PHYS 3313 – Section 001
Lecture #9

Wednesday, Sept. 26, 2012
Dr. Jaehoon Yu

- The Bohr Model of the Hydrogen Atom
- Bohr Radius
- Fine Structure Constant
- The Correspondence Principle
- Characteristic X-ray Spectra
- Atomic Excitation by Electrons

Announcements

- Reading assignments: CH4.6 and CH4.7
- Mid-term exam
  - In class on Wednesday, Oct. 10, in PKH107
  - Covers: CH1.1 to what we finish Wednesday Oct. 3
  - Style: Mixture of multiple choices and free response problems which are more heavily weighted
  - Mid-term exam constitutes 20% of the total
- Conference volunteers, please send e-mail to Dr. Jackson (cbjackson@uta.edu) ASAP!
  - Extra credit of 3 points per each hour served, as good as attending the class!!
- Colloquium today
  - 4pm, SH101
  - Dr. Kaushik De on latest LHC results

Special Project #3

- A total of N_i incident projectile particles of atomic number Z_1 kinetic energy KE scatter on a target of thickness t, atomic number Z_2 and with n atoms per volume. What is the total number of scattered projectile particles at an angle θ? (20 points)
- Please be sure to define all the variables used in your derivation! Points will be deducted for missing variable definitions.
- This derivation must be done on your own. Please do not copy the book or your friends'.
- Due is Monday, Oct. 8.

The Bohr Model of the Hydrogen Atom – The assumptions

- "Stationary" states or orbits must exist in atoms, i.e., orbiting electrons *do not radiate* energy in these orbits. These orbits or stationary states are of a fixed definite energy E.
- The emission or absorption of electromagnetic radiation can occur only in conjunction with a transition between two stationary states. The frequency, f, of this radiation is proportional to the *difference* in energy of the two stationary states:
- $$E = E_1 - E_2 = hf$$
- *where h is Planck's Constant*
  - *Bohr thought this has to do with fundamental length of order ~10^{-10}m*
- Classical laws of physics do not apply to transitions between stationary states.
- The mean kinetic energy of the electron-nucleus system is quantized as $K = nhf_{orb}/2$, where f_{orb} is the frequency of rotation. This is equivalent to the angular momentum of a stationary state to be an integral multiple of $h/2\pi$

How did Bohr Arrived at the angular momentum quantization?

- The mean kinetic energy of the electron-nucleus system is quantized as $K = nhf_{orb}/2$, where f_{orb} is the frequency of rotation. This is equivalent to the angular momentum of a stationary state to be an integral multiple of $h/2\pi$.
- Kinetic energy can be written $K = \frac{nhf}{2} = \frac{1}{2}mv^2$
- Angular momentum is defined as $\left|\vec{L}\right| = \left|\vec{r} \times \vec{p}\right| = mvr$
- The relationship between linear and angular quantifies $v = r\omega;\ \omega = 2\pi f$
- Thus, we can rewrite $K = \frac{1}{2}mvr\omega = \frac{1}{2}L\omega = \frac{1}{2}2\pi Lf = \frac{nhf}{2}$

$$2\pi L = nh \Rightarrow \quad L = n\frac{h}{2\pi} = n\hbar \quad \text{, where } \hbar = \frac{h}{2\pi}$$

Bohr's Quantized Radius of Hydrogen

- The angular momentum is $\left|\vec{L}\right| = \left|\vec{r} \times \vec{p}\right| = mvr = n\hbar$
- So the speed of an orbiting e can be written $v_e = \dfrac{n\hbar}{m_e r}$
- From the Newton's law for a circular motion

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = \frac{m_e v_e^2}{r} \Rightarrow v_e = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}}$$

- So from above two equations, we can get

$$v_e = \frac{n\hbar}{m_e r} = \frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}} \Rightarrow r = \frac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2}$$

Bohr Radius

- The radius of the hydrogen atom for stationary states is

$$r_n = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{m_e e^2} = a_0 n^2$$

Where the **Bohr radius** for a given stationary state is:

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = \frac{\left(8.99\times10^{9}\,N\cdot m^2/C^2\right)\cdot\left(1.055\times10^{-34}\,J\cdot s\right)^2}{\left(9.11\times10^{-31}\,kg\right)\cdot\left(1.6\times10^{-19}\,C\right)^2} = 0.53\times10^{-10}\,m$$

- The smallest diameter of the hydrogen atom is

$$d = 2r_1 = 2a_0 \approx 10^{-10}\,m \approx 1\,\mathring{A}$$

  - OMG!! The fundamental length!!

