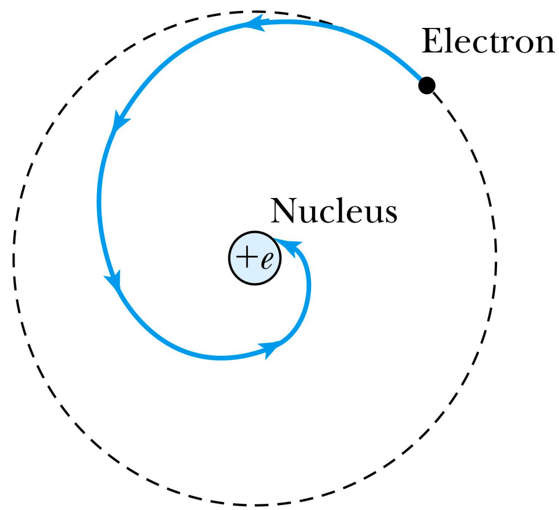
$$\vec{F}_e = \frac{-1}{4\pi\varepsilon_0}\frac{e^2}{r^2}\hat{e}_r = \frac{mv^2}{r}$$

where *v* is the tangential velocity of the electron.

- The total energy is $E = K + V = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{-e^2}{8\pi\varepsilon_0 r}$

The Planetary Model is Doomed

- From classical E&M theory, an accelerated electric charge radiates energy (electromagnetic radiation) which means total energy must decrease. → *Radius r must decrease!!*



Electron crashes into the nucleus!?

- Physics had reached a turning point in 1900 with Planck's hypothesis of the quantum behavior of radiation.

The Bohr Model of the Hydrogen Atom – The assumptions

- "Stationary" states or orbits must exist in atoms, i.e., orbiting electrons *do not radiate* energy in these orbits. These orbits or stationary states are of a fixed definite energy E.
- The emission or absorption of electromagnetic radiation can occur only in conjunction with a transition between two stationary states. The frequency, f, of this radiation is proportional to the *difference* in energy of the two stationary states:
- $$E = E_1 - E_2 = hf$$
- *where h is Planck's Constant*
- Classical laws of physics do not apply to transitions between stationary states.
- The mean kinetic energy of the electron-nucleus system is quantized as $K = nhf_{orb}/2$, where f_{orb} is the frequency of rotation. This is equivalent to the angular momentum of a stationary state to be an integral multiple of h/2 π