PHYS 3313 – Section 001
Lecture #7

Wednesday, Sept. 18, 2013

Dr. Jaehoon Yu

- Relativistic Momentum and Energy
- Relationship between relativistic quantities
- Quantization
- Discovery of the X-ray and the Electron
- Determination of Electron Charge

Announcements

- Reading assignments: CH 3.3 (special topic – the discovery of Helium) and CH3.7
- Colloquium this week: Dr. Samar Mohanty

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Biophotonics revolution: single molecule to whole organism

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4:00p.m Wednesday September 18, 2013
Room 101 Science Hall

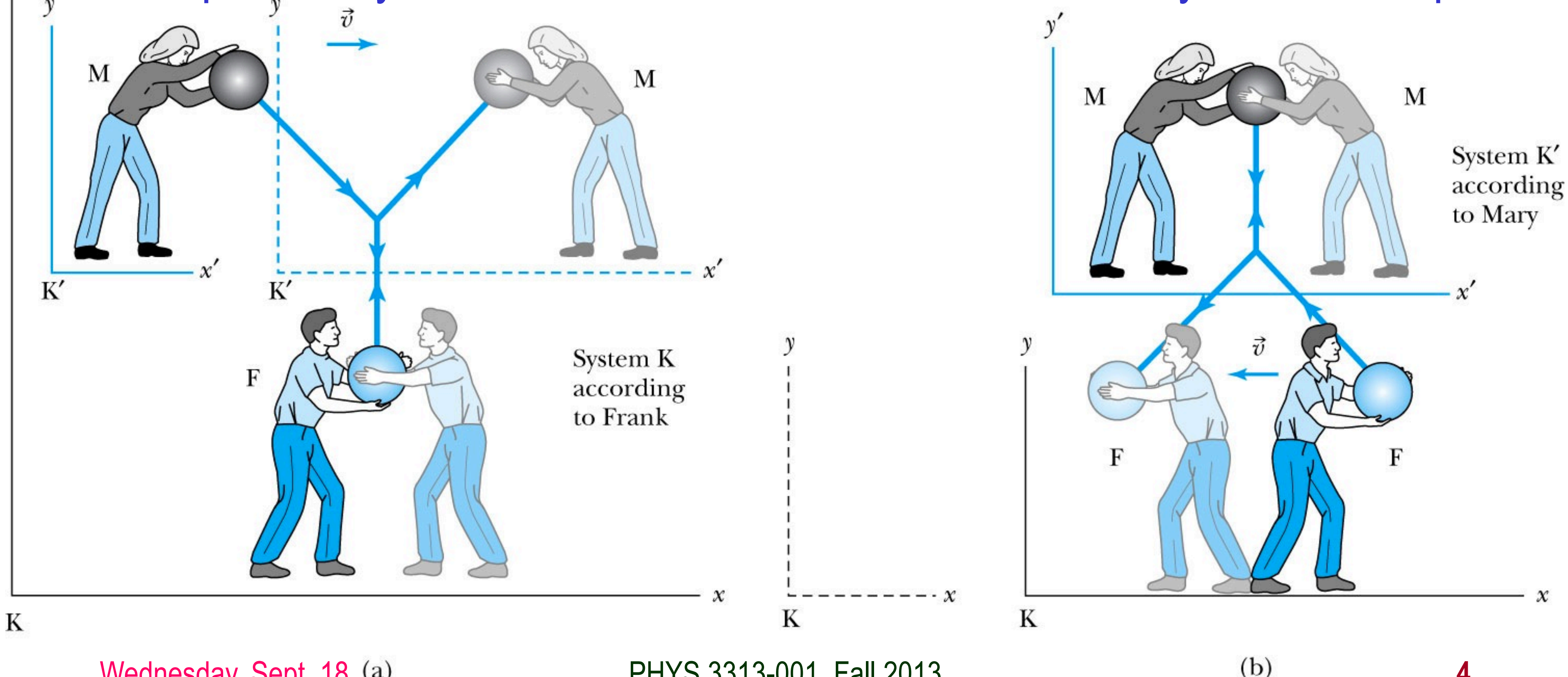
Abstract:

Biophotonics refers to use of light for imaging, control and manipulation of biological samples. Because of non-contact and highly-controllable nature of interaction between photons and bio-samples, it is emerging as one of the most significant interdisciplinary area of research and development from both fundamental and applied point of view. The level of invasiveness of photons can be controlled from non-invasiveness (for imaging applications) and minimal invasiveness (for control/manipulation) to destruction of targeted samples (for therapeutic applications). Use of nanotechnology, nanomaterials and biotechnology is further allowing enhancement of specific interactions (e.g. photomechanical, photothermal, photochemical) between light and bio-samples. I will provide an overview of biophotonics innovations (from single molecule to whole animal level) carried out in our lab, followed by description of our recent focus on construction, control, manipulation and imaging of neural circuits by light. I will briefly describe our present efforts in establishing *Center for neurophotonics* at UTA and activities of the center. In the last half of my talk, I will present our work on advancement in quantitative phase microscopy for high-throughput characterization of materials and may touch upon our exploration of its commercial potential.

Refreshments will be served at 3:30 in the Physics lounge

Relativistic Momentum

- Most fundamental principle used here is the momentum conservation!
- Frank is at rest in system K holding a ball of mass *m*.
- Mary holds a similar ball in system K' that is moving in the *x* direction with velocity *v* with respect to system K.
- At one point they threw the ball at each other with exactly the same speed



Relativistic Momentum

- If we use the definition of momentum, the momentum of the ball thrown by Frank is entirely in the *y* direction

$$p_{Fy} = mu_0$$

- The change of momentum as observed by Frank is

$$\Delta p_F = \Delta p_{Fy} = -2mu_0$$

- Mary measures the initial velocity of her own ball to be

$$u'_{Mx} = 0 \text{ and } u'_{My} = -u_0.$$

- In order to determine the velocity of Mary's ball as measured by Frank we use the velocity transformation equations:

$$u_{Mx} = v \qquad u_{My} = -u_0\sqrt{1 - v^2/c^2}$$

Relativistic Momentum

Before the collision, the momentum of Mary's ball as measured by Frank (in the **F**ixed frame) with the Lorentz velocity X-formation becomes

$$p_{Mx} = mv \qquad p_{My} = -mu_0\sqrt{1 - v^2/c^2}$$

For a perfectly elastic collision, the momentum after the collision is

$$p_{Mx} = mv \qquad p_{My} = +mu_0\sqrt{1 - v^2/c^2}$$

Thus the change in momentum of Mary's ball according to Frank is

$$\Delta p_M = \Delta p_{My} = 2mu_0\sqrt{1-\beta^2} \neq -\Delta p_{Fy}$$

OMG! The linear momentum is not conserved even w/o an external force!!

What do we do?