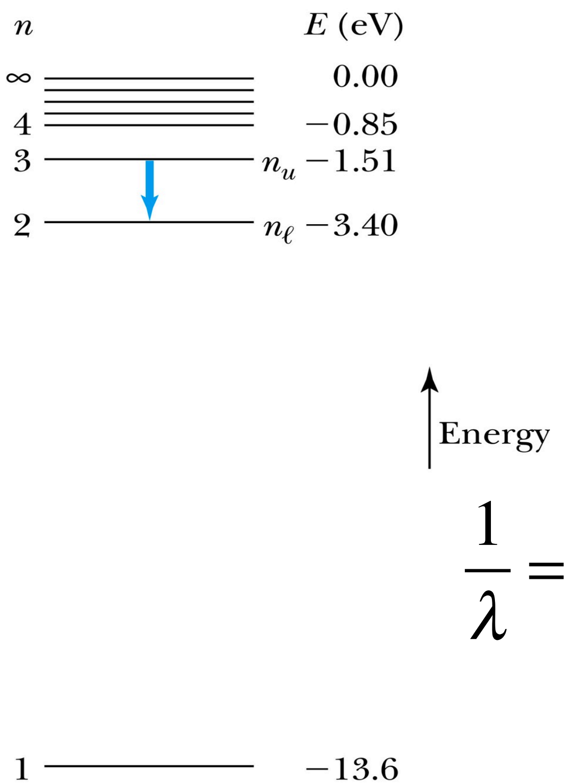
- *n* = 1 gives its lowest energy state (called the "ground" state)

The Hydrogen Atom

- The energies of the stationary states

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 r_n} = -\frac{e^2}{8\pi\varepsilon_0 a_0 n^2} = -\frac{E_0}{n^2} \qquad E_0 = \frac{e^2}{8\pi\varepsilon_0 a_0} = \frac{\left(1.6\times10^{-19}C\right)^2}{8\pi\left(8.99\times10^9\,N\cdot m^2/C^2\right)\cdot\left(0.53\times10^{-10}m\right)} = 13.6eV$$

where E_0 is the ground state energy



- Emission of light occurs when the atom is in an excited state and decays to a lower energy state ($n_u \rightarrow n_\ell$).

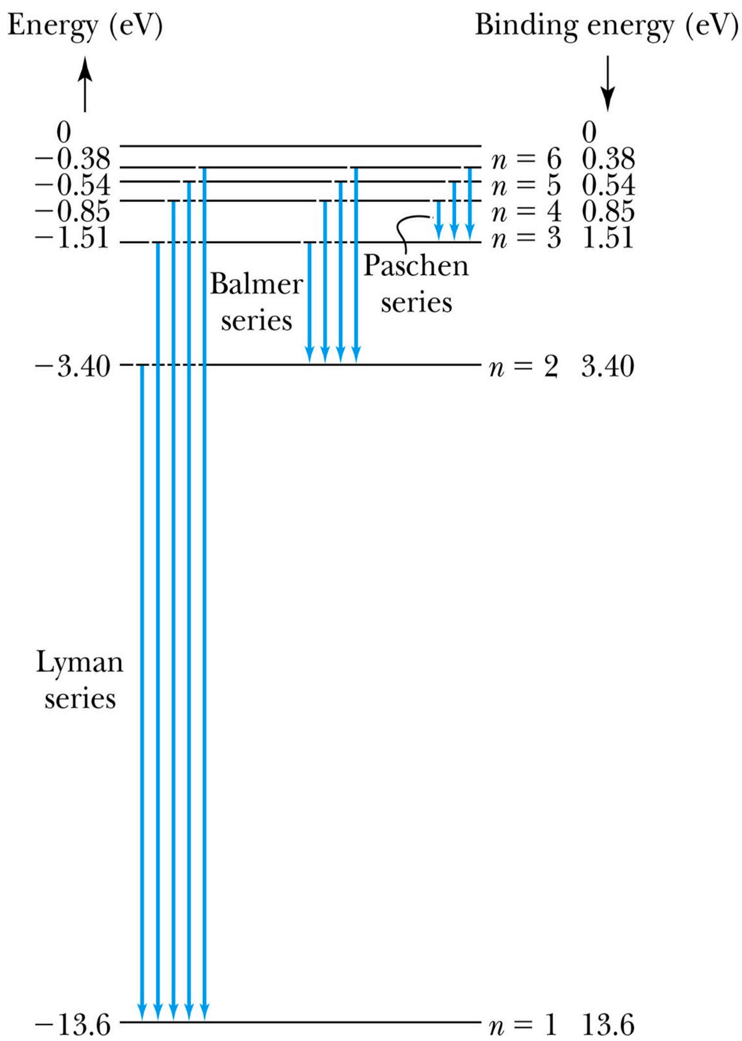
$$hf = E_u - E_l$$

where f is the frequency of a photon.

