

PHYS 3313 – Section 001

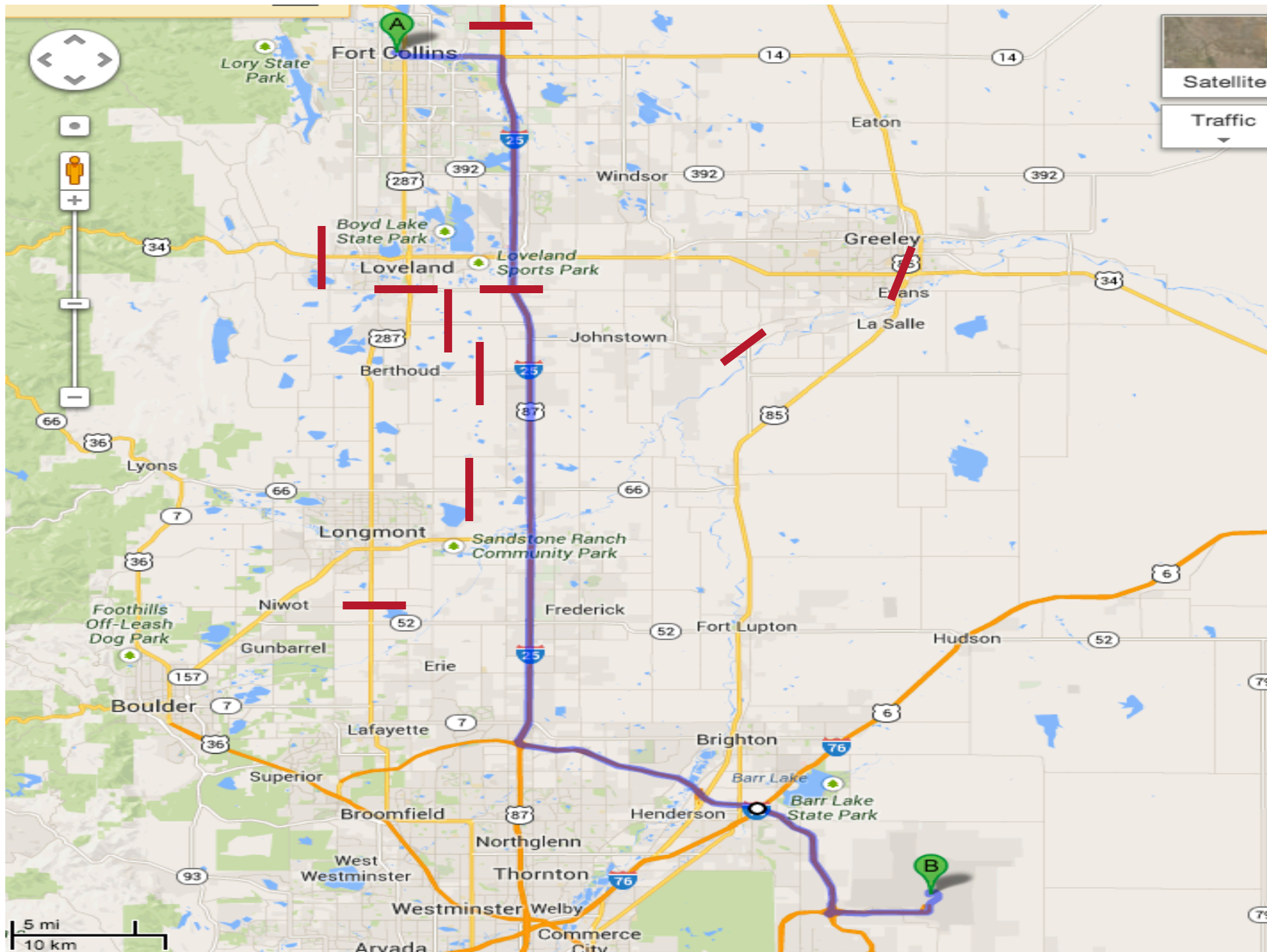
Lecture #6

Monday, Sept. 16, 2013

*Dr. **Jaehoon** **Yu***

- The Twin Paradox
- Space-time Diagram
- The Doppler Effect
- Relativistic Momentum and Energy
- Relationship between relativistic quantities





Reminder: Special Project #2

1. Derive the three Lorentz velocity transformation equations. (10 points)
 2. Derive the three reverse Lorentz velocity transformation equations. (10 points)
 3. Prove that the space-time invariant quantity $s^2 = x^2 - (ct)^2$ is indeed invariant, i.e. $s^2 = s'^2$, in Lorentz Transformation. (5 points)
 4. You must derive each one separately starting from the Lorentz spatial coordinate transformation equations to obtain any credit.
 - Just switching the signs and primes will not cut!
 - Must take the simplest form of the equations, using β and γ .
 5. You MUST have your own, independent answers to the above three questions even if you worked together with others. All those who share the answers will get 0 credit if copied.
- Due for the submission is Wednesday, Sept. 18!



