PHYS 3313 – Section 001
Lecture #14

Monday, Oct. 22, 2012
Dr. Jaehoon Yu

- Infinite Potential Well
- Finite Potential Well
- Penetration Depth
- Degeneracy
- Simple Harmonic Oscillator
- Parabolic Potential
- Barriers and Tunneling

Announcements

- Mid-term results
  - Class average:59.9/96
    - Equivalent to 62.4/100
  - Stop score: 94/96
- Homework #5
  - CH6 end of chapter problems: 3, 5, 11, 14, 22, and 26
  - Due on Monday, Oct. 29, in class
- Mid-term grade discussions during the class time Wednesday in my office, CPB342
  - Last names A – G: 1:00pm – 1:40pm
  - Last names H – Z: 1:40pm – 2:20pm
- LCWS12
  - You are welcome to sit in the talks
- Colloquium this week
  - At 4pm in SH101
  - Dr. Tadashi Ogitsu from Lorentz Livermore National Lab.
- Don't forget the Weinberg lecture at 7:30pm, this Wednesday, Oct. 24!

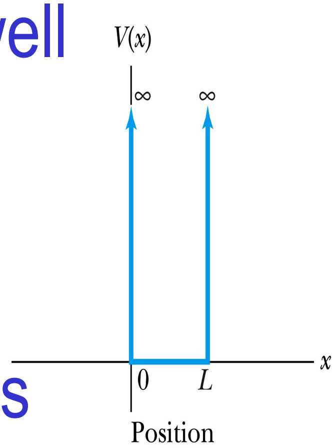
Special project #5

- Show that the Schrodinger equation becomes Newton's second law. (15 points)
- Deadline Monday, Oct. 29, 2012
- You MUST have your own answers!

Infinite Square-Well Potential

- The simplest such system is that of a particle trapped in a box with infinitely hard walls that the particle cannot penetrate. This potential is called an infinite square well and is given by

$$V(x) = \begin{cases} \infty & x \le 0, x \ge L \\ 0 & 0 < x < L \end{cases}$$



- The wave function must be zero where the potential is infinite.
- Where the potential is zero inside the box, the Schrödinger wave equation becomes $\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi$ where $k = \sqrt{2mE/\hbar^2}$.
- The general solution is $\psi(x) = A\sin kx + B\cos kx$.

Quantization

- Since the wave function must be continuous, the boundary conditions of the potential dictate that the wave function must be zero at $x = 0$ and $x = L$. This yields valid solutions for B=0 and for **integer values** of n such that $kL = n\pi$ ➔ $k=n\pi/L$
- The wave function is now $$\psi_n(x) = A\sin\left(\frac{n\pi x}{L}\right)$$
- We normalize the wave function

$$\int_{-\infty}^{+\infty} \psi_n^*(x)\psi_n(x)dx = 1 \qquad A^2\int_0^L \sin^2\left(\frac{n\pi x}{L}\right)dx = 1$$

- The normalized wave function becomes

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

- These functions are identical to those obtained for a vibrating string with fixed ends.

Quantized Energy

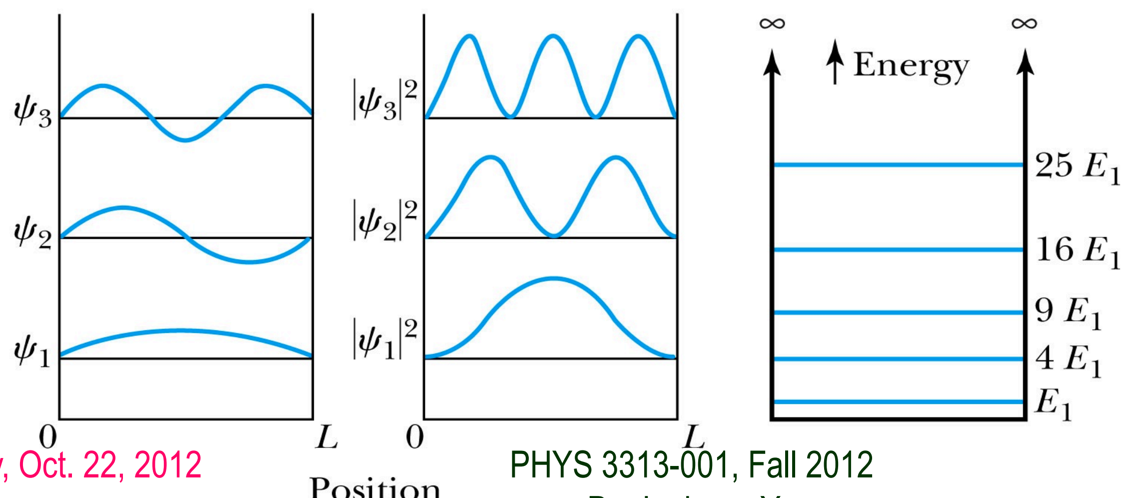
- The quantized wave number now becomes $k_n(x) = \frac{n\pi}{L} = \sqrt{\frac{2mE_n}{\hbar^2}}$
- Solving for the energy yields

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2} \quad (n = 1, 2, 3, \cdots)$$

- Note that the energy depends on the integer values of *n*. Hence the energy is quantized and nonzero.
- The special case of *n* = 1 is called the **ground state energy**.