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{E_u - E_l}{hc} = \frac{E_0}{hc}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) = R_\infty\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$

R_∞ is the **Rydberg constant**. $R_\infty = E_0/hc$

Transitions in the Hydrogen Atom



- **Lyman series:** The atom will remain in the excited state for a short time before emitting a photon and returning to a lower stationary state. All hydrogen atoms exist in n = 1 (invisible).
- **Balmer series:** When sunlight passes through the atmosphere, hydrogen atoms in water vapor absorb the wavelengths (visible).

Fine Structure Constant

- The electron's speed on an orbit in the Bohr model:

$$v_e = \frac{n\hbar}{m_e r_n} = \frac{n\hbar}{m_e \dfrac{4\pi\varepsilon_0 n^2\hbar^2}{m_e e^2}} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_0\hbar}$$

- On the ground state, $v_1 = 2.2 \times 10^6$ m/s ~ less than 1% of the speed of light
- The ratio of v_1 to c is the **fine structure constant, α**.

$$\alpha \equiv \frac{v_1}{c} = \frac{\hbar}{m a_0 c} = \frac{e^2}{4\pi\varepsilon_0\hbar c} =$$

$$\frac{\left(1.6\times10^{-19}C\right)^2}{\left(8.99\times10^9 N\cdot m^2/C^2\right)\cdot\left(1.055\times10^{-34} J\cdot s\right)\cdot\left(3\times10^8 m/s\right)} \approx \frac{1}{137}$$

The Correspondence Principle

Classical electrodynamics + Bohr's atomic model

Determine the properties of radiation

Need a principle to relate the new modern results with classical ones.

Bohr's correspondence principle

In the limits where classical and quantum theories should agree, the quantum theory must produce the classical results.

The Correspondence Principle

- The frequency of the radiation emitted $f_{classical}$ is equal to the orbital frequency f_{orb} of the electron around the nucleus.

$$f_{classical} = f_{obs} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{v}{r} = \frac{1}{2\pi r}\frac{e}{\sqrt{4\pi\varepsilon_0 m_e r}} = \frac{1}{2\pi}\left(\frac{e^2}{4\pi\varepsilon_0 m_e r^3}\right)^{1/2} = \frac{m_e e^4}{4\varepsilon_0^2\hbar^2}\frac{1}{n^3}$$

- The frequency of the transition from $n + 1$ to n is

$$f_{Bohr} = \frac{E_0}{h}\left(\frac{1}{(n)^2} - \frac{1}{(n+1)^2}\right) = \frac{E_0}{h}\frac{n^2 + 2n + 1 - n^2}{n^2(n+1)^2} = \frac{E_0}{h}\left[\frac{2n+1}{n^2(n+1)^2}\right]$$

- For large *n the classical limit*, $f_{Bohr} \approx \frac{2nE_0}{hn^4} = \frac{2E_0}{hn^3}$

Substitute E_0: $f_{Bohr} = \frac{2E_0}{hn^3} = \frac{2}{hn^3}\left(\frac{e^2}{8\pi\varepsilon_0 a_0}\right) = \frac{m_e e^4}{4\varepsilon_0^2\hbar^2}\frac{1}{n^3} = f_{Classical}$

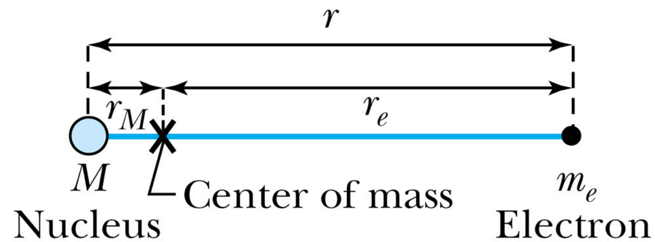
So the frequency of the radiated E between classical theory and Bohr model agrees in large n case!!

