PHYS 3313 – Section 001
Lecture #4

Monday, Sept. 9, 2013
Dr. Jaehoon Yu

- Galilean Transformation
- Do we need Ether?
- Michelson-Morley Experiment
- Einstein's postulates
- Lorentz Transformations
- Time Dilation & Length Contraction

Announcements

- Class e-mail distribution list subscription:
  - Test message went out last week
  - If you have replied, please do so ASAP.
    - If you haven't received my test message, check your spam box or re-register.
- Reading assignments: CH 2.3 and 2.4
- Today's homework problems are (chapter 2 end of the chapter problems):
  - 17, 21, 23, 24, 32, 59, 61, 66, 68, 81 and 96
  - Due is by the beginning of the class, next Monday, Sept. 16
  - Work in study groups together with other students but PLEASE do write your answer in your own way!

Research Projects

1. Each of the 12 research groups picks one research topic
2. Study the topic as a group, looking up references
   - Original theory or Original observation
   - Experimental proofs or Theoretical predictions + subsequent experimental proofs
   - Importance and the impact of the theory/experiment
3. Each member of the group writes a 10 page report, including figures (must not copy!!)
   - 10% of the total grade
   - Can share the theme and facts but you must write your own!
   - Due Mon., Nov. 25, 2013
4. The group presents a 10min power point talk
   - 5% of the total grade
   - Date and time will be announced close to the end of the semester

Group – Research Topic Association

Research Group Number	Research Topic
1	2
2	3
3	11
4	7
5	5
6	1
7	9
8	8
9	4
10	6
11	10
12	12

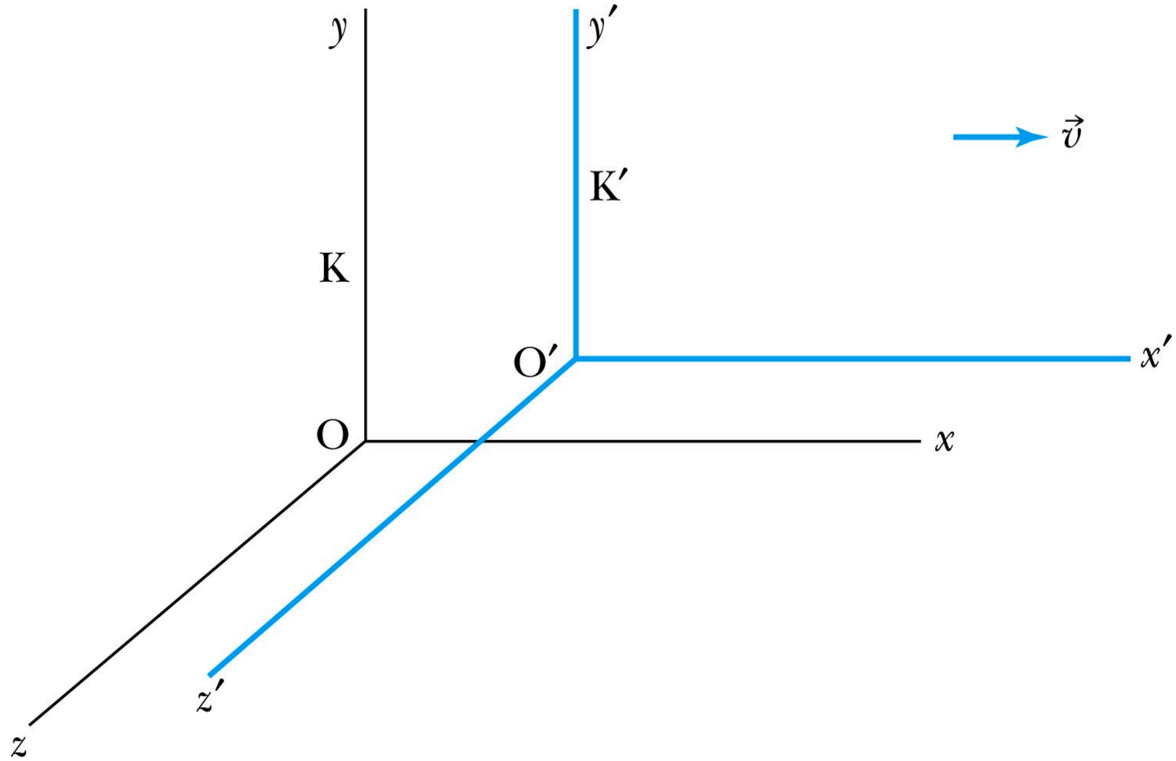
Research Topics

1. Black body radiation
2. Michelson–Morley experiment
3. The Photoelectric effect
4. Special Relativity
5. The property of molecules, Browning Motion
6. Compton Effect
7. Discovery of the electron
8. Radioactivity
9. Rutherford Scattering
10. Super-conductivity
11. The Unification of Electromagnetic and Weak forces
12. The Discovery of the Higgs-like particle