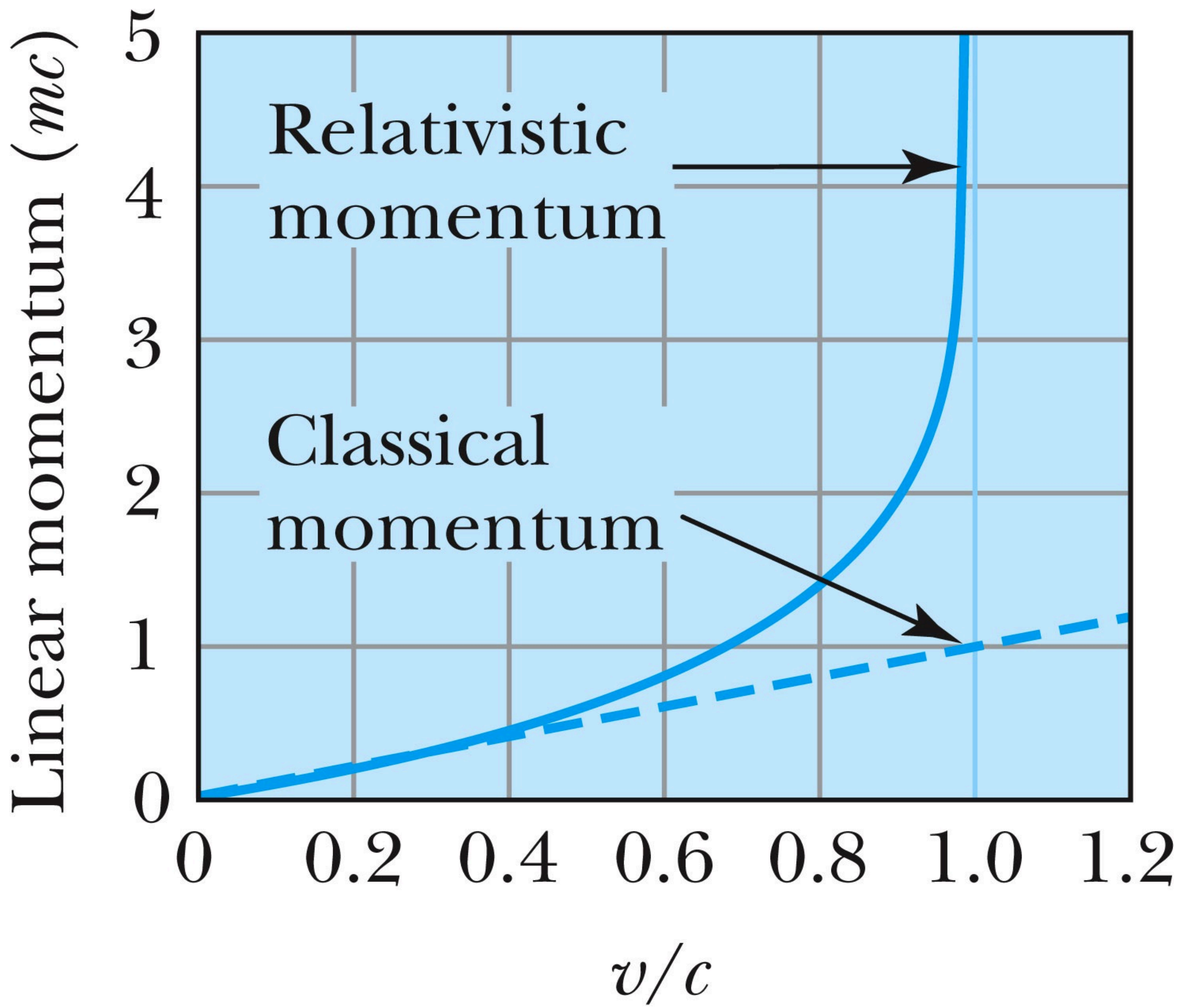
$$\vec{p} = m\frac{d\left(\gamma_u \vec{r}\right)}{dt} = m\gamma_u \vec{u}$$

➔Redefine the momentum in a fashion

➔Something has changed. Mass is now, mγ!! The relativistic mass!!

➔Mass as the fundamental property of matter is called the "rest mass", m_0!

Relativistic and Classical Linear Momentum



How do we keep momentum conserved in a relativistic case?

Redefine the classical momentum in the form:

$$\vec{p} = \Gamma(u) m\vec{u} = \frac{1}{\sqrt{1-u^2/c^2}} m\vec{u}$$

This $\Gamma(u)$ is different than the γ factor since it uses the particle's speed u

➔ What? How does this make sense?

➔ Well the particle itself is moving with a relativistic speed, thus that must impact the measurements by the observer in rest frame!!

Now, the agreed value of the momentum in all frames is:

$$\vec{p} = m\frac{d\vec{r}}{d\tau} = m\frac{d\vec{r}}{dt}\frac{dt}{d\tau} = m\vec{u}\gamma = \frac{1}{\sqrt{1-u^2/c^2}} m\vec{u}$$

Resulting in the new relativistic definition of the momentum:

$$\vec{p} = m\gamma\vec{u}$$

Relativistic Energy

- Due to the new idea of relativistic mass, we must now redefine the concepts of work and energy.
  - Modify Newton's second law to include our new definition of linear momentum, and force becomes:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}\left(\gamma m\vec{u}\right) = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-u^2/c^2}}\right)$$

- The work *W* done by a force **F** to move a particle from rest to a certain kinetic energy is

$$W = K = \int \frac{d}{dt}\left(\gamma m\vec{u}\right)\cdot\vec{u}dt$$

- Resulting relativistic kinetic energy becomes

$$K = \int_0^{\gamma u} ud\left(\gamma u\right) = \gamma mc^2 - mc^2 = \left(\gamma - 1\right)mc^2$$

- Why doesn't this look anything like the classical KE?