Importance of Bohr's Model

- Demonstrated the need for Plank's constant in understanding atomic structure
- Assumption of quantized angular momentum which led to quantization of other quantities, r, v and E as follows
- Orbital Radius: $$r_n = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}n^2 = a_0 n^2$$
- Orbital Speed: $$v = \frac{n\hbar}{mr_n} = \frac{\hbar}{ma_0}\frac{1}{n}$$
- Energy levels: $$E_n = \frac{e^2}{8\pi\varepsilon_0 a_0 n^2} = \frac{E_0}{n^2}$$

Successes and Failures of the Bohr Model

- The electron and hydrogen nucleus actually revolved about their mutual center of mass ➔ reduced mass correction!!



- All we need is to replace m_e with atom's **reduced mass**.

$$\mu_e = \frac{m_e M}{m_e + M} = \frac{m_e}{1 + m_e/M}$$

- The Rydberg constant for infinite nuclear mass, R_∞ is replaced by *R*.

$$R = \frac{\mu_e}{m_e} R_\infty = \frac{1}{1 + m_e/M} R_\infty = \frac{\mu_e e^4}{4\pi c \hbar^3 \left(4\pi\varepsilon_0\right)^2}$$

For H: $R_H = 1.096776 \times 10^7 m^{-1}$

Limitations of the Bohr Model

The Bohr model was a great step of the new quantum theory, but it had its limitations.

1) Works only to single-electron atoms
   - Even for ions ➔ What would change?
   - The charge of the nucleus $$\frac{1}{\lambda} = Z^2 R\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$
2) Could not account for the intensities or the fine structure of the spectral lines
   - Fine structure is caused by the electron spin
3) Could not explain the binding of atoms into molecules

Characteristic X-Ray Spectra and Atomic Number

- Shells have letter names:

  K shell for $n = 1$

  L shell for $n = 2$

  ⋮

- The atom is most stable in its ground state.

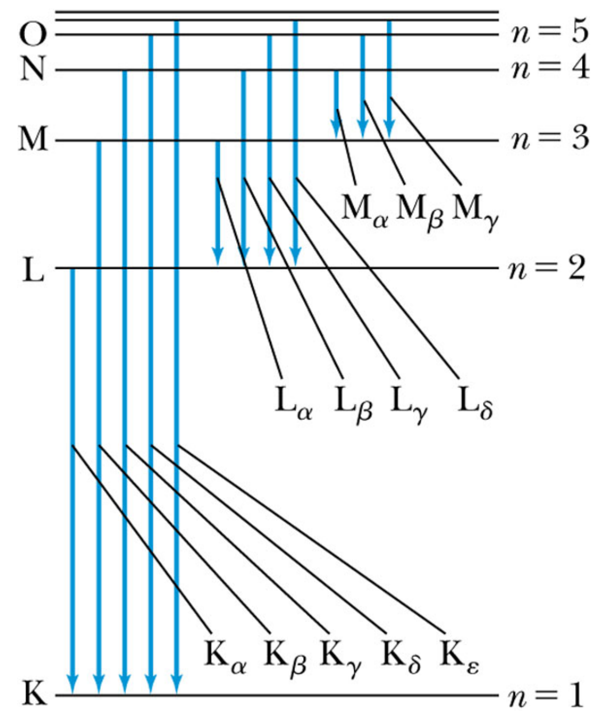
→ An electron from higher shells will fill the inner-shell vacancy at lower energy.

- When a transition occurs in a heavy atom, the radiation emitted is an **x ray**.
- It has the energy E (x ray) $= E_u - E_\ell$.

Atomic Number

L shell to K shell $\longrightarrow$ K_α x ray

M shell to K shell $\longrightarrow$ K_β x ray



- *Atomic number Z = number of protons in the nucleus*
- Moseley found a relationship between the frequencies of the characteristic x ray and Z.

  This holds for the K_α x ray

$$f K_\alpha = \frac{3cR}{4}(Z-1)^2$$

Moseley's Empirical Results

- The x ray is produced from $n = 2$ to $n = 1$ transition.
- In general, the K series of x ray wavelengths are