Newtonian (Classical) Relativity

- It is assumed that Newton's laws of motion must be measured with respect to (relative to) some reference frame.
- A reference frame is called an **inertial frame** if Newton laws are valid in that frame.
- Such a frame is established when a body, not subjected to net external forces, is observed moving in a rectilinear motion at a constant velocity
- ➔ Newtonian Principle of Relativity (Galilean Invariance): If Newton's laws are valid in one reference frame, then they are also valid in another reference frame moving at a uniform velocity relative to the first system.

Inertial Frames K and K'

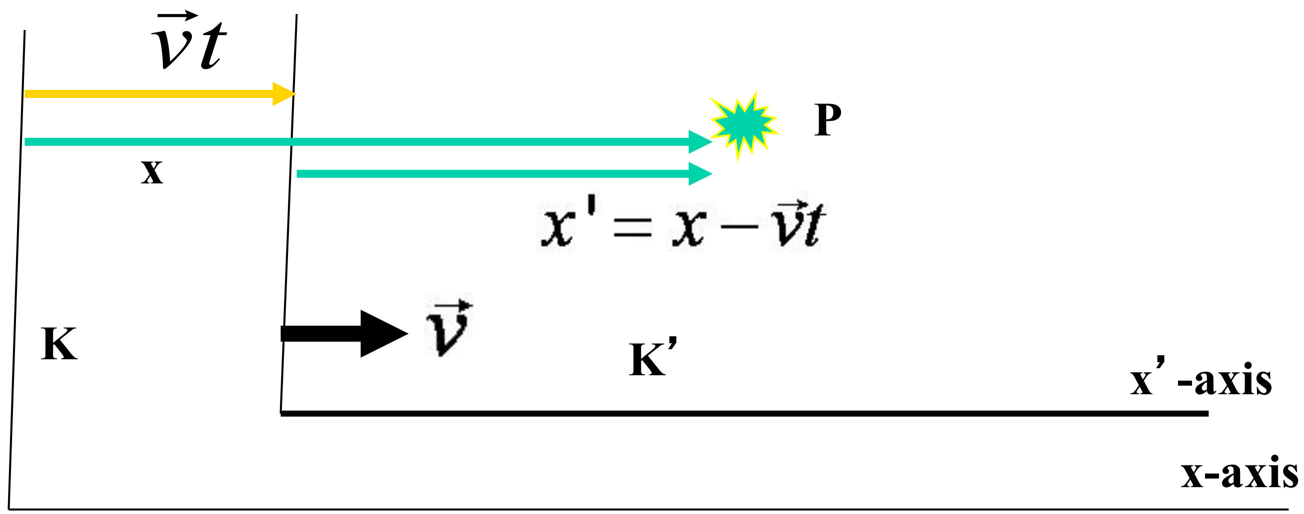


- K is at rest and K' is moving with velocity $\vec{v}$
- All axes are parallel to each other
- K and K' are said to be *INERTIAL COORDINATE SYSTEMS*

The Galilean Transformation

For a point P

- In system K: P = (*x*, *y*, *z*, *t*)
- In system K’ : P = (*x*’ , *y*’ , *z*’ , *t*’)



Conditions of the Galilean Transformation

- Parallel axes between the two inertial reference frames
- K' has a constant relative velocity in the *x*-direction with respect to K

$$x' = x - \vec{v}t$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

- **Time** (*t*) for all observers is a *Fundamental invariant*, i.e., the same for all inertial observers
  - Space and time are separate!!

The Inverse Relations

Step 1. Replace $\vec{v}$ with $-\vec{v}$

Step 2. Replace "primed" quantities with "unprimed" and "unprimed" with "primed"

$$x = x' + \vec{v}t$$

$$y = y'$$

$$z = z'$$

$$t = t'$$

The Transition to Modern Relativity

- Although Newton's laws of motion had the same form under the Galilean transformation, Maxwell's equations did not.
- In 1905, Albert Einstein proposed a fundamental connection between space and time and that Newton's laws are only an approximation.