Big note on Relativistic KE

- Only $K = (\gamma - 1)mc^2$ is right!
- $K = \frac{1}{2}mu^2$ and $K = \frac{1}{2}\gamma mu^2$ are wrong!

Total Energy and Rest Energy

Rewriting the relativistic kinetic energy:

$$\gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = K + mc^2$$

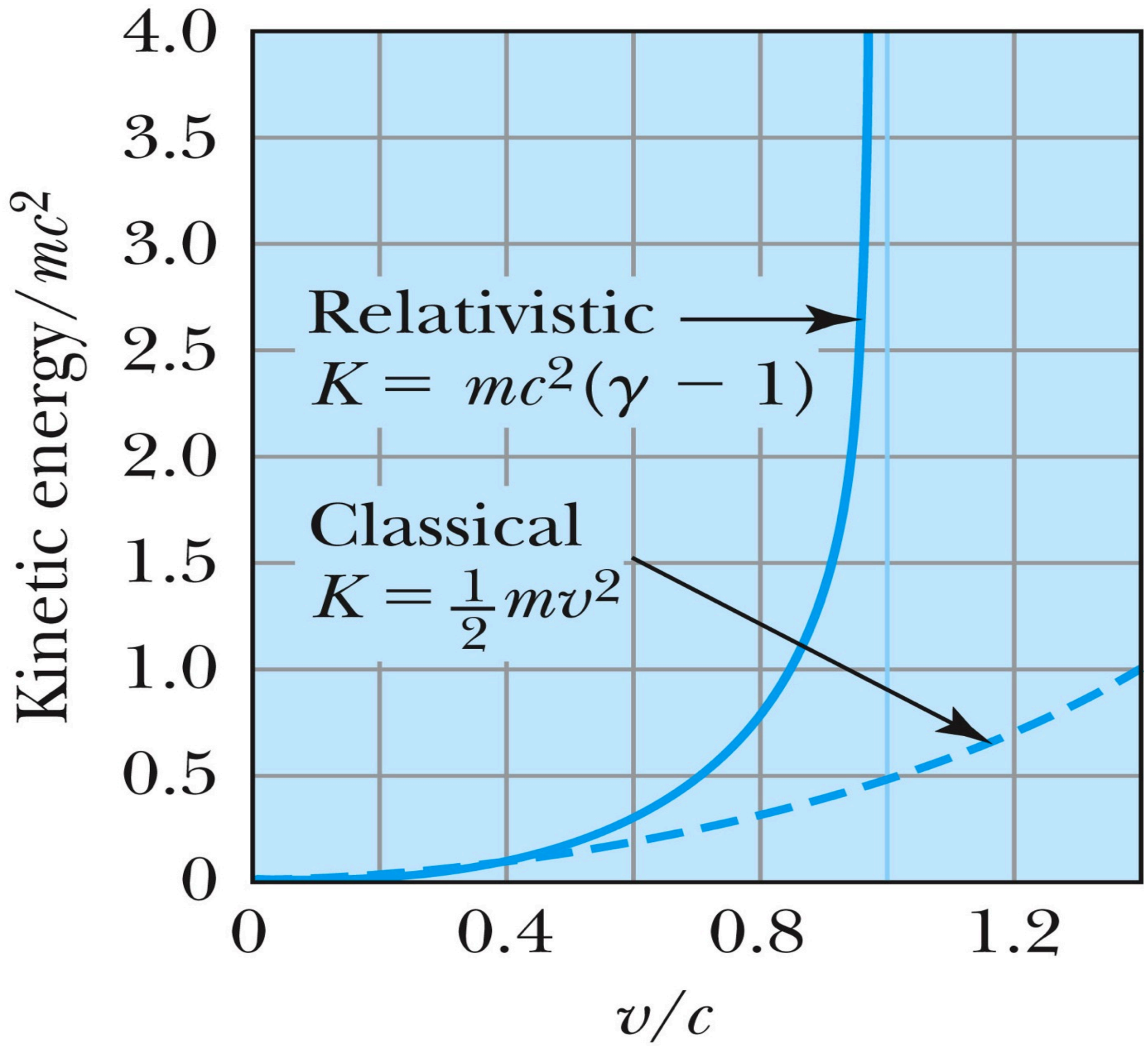
The term mc^2 is called the rest energy and is denoted by E_0.

