PHYS 3313 – Section 001
Lecture #5

Wednesday, Sept. 11, 2013
Dr. Jaehoon Yu

- Time Dilation & Length Contraction
- Relativistic Velocity Addition
- Twin Paradox
- Space-time Diagram
- The Doppler Effect
- Relativistic Momentum and Energy
- Relationship between relativistic quantities

Announcements

- Reading assignments: CH 2.10 (special topic), 2.13 and 2.14
  - Please go through eq. 2.45 through eq. 2.49 and example 2.9
- Reminder for homework #1
  - chapter 2 end of the chapter problems
  - 17, 21, 23, 24, 32, 59, 61, 66, 68, 81 and 96
  - Due is by the beginning of the class, Monday, Sept. 17
  - Work in study groups together with other students but PLEASE do write your answer in your own way!
- Colloquium today
  - Dr. Mingwu Jin, a bio-physicist

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Physics in Medical Imaging

Dr. Mingwu Jin

Candidate for faculty position in Medical Physics
Department of Physics
University of Texas at Arlington

4:00p.m Wednesday September 11, 2013
Room 101 Science Hall

Abstract:

Medical Physics involves the applications of physics to medicine and biology. Major applications are found in both diagnosis and therapy of diseases. Pertinent to the former, various medical imaging modalities provide a non-invasive tool for investigating organic physiology and metabolism and have far-reaching impact on understanding pathogenesis and pathology as well as on finding cures for diseases. Accurate modeling of the physical process in medical imaging and the subsequent signal processing and data analysis can lead to improved image quality, easier interpretation of the imaging results, more accurate quantitative results, and reduced imaging dose or shortened imaging time. The applications of physics in both nuclear medicine and magnetic resonance imaging (MRI) are presented to demonstrate these merits. First, through the better physical and physiological modeling we advanced traditional 3D cardiac single photon emission computed tomography (SPECT) reconstruction to 4D, which can yield much improved image sequences at standard imaging dose. It was further utilized to significantly reduce the imaging dose without compromising the image quality, thus lowering patients' secondary cancer incidences due to ionizing radiation. The spatiotemporal processing strategy of the 4D method can also be exploited for real-time image-guided photothermal therapies. Second, we developed a fast constrained canonical correlation analysis on functional MRI (fMRI) data to robustly detect week brain activations in the medial temporal lobe (MTL) induced by memory activities. These activation patterns held the potential as a biomarker to predict the incipience of Alzheimer's disease. Future work on the development of a medical physics program, image-guided light-induced cancer therapy, and advanced neuroimaging methods for memory studies will be discussed.

Special Project #2

1. Derive the three Lorentz velocity transformation equations. (10 points)
2. Derive the three reverse Lorentz velocity transformation equations. (10 points)
3. Prove that the space-time invariant quantity $s^2=x^2-(ct)^2$ is indeed invariant, i.e. $s^2=s'^2$, in Lorentz Transformation. (5 points)
4. You must derive each one separately starting from the Lorentz spatial coordinate transformation equations to obtain any credit.
   - Just switching the signs and primes will not cut!
   - Must take the simplest form of the equations, using β and γ.
5. You MUST have your own, independent answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied.

- Due for the submission is Wednesday, Sept. 18!

The complete Lorentz Transformations

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \frac{t - \left(vx/c^2\right)}{\sqrt{1-\beta^2}} \qquad t = \frac{t' + \left(vx'/c^2\right)}{\sqrt{1-\beta^2}}$$

- Some things to note
  - What happens when β~0 (or v~0)?
    - The Lorentz x-formation becomes Galilean x-formation
  - Space-time are not separated
  - For non-imaginary x-formations, the frame speed cannot exceed c!

Time Dilation and Length Contraction

Direct consequences of the Lorentz Transformation:

- **Time Dilation**:

  Clocks in a moving inertial reference frame K' run slower with respect to stationary clocks in K.

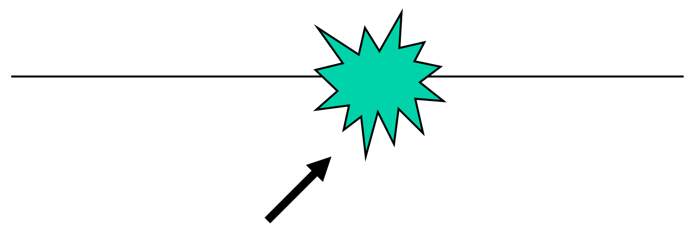
- **Length Contraction**:

  Lengths measured in a moving inertial reference frame K' are shorter with respect to the same lengths stationary in K.