Velocity Addition Summary

- Galilean Velocity addition $v_x = v'_x + v$ where $v_x = \frac{dx}{dt}$ and $v'_x = \frac{dx'}{dt'}$
- From inverse Lorentz transform $dx = \gamma(dx' + vdt')$ and $dt = \gamma(dt' + \frac{v}{c^2}dx')$
- So
$$v_x = \frac{dx}{dt} = \frac{\gamma(dx' + vdt')}{\gamma(dt' + \frac{v}{c^2}dx')} \div \frac{dt'}{dt'} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2} \frac{dx'}{dt'}} = \frac{v'_x + v}{1 + \frac{vv'_x}{c^2}}$$
- Thus

$$v_x = \frac{v'_x + v}{1 + \frac{vv'_x}{c^2}}$$
- What would be the measured speed of light in S frame?

– Since $v'_x = c$ we get

$$v_x = \frac{c + v}{1 + \frac{v^2}{c^2}} = \frac{c^2(c + v)}{c(c + v)} = c$$

Observer in S frame measures c too! Strange but true!

Velocity Addition Example

- Yu Darvish is riding his bike at $0.8c$ relative to observer. He throws a ball at $0.7c$ in the direction of his motion. What speed does the observer see?

$$v_x = \frac{v'_x + v}{1 + \frac{vv'_x}{c^2}}$$

$$v_x = \frac{.7c + .8c}{1 + \frac{.7 \times .8c^2}{c^2}} = 0.962c$$

- What if he threw it just a bit harder?
- Doesn't help—asymptotically approach c , can't exceed (it's not just a postulate it's the law)

Twin Paradox

The Set-up: Twins Mary and Frank at age 30 decide on two career paths: Mary (the **Moving twin**) decides to become an astronaut and to leave on a trip 8 light-years (ly) from the Earth at a great speed and to return; Frank (the **Fixed twin**) decides to stay on the Earth.

The Problem: Upon Mary's return, Frank reasons, that her clocks measuring her age must run slow. As such, she will return younger. However, Mary claims that it is Frank who is moving and consequently his clocks must run slow.

The Paradox: Who is younger upon Mary's return?

Twin Paradox cont'd

- Let's say, Mary is traveling at the speed of $0.8c$
- Frank will measure the Mary's total travel time
 - $T = 8\text{ly}/0.8c = 10\text{yrs}$
- Mary's clock will run slower due to relativity:
 - $T' = \gamma T = 10 / \sqrt{1 - 0.8^2} = 6\text{yrs}$
- Thus, Frank claims that Mary will be 42 years old while he is 50 years old
- Who is correct?



The Resolution

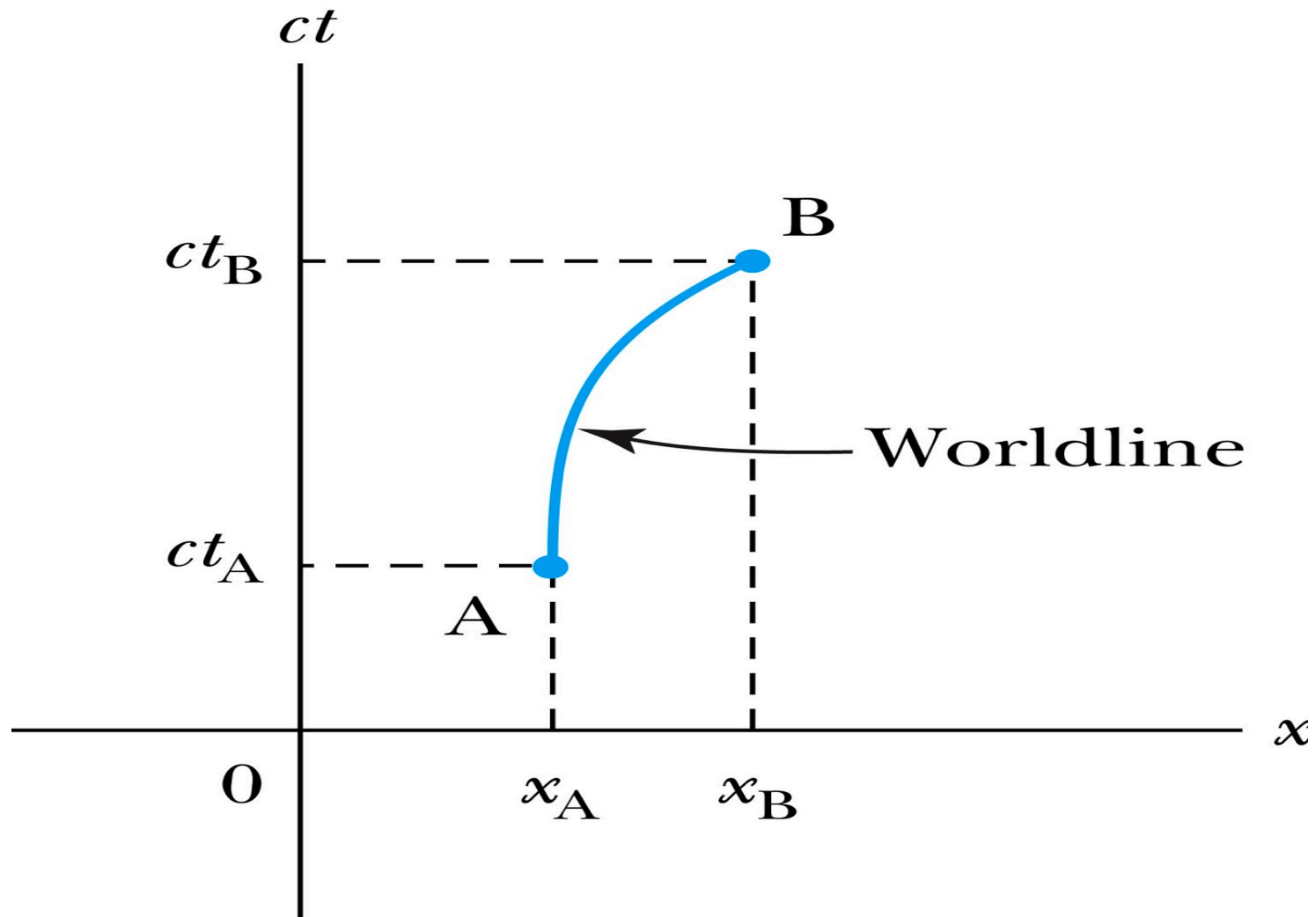
- 1) Frank's clock is in an **inertial system** during the entire trip; however, Mary's clock is not. As long as Mary is traveling at constant speed away from Frank, both of them can argue that the other twin is aging less rapidly.
- 2) When Mary slows down to turn around, she leaves her original inertial system and eventually returns in a completely different inertial system.
- 3) Mary's claim is no longer valid, because she does not remain in the same inertial system. There is also no doubt as to who is in the inertial system. Frank feels no acceleration during Mary's entire trip, but Mary does.
- 4) So Frank is correct! Mary is younger!