They Needed Ether!!

- The wave nature of the light suggested that there existed a propagation medium called the *luminiferous ether* or just **ether**.
- The properties of ether
  - Very low density for planets to move through it without loss of energy
  - Sufficiently high elasticity to support the high velocity of light waves (c=?)

Ether as the Absolute Reference System

- In Maxwell's theory, the speed of light is given by

$$v = c = 1/\sqrt{\varepsilon_0 \mu_0}$$

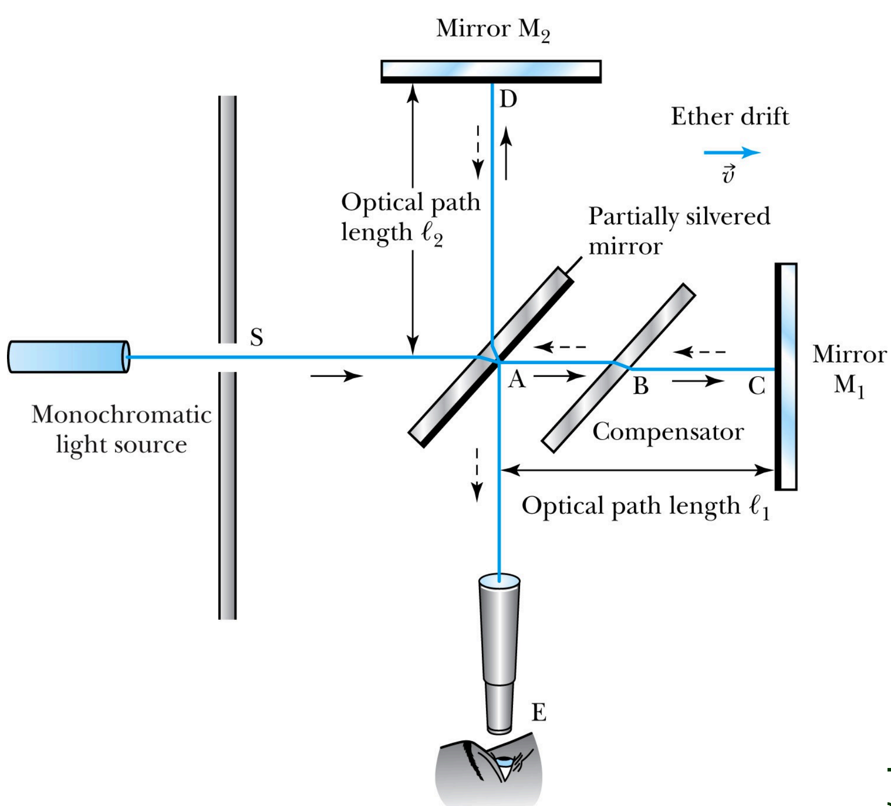
  – The velocity of light between moving systems must be a constant.
    - Can you see why?

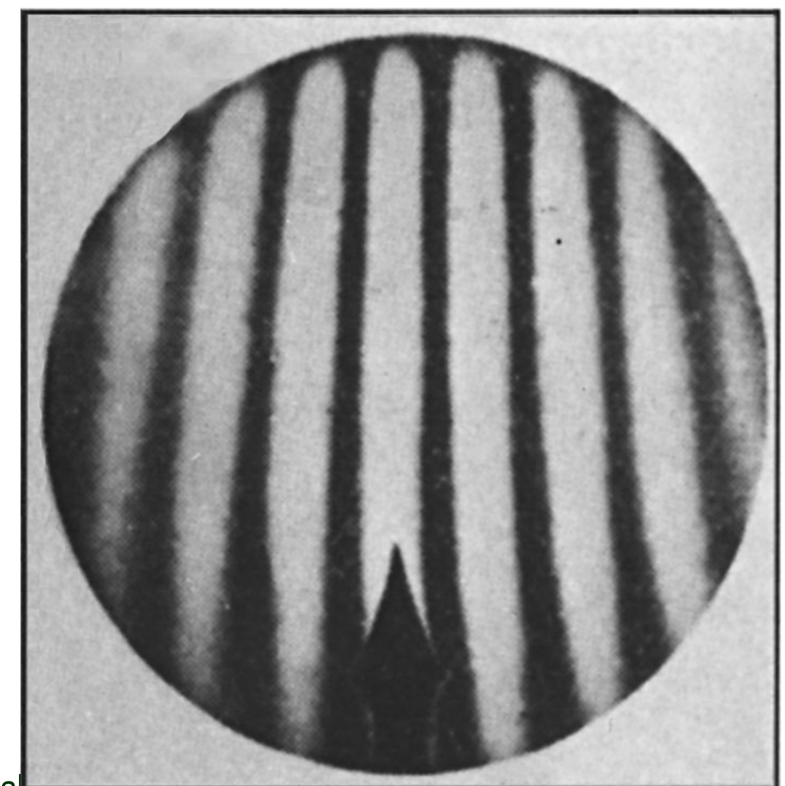
  – Needed a system of medium that keeps this constant!