Time Dilation

To understand *time dilation* the idea of **proper time** must be understood:

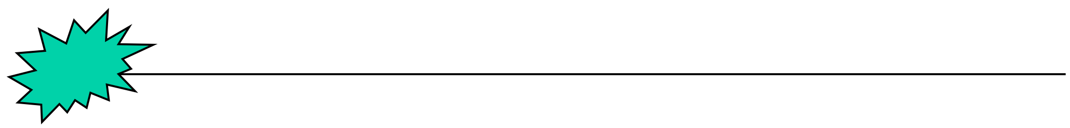
- **proper time**, T_0, is the time difference between two events occurring at the same position in a system as measured by a clock at that position.



Same location (spark "on" then off")

Time Dilation

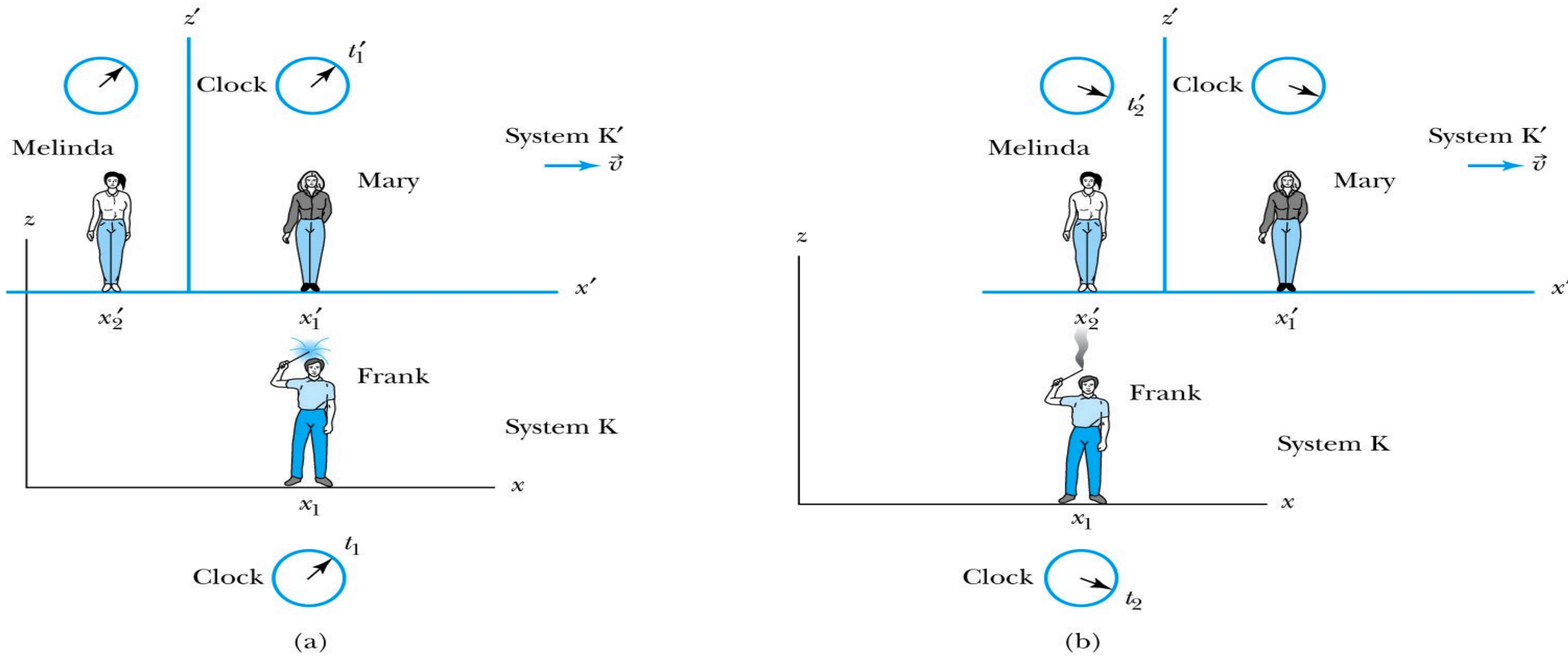
Is this a Proper Time?



spark “on”   then spark “off”

Beginning and ending of the event occur at different positions

Time Dilation with Mary, Frank, and Melinda



Frank's clock is at the same position in system K when the sparkler is lit in (a) ($t=t_1$) and when it goes out in (b) ($t=t_2$). ➔ The proper time $T_0=t_2-t_1$

Mary, in the moving system K', is beside the sparkler when it was lit ($t=t_1'$)

Melinda then moves into the position where and when the sparkler extinguishes ($t=t_2'$)

Thus, Melinda, at the new position, measures the time in system K' when the sparkler goes out in (b).