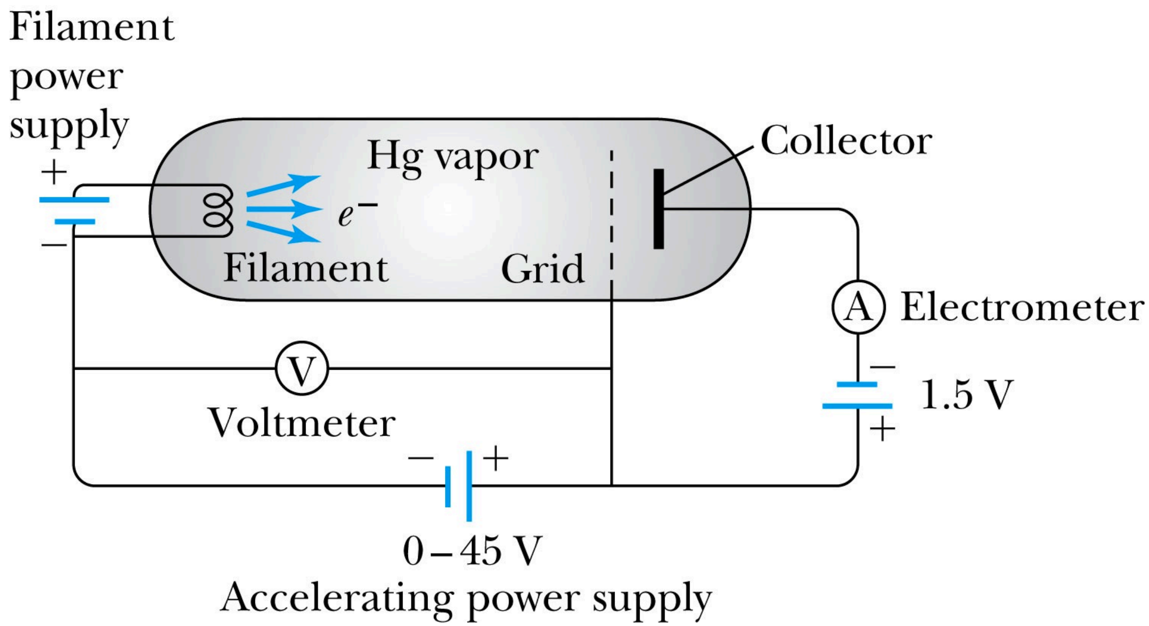
$$\frac{1}{\lambda_K} = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = R(Z-1)^2\left(1 - \frac{1}{n^2}\right)$$

$$\Downarrow$$

Moseley's research clarified the importance of the electron shells for all the elements, not just for hydrogen.

Atomic Excitation by Electrons

- Franck and Hertz studied the phenomenon of ionization.



Accelerating voltage is below 5 V

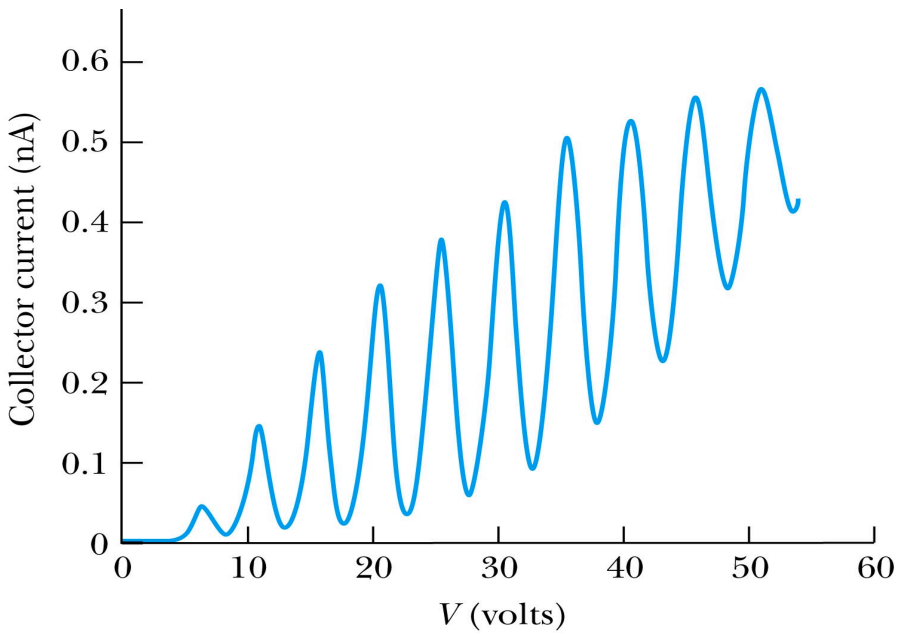
→ electrons did not lose energy

Accelerating voltage is above 5 V

→ sudden drop in the current

Atomic Excitation by Electrons

- Ground state has E_0 to be zero.
  First excited state has E_1.
  The energy difference $E_1 - 0 = E_1$ is the **excitation energy**.



- Hg has an excitation energy of 4.88 eV in the first excited state
- No energy can be transferred to Hg below 4.88 eV because not enough energy is available to excite an electron to the next energy level

- Above 4.88 eV, the current drops because scattered electrons no longer reach the collector until the accelerating voltage reaches 9.8 eV and so on.