PHYS 3313 – Section 001
Lecture #17

Monday, Nov. 1, 2012
Dr. Jaehoon Yu

- Alpha Particle Decay
- Use of Schrodinger Equation on Hydrogen Atom
- Solutions for Schrodinger Equation for Hydrogen Atom
- Quantum Numbers
- Principal Quantum Number

Announcements

- Thank you for those of you volunteered for LCWS12
- Reminder: homework #6
  - CH6 end of chapter problems: 34, 39, 46, 62 and 65
  - Due on Monday, Nov. 12, in class
- Quiz #3
  - Beginning of the class Wednesday, Nov. 7.
  - Covers CH5 through what we finish today
- Colloquium this week
  - At 4pm, Wednesday, Nov. 7, in SH101
  - Dr. Nick White of Goddard Space Center, NASA

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Science at the Goddard Space Flight Center

Dr. Nick White
Goddard Space Flight Center
4:00 pm Wednesday November 7, 2012 room 101 SH

Abstract:

The Sciences and Exploration Directorate of the NASA Goddard Space Flight Center (GSFC) is the largest Earth and space science research organization in the world. Its scientists advance understanding of the Earth and its life-sustaining environment, the Sun, the solar system, and the wider universe beyond. Researchers in the Sciences and Exploration Directorate work with engineers, computer programmers, technologists, and other team members to develop the cutting-edge technology needed for space-based research. Instruments are also deployed on aircraft, balloons, and Earth's surface. I will give an overview of the current research activities and programs at GSFC including the James Web Space Telescope (JWST), future Earth Observing programs, experiments that are exploring our solar system and studying the interaction of the Sun with the Earth's magnetosphere.

Refreshments will be served at 3:30p.m in the Physics Lounge

Research Projects

1. Each of the 10 research groups picks one research topic
2. Study the topic as a group, looking up references
   - Original theory or Original observation
   - Experimental proofs or Theoretical prediction + subsequent experimental proofs
   - Importance and the impact of the theory/experiment
   - Conclusions
3. Each member of the group writes a 10 (max) page report, including figures (must not copy!!)
   - 10% of the total grade
   - Can share the theme and facts but you must write your own!
   - ***<u>Due Mon., Nov. 26, 2012</u>***
4. Each group presents a 10min power point talk
   - 5% of the total grade
   - Date and time will be announced close to the end of the semester

Research Topics

1. Black body radiation
2. Michelson–Morley experiment
3. The Photoelectric effect
4. Special Relativity
5. The property of molecules, Browning Motion
6. Compton Effect
7. Radioactive
8. Rutherford Scattering
9. Super-conductivity
10. The Unification of Electromagnetic and Weak forces