Space-time

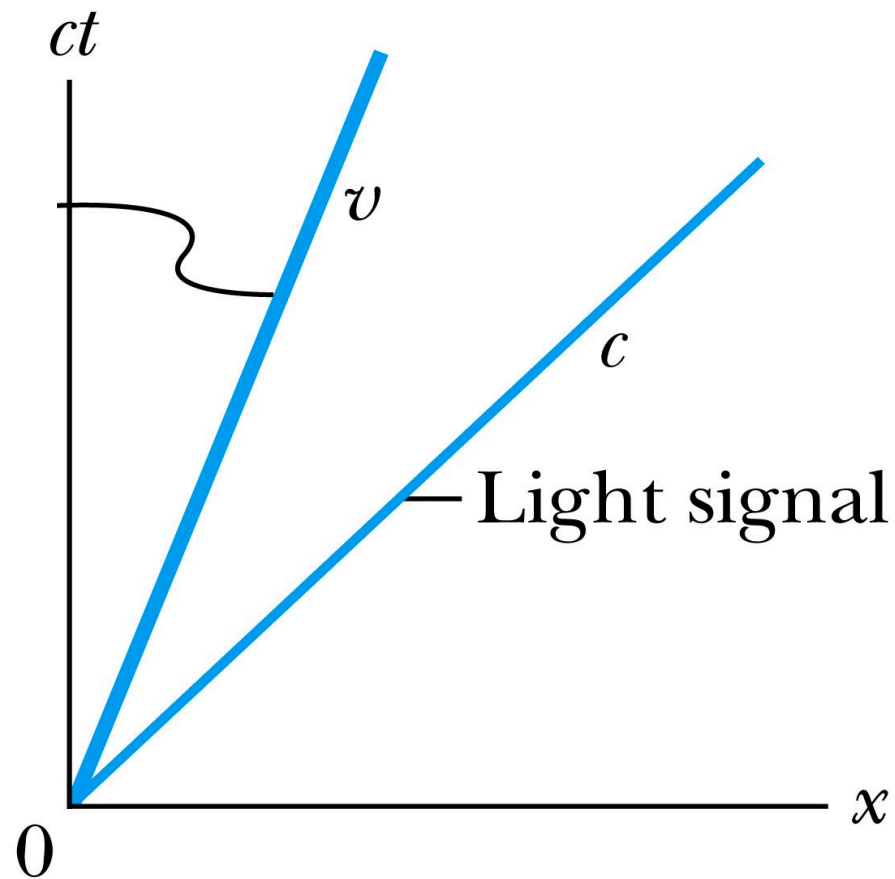
- When describing events in relativity, it is convenient to represent events on a **space-time** diagram.
- In this diagram one spatial coordinate x specifies the position, and instead of time t , ct is used as the other coordinate so that both coordinates will have the same dimensions of length.
- Space-time diagrams were first used by H. Minkowski in 1908 and are often called **Minkowski diagrams**. Paths in Minkowski space-time are called **worldlines**.