According to Mary and Melinda…

- Mary and Melinda measure the two times for the sparkler to be lit and to go out in system K' as times t_1'and t_2' so that by the Lorentz transformation:

$$t'_2 - t'_1 = \frac{(t_1 - t_2) - (v/c^2)(x_1 - x_2)}{\sqrt{1-\beta^2}}$$

  - Note here that Frank records $x_2 - x_1 = 0$ in K with a proper time: $T_0 = t_2 - t_1$ or

$$T' = t'_2 - t'_1 = \frac{T}{\sqrt{1-\beta^2}} = \gamma T$$

Time Dilation: Moving Clocks Run Slow

1) $T' > T_0$ or the time measured between two events at *different positions* is greater than the time between the same events at *one position:* ***time dilation.***

The proper time is always the shortest time!!

2) The events do not occur at the same space and time coordinates in the two systems

3) System K requires 1 clock and K' requires 2 clocks.

Time Dilation Example: muon lifetime

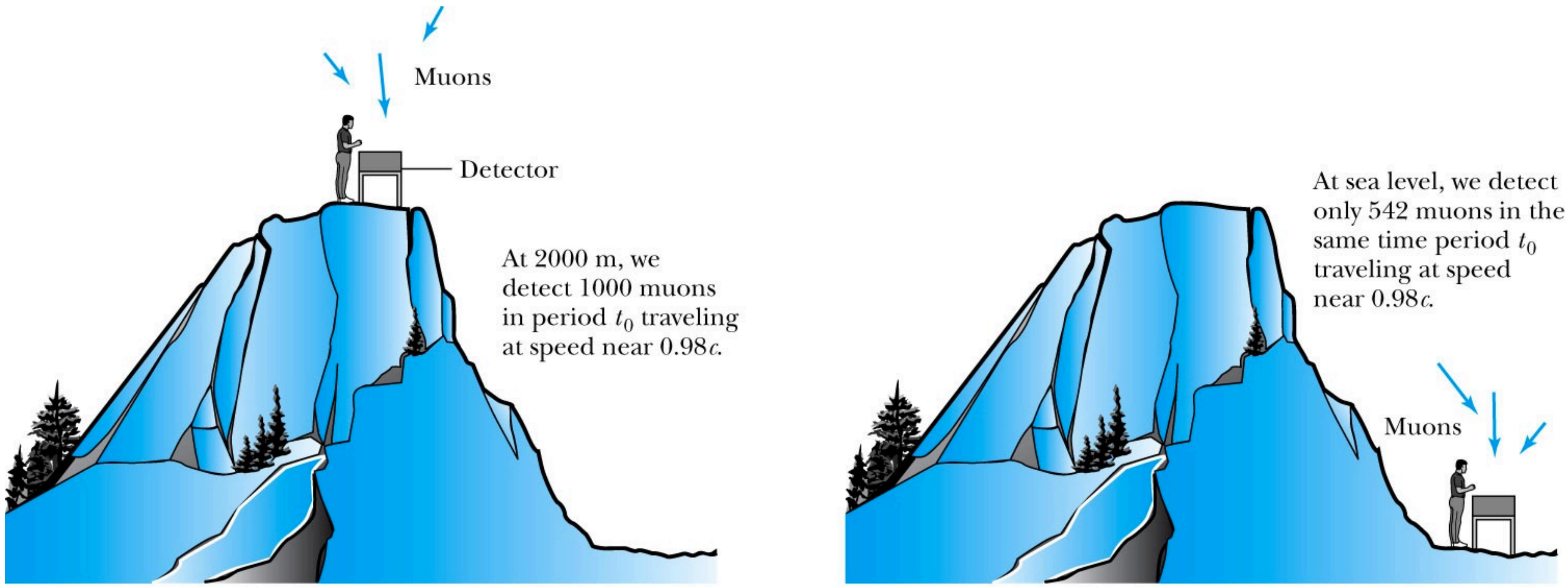
- Muons are essentially heavy electrons (~200 times heavier)
- Muons are typically generated in collisions of cosmic rays in upper atmosphere and, unlike electrons, decay ($t_0 = 2.2$ μsec)
- For a muon incident on Earth with v=0.998c, an observer on Earth would see what lifetime of the muon?
- 2.2 μsec?