$$E_0 = mc^2$$

The sum of the kinetic energy and rest energy is interpreted as the total energy of the particle.

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = \frac{E_0}{\sqrt{1-u^2/c^2}} = K + E_0$$

Relativistic and Classical Kinetic Energies



Relationship of Energy and Momentum

$$p = \gamma m u = \frac{mu}{\sqrt{1 - u^2/c^2}}$$

We square this result, multiply by c^2, and rearrange the result.

$$p^2c^2 = \gamma^2 m^2 u^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2}\right) = \gamma^2 m^2 c^4 \beta^2$$

$$\beta^2 = 1 - \frac{1}{\gamma^2} \Rightarrow p^2c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2}\right) = \gamma^2 m^2 c^4 - m^2 c^4$$

Rewrite

$$p^2c^2 = E^2 - E_0^2$$

Rewrite

$$E^2 = p^2c^2 + E_0^2 = p^2c^2 + m^2c^4$$

Massless Particles have a speed equal to the speed of light c

- Recall that a photon has "zero" rest mass and the equation from the last slide reduces to: E = pc and we may conclude that:

$$E = \gamma mc^2 = pc = \gamma muc$$

- Thus the velocity, u, of a massless particle must be c since, as $m \rightarrow 0, \gamma \rightarrow \infty$ and it follows that: u = c.

Units of Work, Energy and Mass

- The work done in accelerating a charge through a potential difference V is $W = qV$.
  - For a proton, with the charge $e = 1.602 \times 10^{-19}$ C being accelerated across a potential difference of 1 V, the work done is

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$W = (1.602 \times 10^{-19})(1 \text{ V}) = 1.602 \times 10^{-19} \text{ J}$$

- eV is also used as a unit of energy.

Other Units

1) Rest energy of a particle:
   Example: Rest energy, E_0, of proton

$$E_0(proton) = m_p c^2 = \left(1.67\times10^{-27}\,kg\right)\cdot\left(3.00\times10^{8}\,m/s\right) = 1.50\times10^{-10}\,J$$

$$= 1.50\times10^{-10}\,J\cdot\frac{1\,eV}{1.602\times10^{-19}\,J} = 9.38\times10^{8}\,eV$$

2) **Atomic mass unit** (amu): Example: carbon-12

$$M\left({}^{12}C\ \text{atom}\right) = \frac{12\,g/mole}{6.02\times10^{23}\,atoms/mole}$$

$$= 1.99\times10^{-23}\,g/atom$$

$$M\left({}^{12}C\ \text{atom}\right) = 1.99\times10^{-26}\,kg/atom = 12\,u/atom$$

Binding Energy

- The potential energy associated with the force keeping the system together ➔ E_B.
- The difference between the rest energy of the individual particles and the rest energy of the combined bound system.

$$M_{\text{bound system}}c^2 + E_B = \sum_i m_i c^2$$

$$E_B = \sum_i m_i c^2 - M_{\text{bound system}}c^2$$

What does the word “Quantize” mean?