The Space-time Diagram

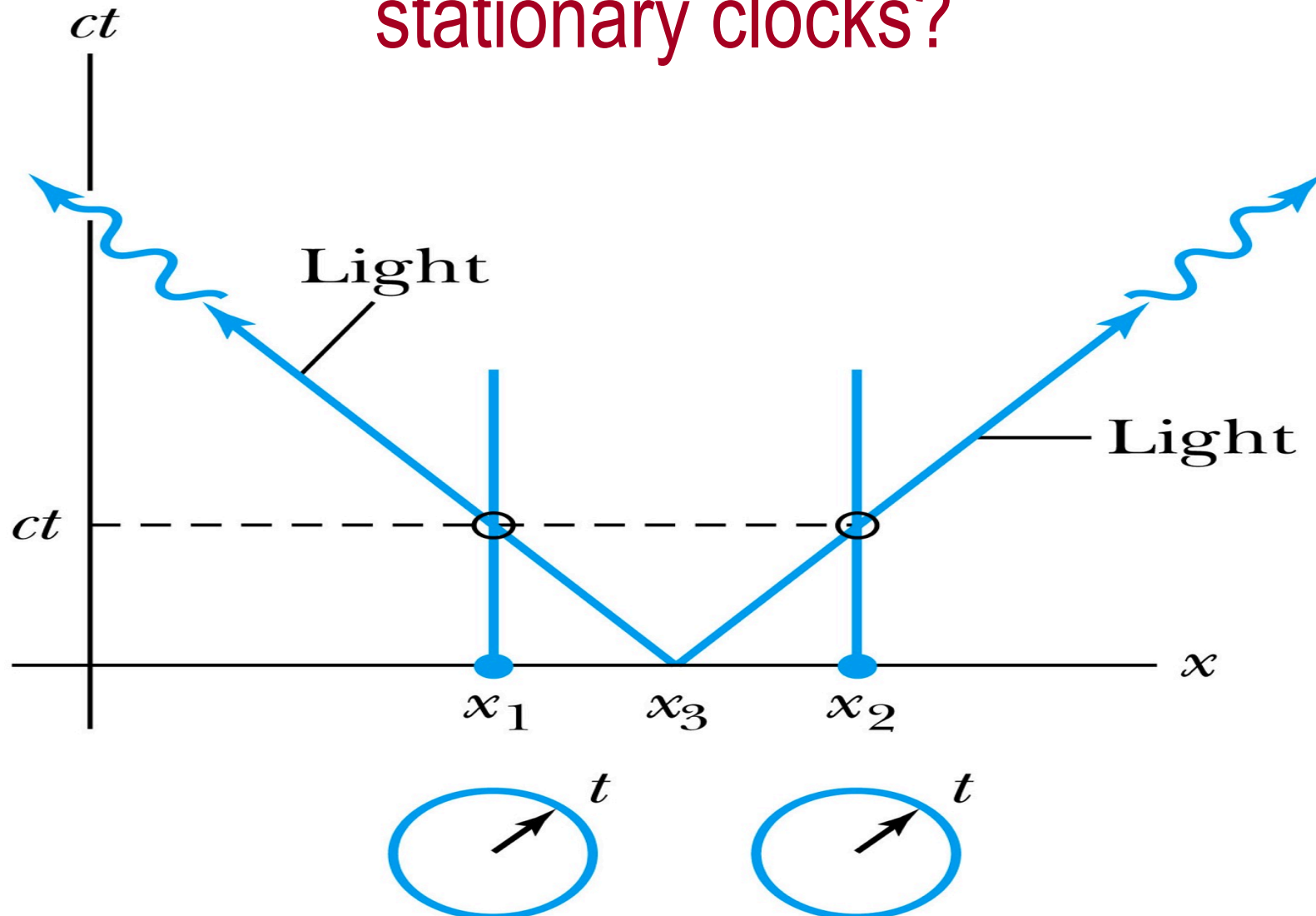


Particular Worldlines

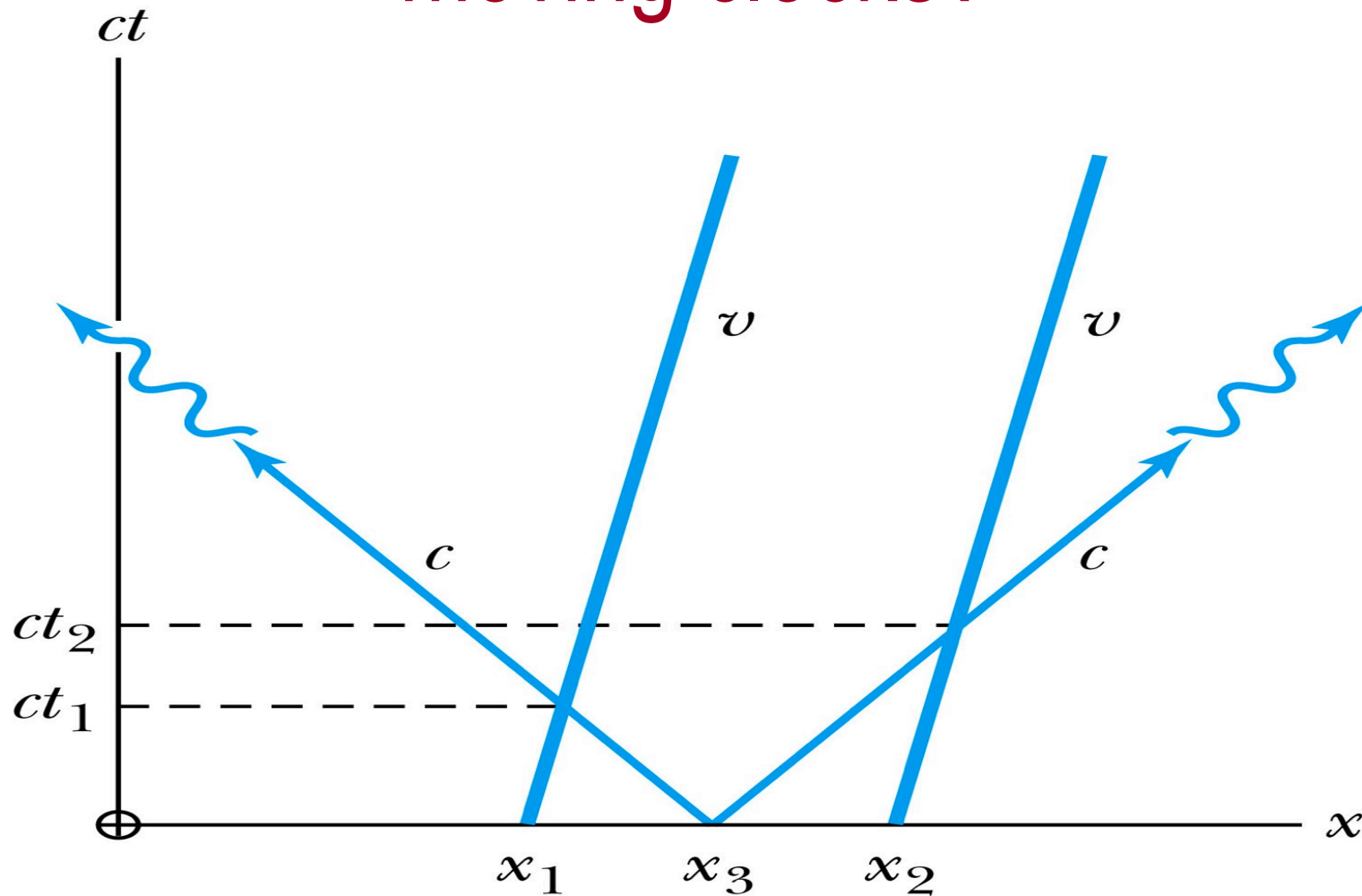
- How does the worldline for a spaceship running at the velocity $v(<c)$ look?
- How does the worldline for light signal look?