- Ether proposed as the absolute reference system in which the speed of light is constant and from which other measurements could be made.
- The Michelson-Morley experiment was an attempt to show the existence of ether.

The Michelson-Morley Experiment

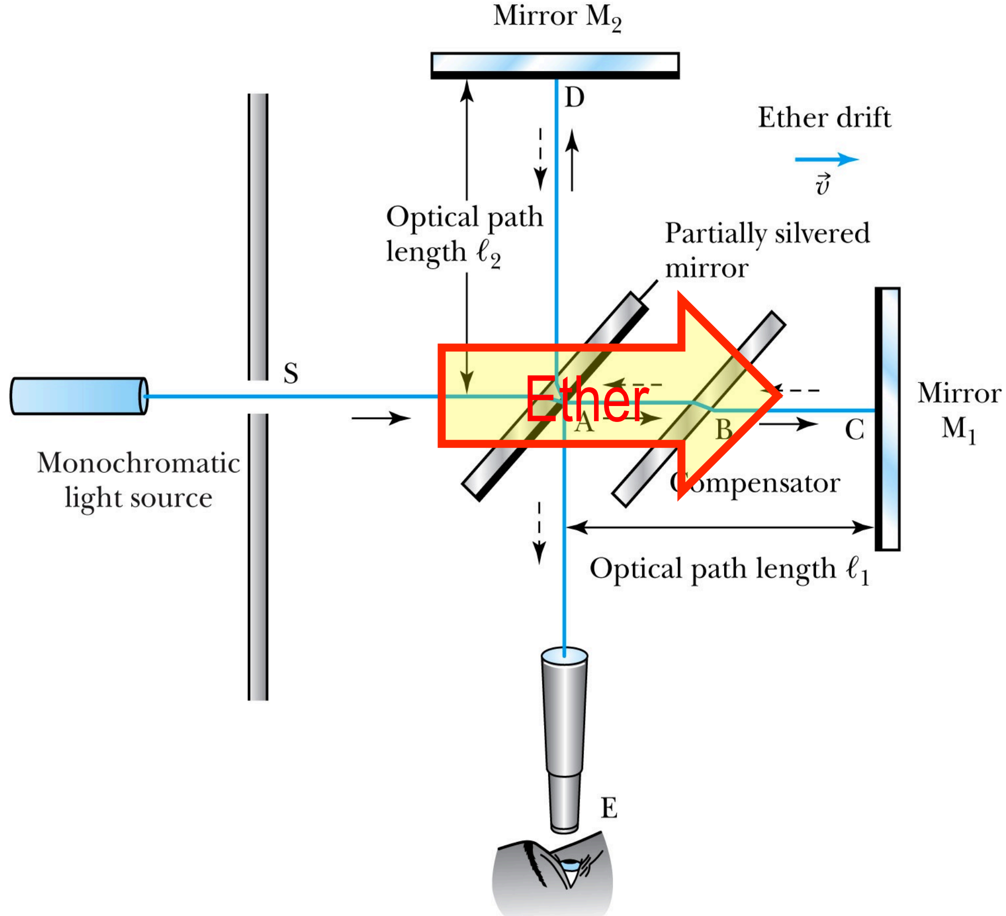
- Albert Michelson (1852–1931) built an extremely precise device called the *interferometer* to measure the phase difference between two light waves traveling in orthogonal directions.





How does Michelson's Interferometer work?

1. AC is parallel to the motion of the Earth inducing an "ether wind"
2. Light from source S is split by mirror A and travels to mirrors C and D in mutually perpendicular directions
3. After the reflection the beams recombine at A slightly out of phase due to the "ether wind" as viewed by telescope E.