- Dictionary: To restrict to discrete values
- To consist of indivisible discrete quantities instead of continuous quantities
  - Integer is a quantized set with respect to real numbers
- Some examples of quantization?
  - Digital photos
  - Lego blocks
  - Electric charge
  - Photon (a quanta of light) energy
  - Angular momentum
  - Etc…

Discovery of the X Ray and the Electron

- X rays were discovered by Wilhelm Röntgen in 1895.
  - Observed X rays emitted by cathode rays bombarding glass
- Electrons were discovered by J. J. Thomson.
  - Observed that cathode rays were charged particles

Cathode Ray Experiments

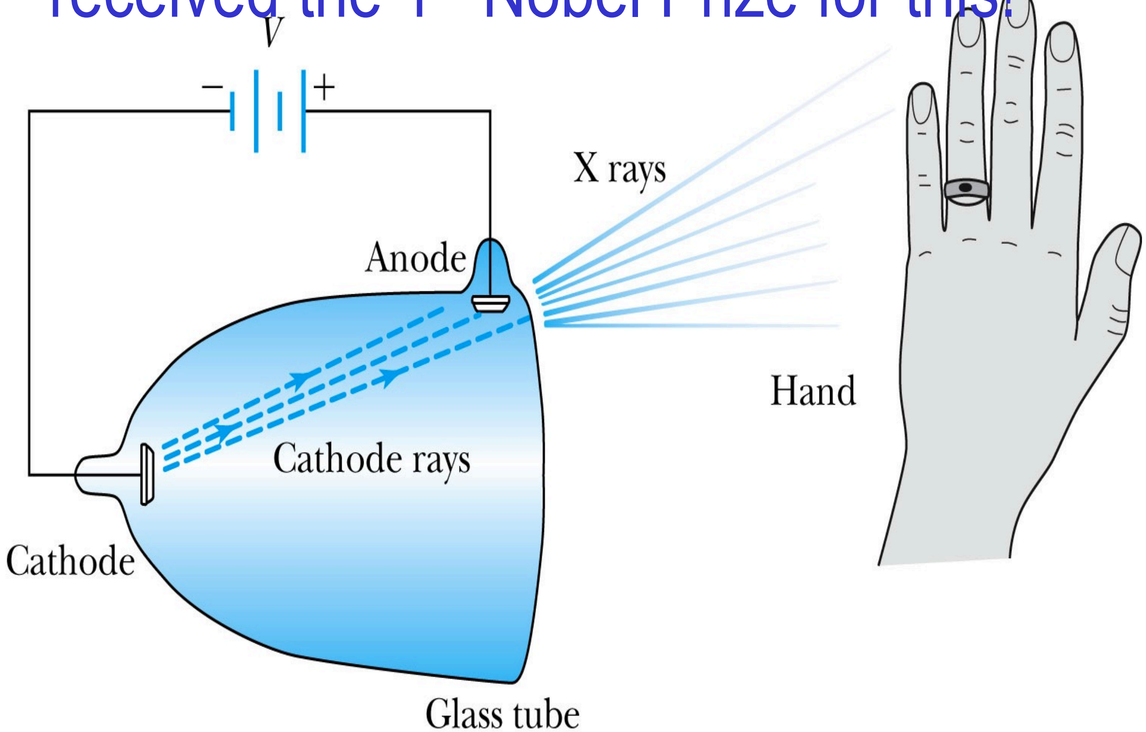
- In the 1890s scientists and engineers were familiar with cathode rays, generated from one of the metal plates in an evacuated tube across a large electric potential
- People thought cathode rays had something to do with atoms.
- It was known that cathode rays could penetrate matter and their properties were under intense investigation during the 1890s.

Observation of x Rays

- Wilhelm Röntgen studied the effect of cathode rays passing through various materials.
- He noticed that a nearby phosphorescent screen glowed during some of these experiments.
- These rays were unaffected by magnetic fields and penetrated materials more than cathode rays.
- He called them **x rays** and deduced that they were produced by the cathode rays bombarding the glass walls of his vacuum tube