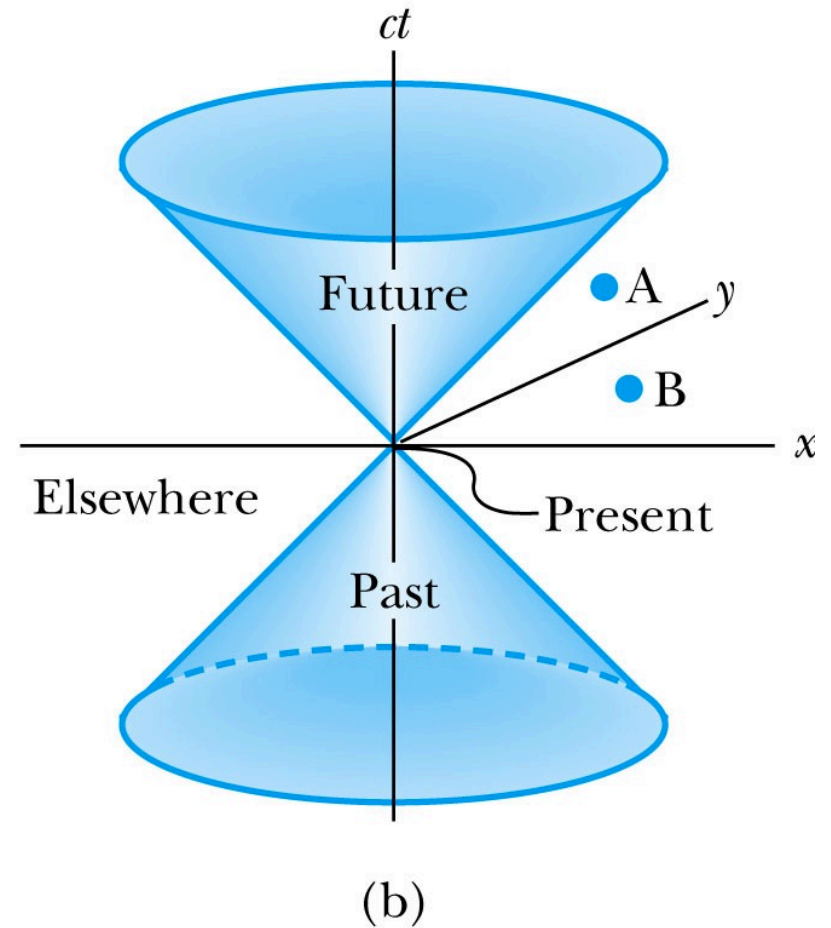
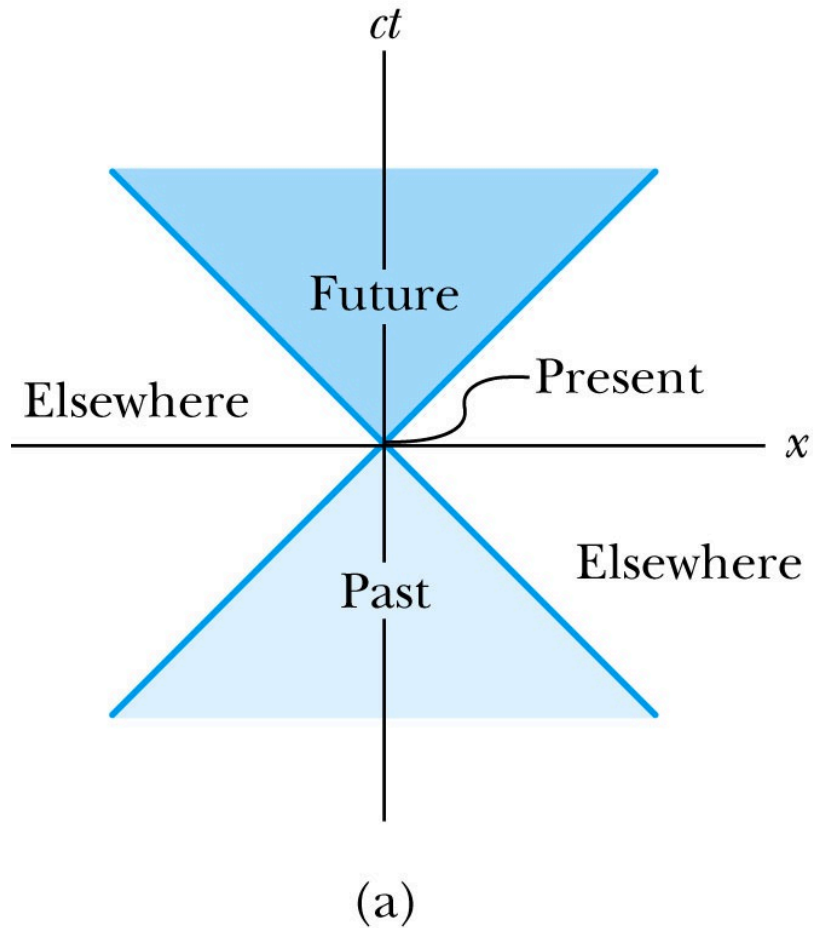
How about the time measured by two stationary clocks?



How about the time measured by moving clocks?



The Light Cone



Invariant Quantities: The Space-time Interval

- Since all observers “see” the same speed of light, then all observers, regardless of their velocities, must see spherical wave fronts. Thus the quantity:

$$s^2 = x^2 - c^2 t^2 = (x')^2 - c^2 (t')^2 = (s')^2$$

is invariant.

- For any two events, the space-time interval $\Delta s^2 = \Delta x^2 - \Delta(ct)^2$ between the two events is **invariant** in any inertial frame.



Space-time Invariants

There are three possibilities for the invariant quantity Δs^2 :

$$\Delta s^2 = \Delta x^2 - \Delta (ct)^2$$

1) $\Delta s^2 = 0$: $\Delta x^2 = c^2 \Delta t^2$: **light-like** separation

- Two events can be connected only by a light signal.