Research Presentations

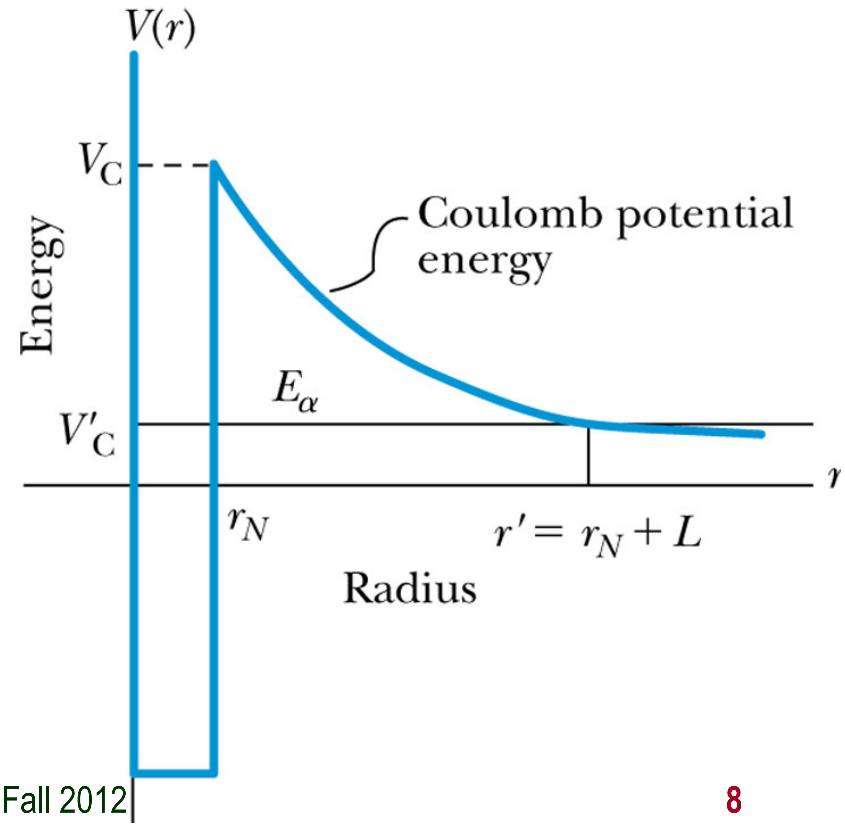
- Each of the 10 research groups makes a 10min presentation
  - 8min presentation + 2min Q&A
  - All presentations must be in power point
  - I must receive all final presentation files by 8pm, Sunday, Dec. 2
    - No changes are allowed afterward
  - The representative of the group makes the presentation with all group members participate in the Q&A session
- Date and time: Determined by drawing
  - In class Monday, Dec. 3 or in class Wednesday, Dec. 5
- Important metrics
  - Contents of the presentation: 60%
    - Inclusion of all important points as mentioned in the report
    - The quality of the research and making the right points
  - Quality of the presentation itself: 15%
  - Presentation manner: 10%
  - Q&A handling: 10%
  - Staying in the allotted presentation time: 5%
  - Judging participation and sincerity: 5%

Group – Research Topic Association

Research Group	Research Topic	Presentation Date
1	6	12/5-4
2	5	12/5-5
3	7	12/5-1
4	2	12/3-2
5	1	12/3-3
6	9	12/3-5
7	10	12/3-1
8	4	12/5-3
9	3	12/5-2
10	8	12/3-4

Alpha-Particle Decay

- May nuclei heavier than Pb emits alpha particles (nucleus of He)! The phenomenon of tunneling explains the alpha-particle decay of heavy, radioactive nuclei.
- Inside the nucleus, an alpha particle feels the strong, short-range attractive nuclear force as well as the repulsive Coulomb force.
- The nuclear force dominates inside the nuclear radius where the potential is approximately a square well.
- The Coulomb force dominates outside the nuclear radius.
- The potential barrier at the nuclear radius is several times greater than the energy of an alpha particle (~5MeV).
- According to quantum mechanics, however, the alpha particle can "tunnel" through the barrier. Hence this is observed as radioactive decay.



Application of the Schrödinger Equation to the Hydrogen Atom

- The approximation of the potential energy of the electron-proton system is the Coulomb potential:

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r}$$

- To solve this problem, we use the three-dimensional time-independent Schrödinger Equation.

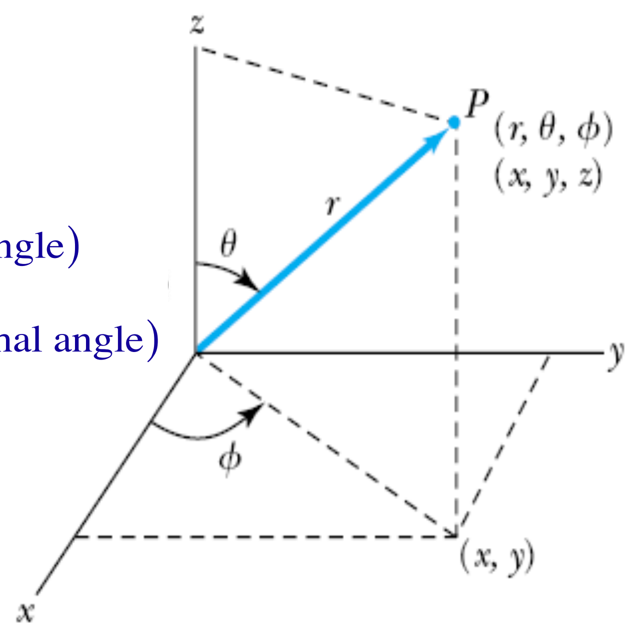
$$-\frac{\hbar}{2m}\frac{1}{\psi(x,y,z)}\left(\frac{\partial^2\psi(x,y,z)}{\partial x^2}+\frac{\partial^2\psi(x,y,z)}{\partial y^2}+\frac{\partial^2\psi(x,y,z)}{\partial z^2}\right) = E - V(r)$$