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 16$$

- t=35 μsec
- Moving clocks run slow so when an outside observer measures, they see a longer time than the muon itself sees.

Experimental Verification of Time Dilation

Arrival of Muons on the Earth's Surface



(a) (b)

The number of muons detected with speeds near 0.98*c* is much different (a) on top of a mountain than

(b) at sea level, because of the muon's decay. The experimental result agrees with our time dilation equation.

Length Contraction

To understand *length contraction* the idea of **proper length** must be understood:

- Let an observer in each system K and K' have a meter stick at rest in ***their own system*** such that each measures the same length at rest.
- The length as measured at rest at the same time is called the **proper length**.

Length Contraction cont'd

Each observer lays the stick down along his or her respective *x* axis, putting the left end at x_ℓ (or x'_ℓ) and the right end at x_r (or x'_r).

- Thus, in the rest frame K, Frank measures his stick to be:

$$L_0 = x_r - x_l$$

- Similarly, in the moving frame K', Mary measures her stick at rest to be:

$$L'_0 = x'_r - x'_l$$

- Frank in his rest frame measures the moving length in Mary's frame moving with velocity v.
- Thus using the Lorentz transformations Frank measures the length of the stick in K' as:

$$x'_r - x'_l = \frac{(x_r - x_l) - v(t_r - t_l)}{\sqrt{1-\beta^2}}$$

Where both ends of the stick must be measured simultaneously, i.e, $t_r = t_\ell$

Here Mary's proper length is $L'_0 = x'_r - x'_\ell$

and Frank's measured length is $L = x_r - x_\ell$

Measurement in Rest Frame

The observer in the rest frame measures the moving length as L given by

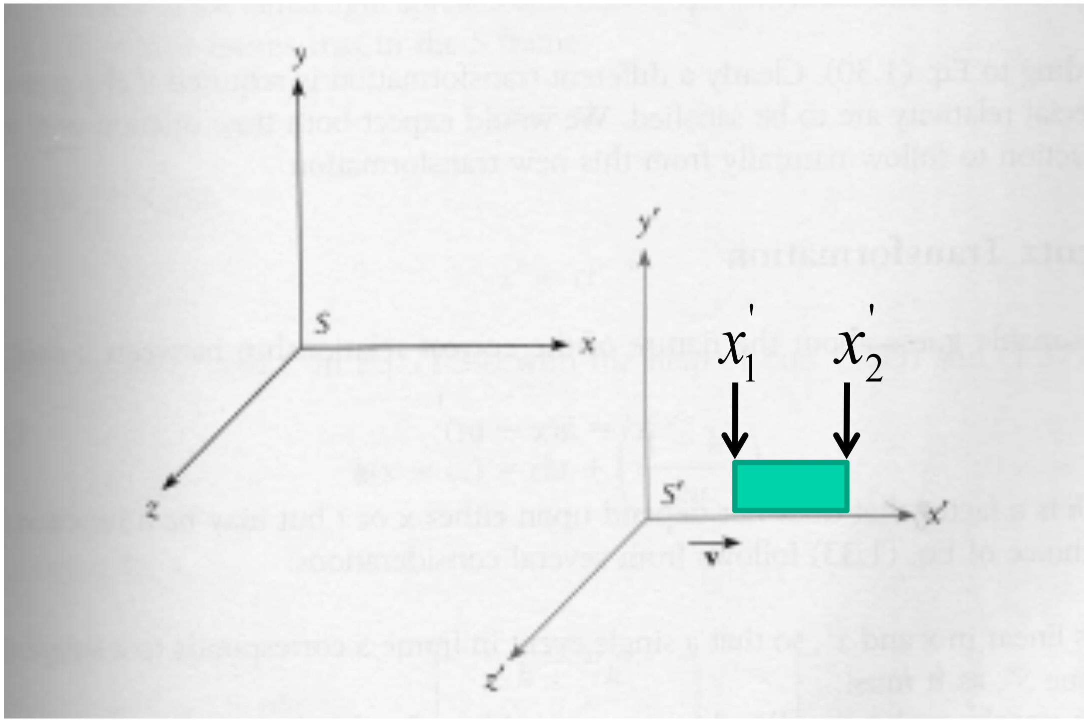
$$L_0' = \frac{L}{\sqrt{1-\beta^2}} = \gamma L$$