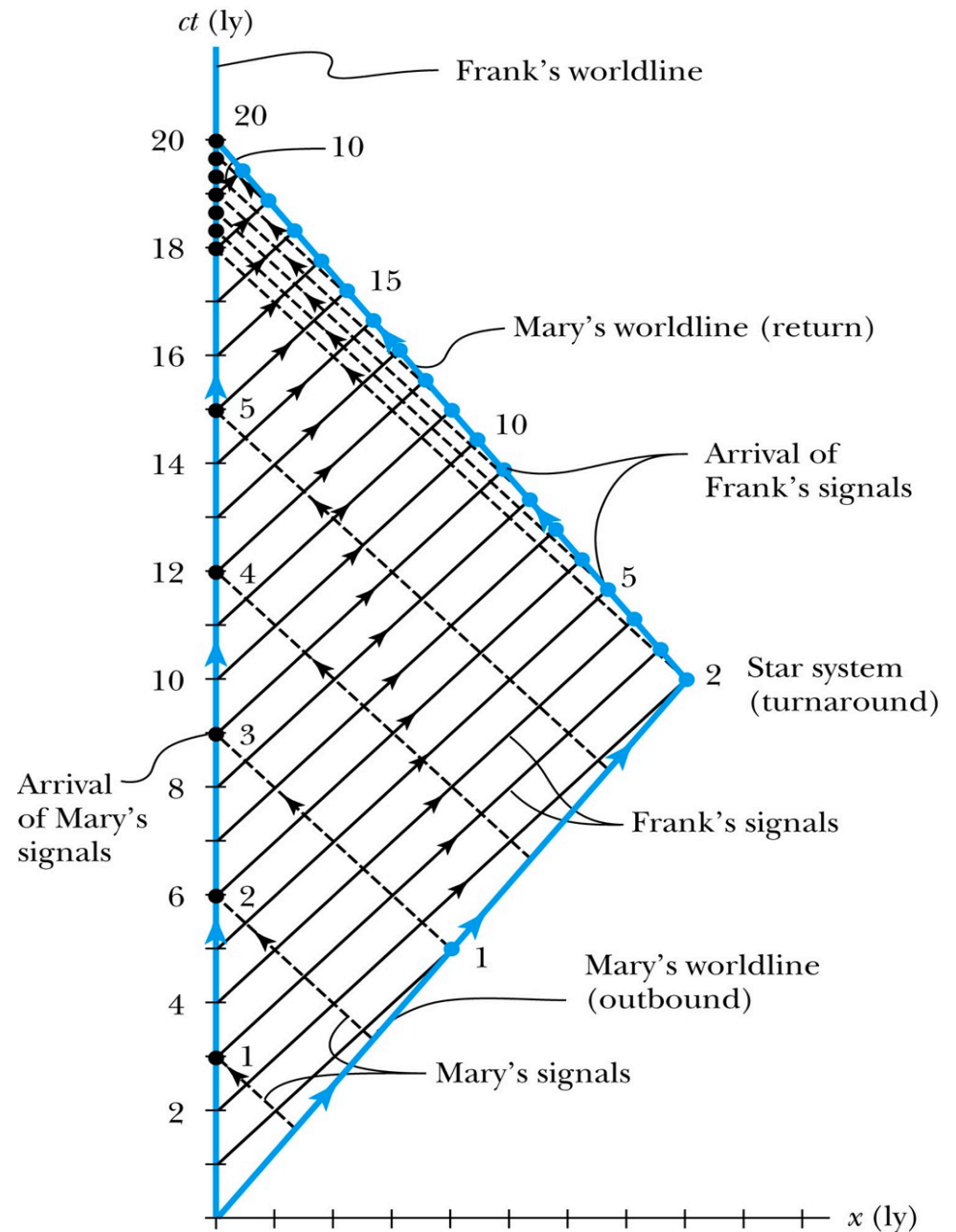
2) $\Delta s^2 > 0$: $\Delta x^2 > c^2 \Delta t^2$: **space-like** separation

- No signal can travel fast enough to connect the two events.
The events are not causally connected!!

3) $\Delta s^2 < 0$: $\Delta x^2 < c^2 \Delta t^2$: **time-like** separation

- Two events can be causally connected.
- These two events cannot occur simultaneously!