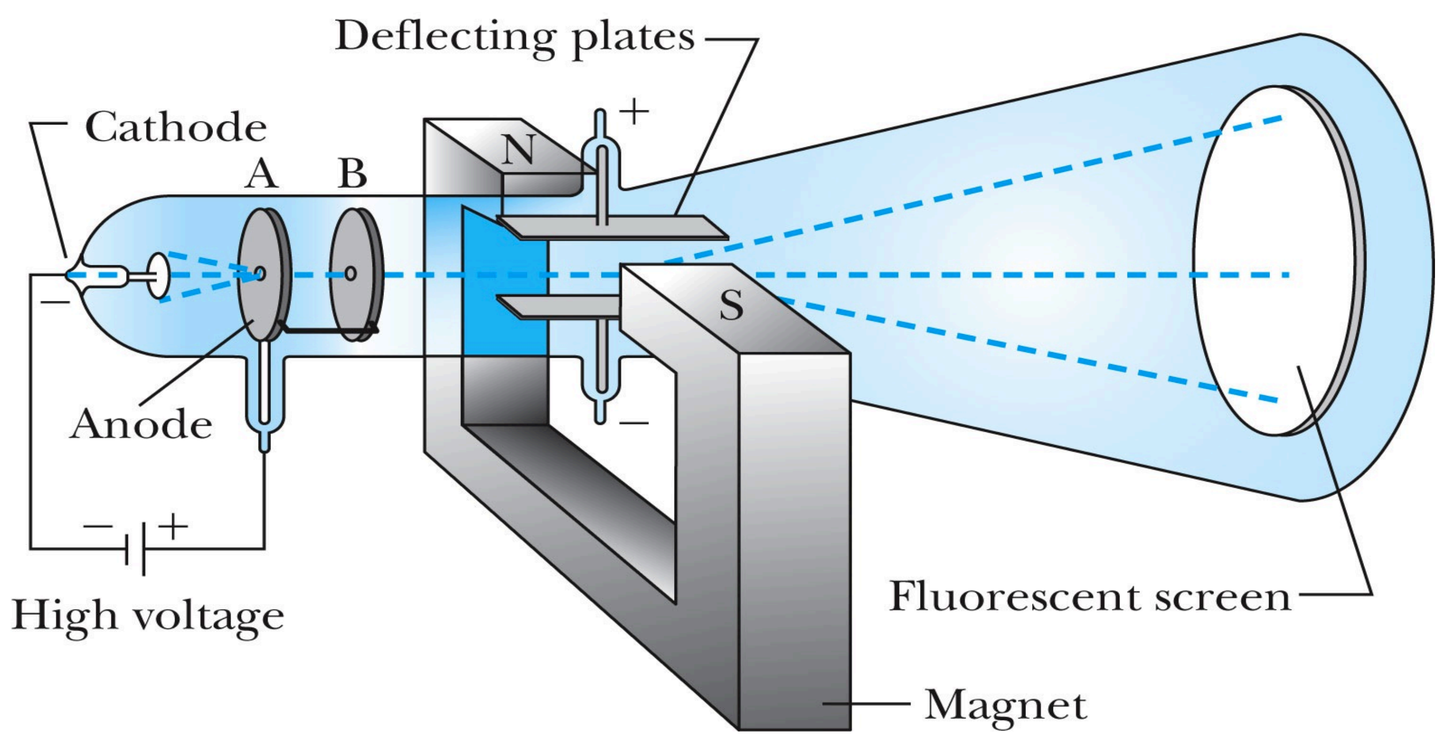
Röntgen's X Ray Tube

- Röntgen produced X-ray by allowing cathode rays to impact the glass wall of the tube.
- Took image the bones of a hand on a phosphorescent screen.
- Tremendous contribution to medical imaging, and Röntgen received the 1st Nobel Prize for this!