The analysis – Galilean X-formation

- Travel time t_1 for round trip over AC (the ether direction) is

$$t_1 = \frac{l_1}{c+v} + \frac{l_1}{c-v} = \frac{2l_1 c}{c^2 - v^2} = \frac{2l_1}{c}\frac{1}{1 - v^2/c^2}$$

- Travel time t_2 for round trip over AD (perpendicular direction to ether) is

$$t_2 = \frac{2l_2}{\sqrt{c^2 - v^2}} = \frac{2l_2}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

- The time difference is

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{l_2}{\sqrt{1 - v^2/c^2}} - \frac{l_1}{1 - v^2/c^2}\right)$$

The analysis

- After rotating the machine by 90°, the time difference becomes

$$\Delta t' = t_2' - t_1' = \frac{2}{c}\left(\frac{l_2}{1 - v^2/c^2} - \frac{l_1}{\sqrt{1 - v^2/c^2}}\right)$$

- The difference of the time differences

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{l_1 + l_2}{1 - v^2/c^2} - \frac{l_1 + l_2}{\sqrt{1 - v^2/c^2}}\right) = \frac{2}{c}(l_1 + l_2)\left(\frac{1}{1 - v^2/c^2} - \frac{1}{\sqrt{1 - v^2/c^2}}\right)$$

- Since v (the Earth's speed) is 10^{-4} of c, we can do binomial expansion of the above

$$\Delta t' - \Delta t = \frac{2}{c}(l_1 + l_2)\left[\left(1 + \frac{v^2}{c^2} + \cdots\right) - \left(1 + \frac{v^2}{2c^2} + \cdots\right)\right] \approx \frac{v^2}{c^3}(l_1 + l_2)$$

The Results

- Using the Earth's orbital speed as:

$$V = 3 \times 10^4 \ m/s$$

together with

$$\ell_1 \approx \ell_2 = 1.2m$$

The time difference becomes

$$\Delta t' - \Delta t \approx V^2 (\ell_1 + \ell_2)/c^3 = 8 \times 10^{-17} s$$

- Although a very small number, it was within the experimental range of measurement for light waves.
- Later with Morley, they increased the path lengths to 11m and improved precision better than a factor of 10
- Yet, Michelson FAILED to "see" the expected interference pattern

Conclusions of Michelson Experiment

- Michelson noted that he should be able to detect a phase shift of light due to the time difference between path lengths but found none.
- He therefore concluded that the hypothesis of the stationary ether must be incorrect.
- After several repeats and refinements with assistance from Edward Morley (1893-1923), again *a null result.*
- ***Thus, ether does not seem to exist!***
- Many explanations ensued afterward but none worked out!
- This experiment shattered the popular belief of light being waves

The Lorentz-FitzGerald Contraction

- Another hypothesis proposed independently by both H. A. Lorentz and G. F. FitzGerald suggested that the length ℓ_1, in the direction of the motion was *contracted* by a factor of

$$\sqrt{1-v^2/c^2}$$

- Thus making the path lengths equal to account for the zero phase shift.
  - This, however, was an ad hoc assumption that could not be experimentally tested.

Einstein's Postulates

- Fundamental assumption: Maxwell's equations must be valid in all inertial frames
- **The principle of relativity**: The laws of physics are the same in all inertial systems. There is no way to detect absolute motion, and no preferred inertial system exists
  - Published a paper in 1905 at the age 26
  - Believed to be fundamental
- **The constancy of the speed of light**: Observers in all inertial systems measure the same value for the speed of light in vacuum.

The Lorentz Transformations

General linear transformation relationship between P=(x, y, z, t) in frame S and P'=(x',y',z',t') in frame S' ➔ these assume measurements are made in S frame and transferred to S' frame

- preserve the constancy of the speed of light between inertial observers
- account for the problem of simultaneity between these observers

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad y' = y \quad z' = z \quad t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1 - v^2/c^2}}$$

- With the definitions $\beta \equiv v/c$ and $\gamma \equiv 1\Big/\sqrt{1-\beta^2}$

$$x' = \gamma\left(x - \beta ct\right) \quad y' = y \quad z' = z \quad t' = \gamma\left(1 - \beta x/c\right)$$