The Twin Paradox in Space-Time



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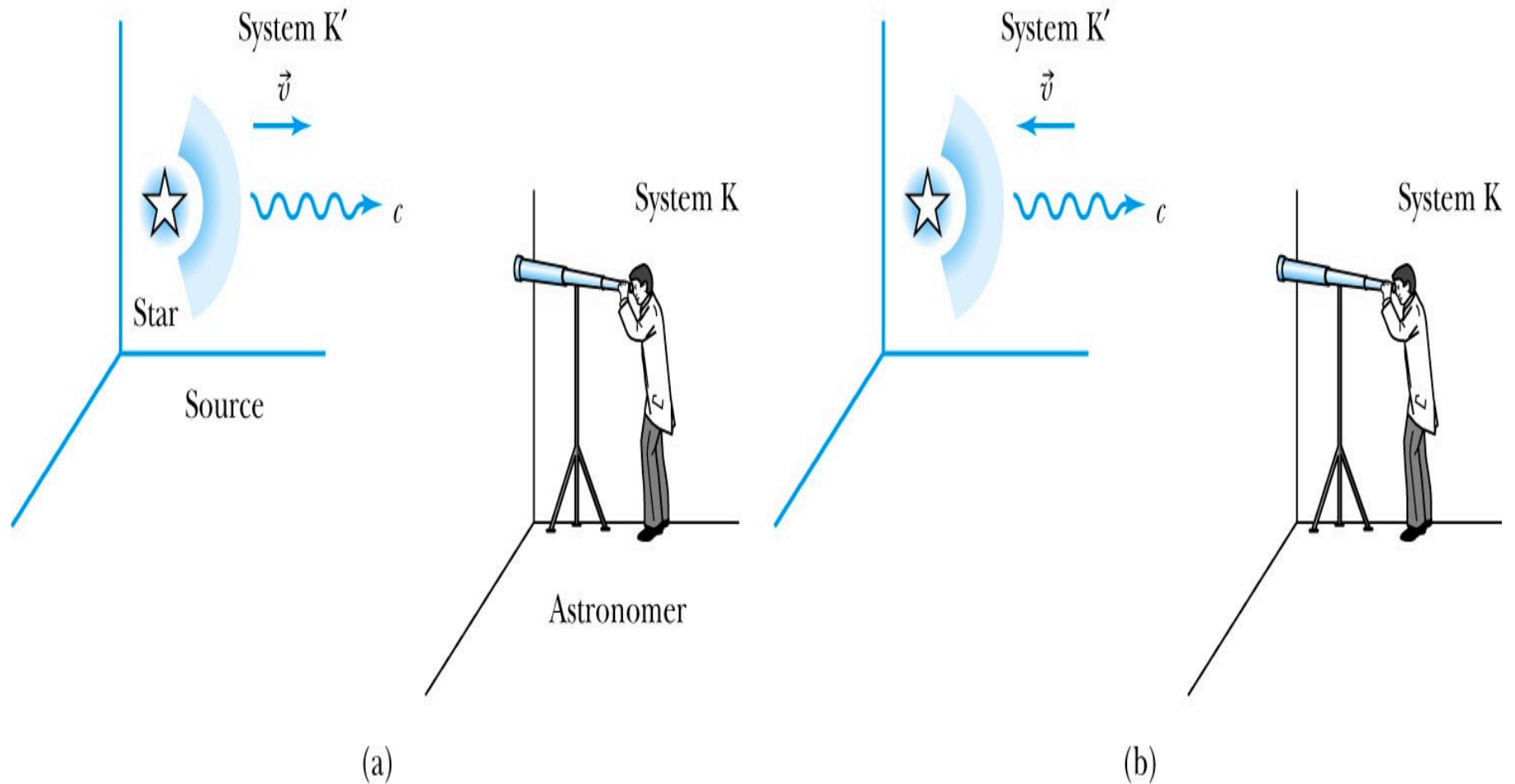
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The Doppler Effect

- The Doppler effect of sound
 - *Increased frequency* of sound as a source approaches a receiver
 - *Decreased frequency* as the source recedes.
- Also, the same change in sound frequency occurs when the source is fixed and the receiver is moving.
 - The change in frequency of the sound wave depends on whether the source or receiver is moving.
- Does this violate the principle of relativity?
 - No
 - Why not?
 - Sound wave is in a special frame of media such as air, water, or a steel plate in order to propagate;
- Light does not need a medium to propagate!



Recall the Doppler Effect



The Relativistic Doppler Effect

Consider a source of light (a star) and a receiver (an astronomer) approaching one another with a relative velocity v .

- 1) Consider the receiver in system K and the light source in system K' moving toward the receiver with velocity v .
- 2) The source emits n waves during the time interval T .
- 3) Because the speed of light is always c and the source is moving with velocity v , the total distance between the front and rear of the wave emitted during the time interval T is:

$$\text{Length of wave train} = cT - vT$$



The Relativistic Doppler Effect (con't)

Because there are n waves emitted by the source in time period T , the wavelength measured by the stationary receiver is

$$\lambda = \frac{cT - vT}{n}$$

And the resulting frequency measured by the receiver is

$$f = \frac{cn}{cT - vT}$$

The number of waves emitted in the moving frame of the source is $n = f_0 T'_0$ with the proper time $T'_0 = T/\gamma$ we obtain the measured frequency by the receiver as

$$f = \frac{cf_0 T/\gamma}{cT - vT} = \frac{1}{1 - v/c} \frac{f_0}{\gamma} = \frac{\sqrt{1 - \beta^2}}{1 - \beta} f_0 = \sqrt{\frac{(1 - \beta)(1 + \beta)}{(1 - \beta)^2}} f_0 = \sqrt{\frac{1 + \beta}{1 - \beta}} f_0$$



Results of Relativistic Doppler Effect

When source/receiver is approaching with $\beta = v/c$ the resulting frequency is

$$f = \sqrt{\frac{1+\beta}{1-\beta}} f_0$$

Higher than the actual source's frequency, blue shift!!

When source/receiver is receding with $\beta = v/c$ the resulting frequency is

$$f = \sqrt{\frac{1-\beta}{1+\beta}} f_0$$

Lower than the actual source's frequency, red shift!!

If we use $+\beta$ for approaching source/receiver and $-\beta$ for receding source/receiver, relativistic Doppler Effect can be expressed

$$f = \sqrt{\frac{1+\beta}{1-\beta}} f_0$$

What can we use this for?

$$f \cos \theta = \frac{f_0 (\cos \theta' + \beta)}{\sqrt{1-\beta^2}}$$