but since both Mary and Frank in their respective frames measure $L'_0 = L_0$

$$L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma}$$

and $L_0 > L$, i.e. the moving stick shrinks

Length Contraction Summary



- Proper length (length of object in its own frame:

$$L_0 = x_2' - x_1'$$

- Length of object in observer's frame:

$$L = x_2 - x_1$$

$$L_0' = L_0 = x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt) = \gamma(x_2 - x_1)$$

$L_0 = \gamma L \qquad L = L_0 / \gamma$ $\qquad \gamma > 1$ so the length is shorter in the direction of motion (length contraction!)

More about Muons

- Rate: 1/cm^2/minute at Earth's surface (so for a person with 600 cm^2 that would be 600/60=10 muons/sec passing through!)
- They are typically produced in atmosphere about 6 km above surface of Earth and often have velocities that are a substantial fraction of speed of light, v=.998 c for example and life time 2.2 μsec $$vt_0 = 2.994 \times 10^8 \frac{m}{\sec} \cdot 2.2 \times 10^{-6} \sec = 0.66 km$$
- How do they reach the Earth if they only go 660 m and not 6000 m?
- The time dilation stretches life time to t=35 μsec not 2.2 μsec, thus they can travel 16 times further, or about 10 km, implying they easily reach the ground
- But riding on a muon, the trip takes only 2.2 μsec, so how do they reach the ground???
- Muon-rider sees the ground moving towards him, so the length he has to travel contracts and is only $L_0 / \gamma = 6/16 = 0.38 km$
- At 1000 km/sec, it would take 5 seconds to cross U.S. , pretty fast, but does it give length contraction? $L = .999994 L_0$ {not much contraction} (for v=0.9c, the length is reduced by 44%)

Addition of Velocities

How do we add velocities in a relativistic case?

Taking differentials of the Lorentz transformation, relative velocities may be calculated:

$$dx = \gamma\left(dx' + v\,dt'\right)$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \gamma\left[dt' + \left(v/c^2\right)dx'\right]$$

So that…

defining velocities as: $u_x = dx/dt$, $u_y = dy/dt$, $u'_x = dx'/dt'$, etc. it can be shown that:

$$u_x = \frac{dx}{dt} = \frac{\gamma(dx' + v\,dt')}{\gamma[dt' + (v/c^2)\,dx']} = \frac{u'_x + v}{1 + (v/c^2)u'_x}$$

With similar relations for u_y and u_z:

$$u_y = \frac{u'_y}{\gamma\left[1 + (v/c^2)u_x'\right]} \qquad u_z = \frac{u'_z}{\gamma\left[1 + (v/c^2)u_x'\right]}$$

The Lorentz Velocity Transformations

In addition to the previous relations, the **Lorentz velocity transformations** for u'_x, u'_y, and u'_z can be obtained by switching primed and unprimed and changing v to $-v$:

$$u'_x = \frac{u_x - v}{1 - (v/c^2)u_x}$$

$$u'_y = \frac{u_y}{\gamma\left[1 - (v/c^2)u_x\right]}$$

$$u'_z = \frac{u_z}{\gamma\left[1 - (v/c^2)u_x\right]}$$

Velocity Addition Summary

- Galilean Velocity addition $v_x = v_x' + v$ where $v_x = \frac{dx}{dt}$ and $v_x' = \frac{dx'}{dt'}$
- From inverse Lorentz transform $dx = \gamma(dx' + vdt')$ and $dt = \gamma(dt' + \frac{v}{c^2}dx')$
- So $$v_x = \frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{\gamma(dt' + \frac{v}{c^2}dx')} \div \frac{dt'}{dt'} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2}\frac{dx'}{dt'}} = \frac{v_x' + v}{1 + \frac{vv_x'}{c^2}}$$
- Thus $$\boxed{v_x = \frac{v_x' + v}{1 + \frac{vv_x'}{c^2}}}$$
- What would be the measured speed of light in S frame?
  - Since $v_x' = c$ we get $$v_x = \frac{c + v}{1 + \frac{v^2}{c^2}} = \frac{c^2(c+v)}{c(c+v)} = c$$

Observer in S frame measures c too! Strange but true!