Properties of the Relativistic Factor γ

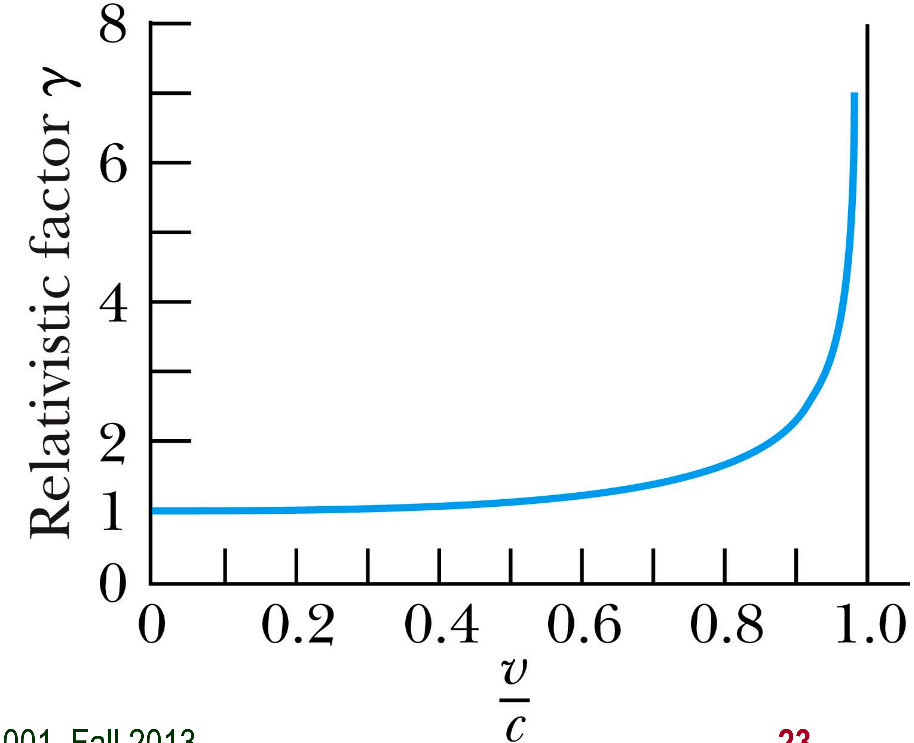
What is the property of the relativistic factor, γ?

Is it bigger or smaller than 1?

Recall Einstein's postulate, $\beta = v/c < 1$ for all observers

- $\gamma = 1$ only when $v = 0$

$$\gamma = 1 \big/ \sqrt{1-\beta^2} \geq 1$$



The complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}} \qquad t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

- Some things to note
  - What happens when β~0 (or v~0)?
    - The Lorentz x-formation becomes Galilean x-formation
  - Space-time are not separated
  - For non-imaginary x-formations, the frame speed cannot exceed c!

Time Dilation and Length Contraction

Direct consequences of the Lorentz Transformation:

- **Time Dilation**:

  Clocks in a moving inertial reference frame K' run slower with respect to stationary clocks in K.

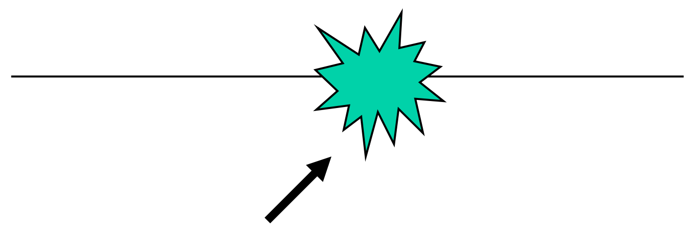
- **Length Contraction**:

  Lengths measured in a moving inertial reference frame K' are shorter with respect to the same lengths stationary in K.

Time Dilation

To understand *time dilation* the idea of **proper time** must be understood:

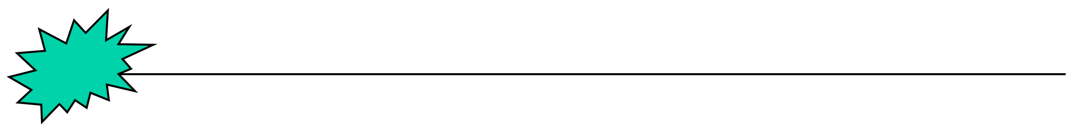
- **proper time**, T_0, is the time difference between two events occurring at the same position in a system as measured by a clock at that position.



Same location (spark “on” then off”)

Time Dilation

Is this a Proper Time?



spark "on"   then spark "off"

Beginning and ending of the event occur at different positions