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

$$\psi_n^* \psi_n = |\psi_n|^2 = \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right)$$



$$E_3 = \frac{9\pi^2 \hbar^2}{2mL^2}$$

$$E_2 = \frac{2\pi^2 \hbar^2}{mL^2}$$

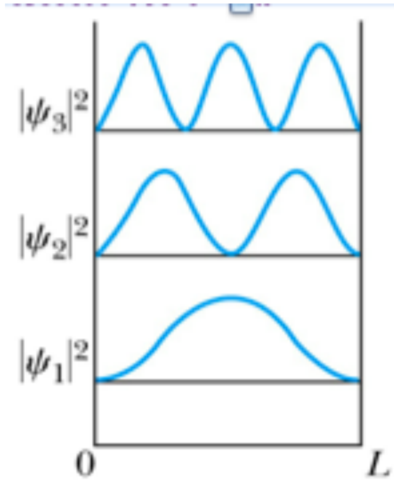
$$E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$$

How does this correspond to Classical Mech.?

- What is the probability of finding a particle in a box of length L? $\frac{1}{L}$
- Bohr's correspondence principle says that QM and CM must correspond to each other! When?
  - When n becomes large, the QM approaches to CM
- So when n→∞, the probability of finding a particle in a box of length L is

$$P = \int_0^L \psi_n^*(x)\psi_n(x)dx = \frac{2}{L}\int_0^L \sin^2\left(\frac{n\pi x}{L}\right)dx \approx \frac{2}{L}\int_0^{n\pi} \sin^2(y)dy = \frac{2}{L}\cdot\frac{1}{2} = \frac{1}{L}$$

- Which is identical to the CM probability!!
- One can also see this from the plot of P!



Ex 6.8: Expectation values inside a box

Determine the expectation values for x, x^2, p and p^2 of a particle in an infinite square well for the first excited state.

What is the wave function of the first excited state? n=? 2

$$\psi_{n=2}(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$$

$$\langle x\rangle_{n=2}=\frac{2}{L}\int_{-\infty}^{+\infty}\psi^*_{n=2}(x)x\psi_{n=2}(x)=\frac{2}{L}\int_0^L x\sin^2\left(\frac{2\pi x}{L}\right)dx=\frac{L}{2}$$

$$\langle x^2\rangle_{n=2}=\frac{2}{L}\int_0^L x^2\sin^2\left(\frac{2\pi x}{L}\right)dx=0.32L^2$$

$$\langle p\rangle_{n=2}=\frac{2}{L}\int_0^L \sin\left(\frac{2\pi x}{L}\right)(-i\hbar)\frac{\partial}{\partial x}\left[\sin\left(\frac{n\pi x}{L}\right)\right]dx=-i\hbar\frac{2}{L}\frac{2\pi}{L}\int_0^L\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi x}{L}\right)dx=0$$

$$\langle p^2\rangle_{n=2}=\frac{2}{L}\int_0^L \sin\left(\frac{2\pi x}{L}\right)(-i\hbar)^2\frac{\partial^2}{\partial x^2}\left[\sin\left(\frac{2\pi x}{L}\right)\right]dx=\hbar^2\frac{2}{L}\left(\frac{2\pi}{L}\right)^2\int_0^L\sin^2\left(\frac{2\pi x}{L}\right)dx=\frac{4\pi^2\hbar^2}{L^2}$$

$$E_2=\frac{4\pi^2\hbar^2}{2mL^2}=\frac{\langle p^2\rangle_{n=2}}{2m}$$

Ex 6.9: Proton Transition Energy

A typical diameter of a nucleus is about 10^{-14}m. Use the infinite square-well potential to calculate the transition energy from the first excited state to the ground state for a proton confined to the nucleus.

The energy of the state n is $E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$

What is n for the ground state? n=1

$$E_1 = \frac{\pi^2 \hbar^2}{2mL^2} = \frac{\pi^2 \hbar^2 c^2}{2mc^2 L^2} = \frac{1}{mc^2} \frac{\pi^2 \cdot (197.3 eV \cdot nm)^2}{2 \cdot (10^5 nm)} = \frac{1.92 \times 10^{15} eV^2}{938.3 \times 10^6 eV} = 2.0 MeV$$

What is n for the 1st excited state? n=2

$$E_2 = 2^2 \frac{\pi^2 \hbar^2}{2mL^2} = 8.0 MeV$$

So the proton transition energy is

$$\Delta E = E_2 - E_1 = 6.0 MeV$$