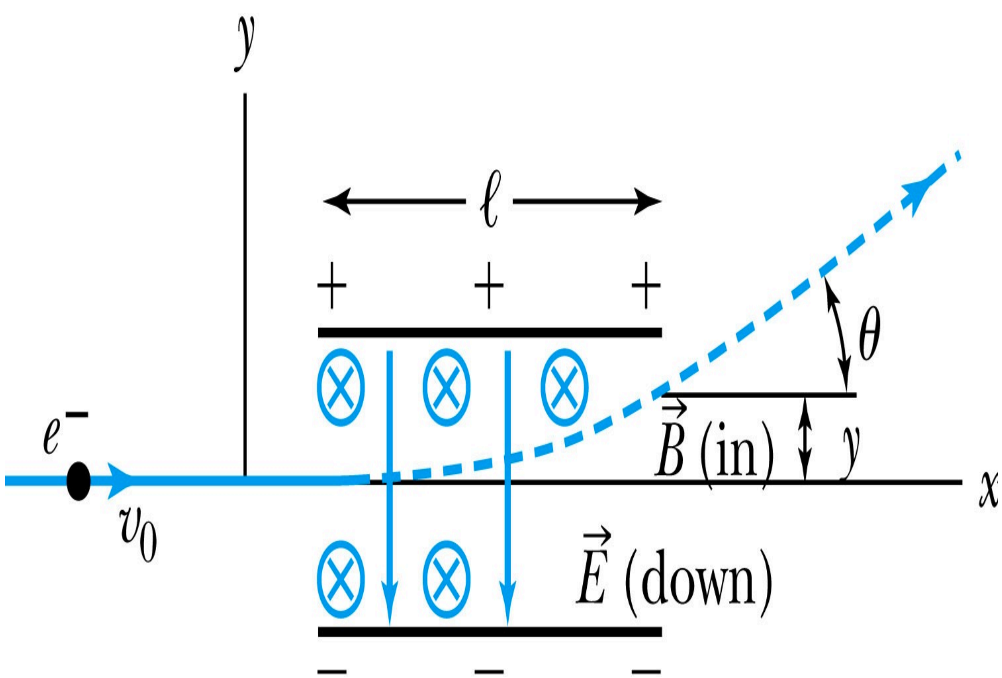
J.J. Thomson's Cathode-Ray Experiment

- Thomson showed that the cathode rays were negatively charged particles (electrons)! How?
  - By deflecting them in electric and magnetic fields.



Thomson's Experiment

- Thomson measured the ratio of the electron's charge to mass by sending electrons through a region containing a magnetic field perpendicular to an electric field.



- Measure the deflection angle with only E!
- Turn on and adjust B field till no deflection!
- What do we know?
  - ℓ, B, E and θ
- What do we not know?
  - v_0, q and m

Calculation of *q/m*

- An electron moving through the electric field w/o magnetic field is accelerated by the force: $F_y = ma_y = qE$
- Electron angle of deflection: $\tan\theta = \frac{v_y}{v_x} = \frac{a_y t}{v_0} = \frac{qE}{m}\frac{l/v_0}{v_0} = \frac{qE}{m}\frac{l}{v_0^2}$
- Adjust the perpendicular magnetic field till it balances E and keeps electrons from deflecting in y-direction

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} = 0$$

$$F_y = -qE + qv_x B = 0 \quad \Rightarrow qE = qv_x B \quad \Rightarrow v_x = \frac{E}{B} = v_0$$

- Charge to mass ratio:

$$\tan\theta = \frac{qE}{m}\frac{l}{v_0^2} \Rightarrow \quad \frac{q}{m} = \frac{v_0^2\tan\theta}{El} = \frac{(E/B)^2\tan\theta}{El} = \frac{E\tan\theta}{B^2 l}$$

Ex 3.1: Thomson's experiment

- In an experiment similar to Thomson's, we use deflecting plates 5.0cm in length with an electric field of $1.2\mathrm{x}10^4$V/m. Without the magnetic field, we find an angular deflection of 30^o, and with a magnetic field of $8.8\mathrm{x}10^{-4}$T we find no deflection. What is the initial velocity of the electron and its q/m?
- First v_0 using E and B, we obtain:

$$v_0 = v_x = \frac{E}{B} = \frac{1.2\times10^4}{8.8\times10^{-4}} = 1.4\times10^7\,m/s$$

- q/m is then

$$\frac{q}{m} = \frac{E\tan\theta}{B^2 l} = \frac{1.2\times10^4\tan30^\circ}{\left(8.8\times10^{-4}\right)^2\cdot0.5} = 1.8\times10^{11}\,C/kg$$

- What is the actual value of q/m using the known quantities?

$$\frac{q}{m} = \frac{1.6022\times10^{-19}}{9.1094\times10^{-31}} = 1.759\times10^{-11}\,C/kg$$