- For Hydrogen-like atoms with one electron (He^+ or Li^{++})
- Replace e^2 with Ze^2 (Z is the atomic number)
- Use appropriate reduced mass μ

Application of the Schrödinger Equation

- The potential (central force) $V(r)$ depends on the distance r between the proton and electron.

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$
$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \cos^{-1}\frac{z}{r} \text{(polar angle)}$$
$$\phi = \tan^{-1}\frac{y}{x} \text{(azimuthal angle)}$$



- Transform to spherical polar coordinates to exploit the radial symmetry.
- Insert the Coulomb potential into the transformed Schrödinger equation.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} + \frac{2\mu}{\hbar^2}(E - V)\psi = 0$$

Application of the Schrödinger Equation

- The wave function ψ is a function of r, θ and ϕ .

$\longrightarrow$ The equation is separable into three equations of independent variables

$\longrightarrow$ The solution may be a product of three functions.

$$\longrightarrow \quad \psi(r,\theta,\phi) = R(r) f(\theta) g(\phi)$$

- We can separate the Schrodinger equation in polar coordinate into three separate differential equations, each depending only on one coordinate: r, θ, or ϕ .

Solution of the Schrödinger Equation for Hydrogen

- Substitute ψ into the polar Schrodinger equation and separate the resulting equation into three equations: *R*(*r*), *f*(*θ*), and *g*(φ).

Separation of Variables

- The derivatives in Schrodinger eq. can be written as

$$\frac{\partial\psi}{\partial r} = fg\frac{\partial R}{\partial r} \qquad \frac{\partial\psi}{\partial\theta} = Rg\frac{\partial f}{\partial\theta} \qquad \frac{\partial^2\psi}{\partial\phi^2} = Rf\frac{\partial^2 g}{\partial\phi^2}$$

- Substituting them into the polar coord. Schrodinger Eq.

$$\frac{fg}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{Rg}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{Rf}{r^2\sin^2\theta}\frac{\partial^2 g}{\partial\phi^2} + \frac{2\mu}{\hbar^2}(E-V)Rgf = 0$$

- Multiply both sides by $r^2 \sin^2 \theta / Rfg$

$$\frac{\sin^2\theta}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{\sin\theta}{f}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{g}\frac{\partial^2 g}{\partial\phi^2} + \frac{2\mu}{\hbar^2}r^2\sin^2\theta(E-V) = 0$$

Reorganize

$$-\frac{\sin^2\theta}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) - \frac{2\mu}{\hbar^2}r^2\sin^2\theta(E-V) - \frac{\sin\theta}{f}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) = \frac{1}{g}\frac{\partial^2 g}{\partial\phi^2}$$

Solution of the Schrödinger Equation

- Only r and θ appear on the left-hand side and only ϕ appears on the right-hand side of the equation
- The left-hand side of the equation cannot change as ϕ changes.
- The right-hand side cannot change with either r or θ.
- Each side needs to be equal to a constant for the equation to be true in all cases. Set the constant $-m_\ell^2$ equal to the right-hand side of the reorganized equation

$$\frac{d^2 g}{d\phi^2} = -m_l^2 g$$ -------- **azimuthal equation**

- It is convenient to choose a solution to be $e^{im_l\phi}$.