PHYS 3313 – Section 001
Lecture #12

Monday, Oct. 21, 2013
Dr. Jaehoon Yu

- The Schrödinger Wave Equation
- Probability Density
- Wave Function Normalization
- Time-Independent Schrödinger Wave Equation
- Expectation Values
- Operators – Position, Momentum and Energy

Announcements

- Mid-term exam results
  - Class average: 45.2/96
    - Equivalent to 47/100
  - Top score: 96/96
- Mid-term grade discussion moved to Monday, Oct. 28
- Reminder Homework #4
  - End of chapter problems on CH5: 8, 10, 16, 24, 26, 36 and 47
  - Due: This Wednesday, Oct. 23
- Reading assignments
  - CH6.1 – 6.7 + the special topic
- Colloquia this week
  - 4pm today, SH101, Dr. C. Guo of U. of Rochester
  - 4pm Wednesday, SH101, Dr. X. Chu, U. of Colorado

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Dr. Chunlei Guo

The Institute of Optics, University of Rochester

Candidate for Faculty Position:

The Richard N. Claytor
Distinguished Professorship in Optics

Monday October 21 at 4:00 pm in Room 101 SH

The Black and Colored Metals and Applications

Abstract:

Advances of high-intensity femtosecond laser techniques have opened new frontiers in studying exotic states of matter when it is exposed to ultrashort high-intensity laser pulse irradiation. In this talk, I will discuss a number of our ongoing research topics in this area, especially our recent discoveries of the so-called black and colored metals. I will also discuss various applications of the black and colored metals, from making more efficient light sources to turning regular materials superwicking.

<u>Refreshments will be served at 3:30p.m in the Physics lounge room 106 SH</u>

Physics Department
The University of Texas at Arlington

COLLOQUIUM

Dr. Xinzhao Chu

The University of Colorado at Boulder

Candidate for Faculty Position:
The Richard N. Claytor
Distinguished Professorship in Optics

Wednesday October 23 at 4:00 pm in Room 101 SH

Photons from Atomic World to the Space

Abstract:

Living and working with photons every day, I do believe that photons are a key element to human society and scientific research. Photons have allowed people to explore the detailed structures of a single atom and its interaction with radiation fields; photons have enabled many fancy technologies and products used in our daily life; and photons have extended human's eyes to peek into the Universe – just a few examples among millions of photon "fairy tales". In this colloquium I will share my experience of using photons to explore the unknowns from atomic dimension to the space, and discuss the future of optics and photonics – how this very active science branch could bring prosperity to our world. In particular, I would like to tell the stories of how we use photons to unlock the secrets of Antarctica – the last frontier on Earth!

Refreshments will be served at 3:30p.m in the Physics lounge room 106 SH

Research Projects

1. Each of the 12 research groups picks one research topic
2. Study the topic as a group, looking up references
   - Original theory or Original observation
   - Experimental proofs or Theoretical predictions + subsequent experimental proofs
   - Importance and the impact of the theory/experiment
3. Each member of the group writes a 10 page report, including figures (must not copy!!)
   - 10% of the total grade
   - Can share the theme and facts but you must write your own!
   - Due Mon., Nov. 25, 2013
4. The group presents a 10min power point talk
   - 5% of the total grade
   - Date and time will be announced close to the end of the semester

Group – Research Topic Association

Research Group Number	Research Topic
1	2
2	3
3	11
4	7
5	5
6	1
7	9
8	8
9	4
10	6
11	10
12	12

Research Topics

1. Black body radiation
2. Michelson–Morley experiment
3. The Photoelectric effect
4. Special Relativity
5. The property of molecules, Browning Motion
6. Compton Effect
7. Discovery of the electron
8. Radioactivity
9. Rutherford Scattering
10. Super-conductivity
11. The Unification of Electromagnetic and Weak forces
12. The Discovery of the Higgs-like particle

Special Project #4

- Prove that the wave function $\Psi = A[\sin(kx-\omega t) + i\cos(kx-\omega t)]$ is a good solution for the time-dependent Schrödinger wave equation. Do NOT use the exponential expression of the wave function. (10 points)
- Determine whether or not the wave function $\Psi = Ae^{-\alpha|x|}$ satisfy the time-dependent Schrödinger wave equation. (10 points)
- Due for this special project is Monday, Oct. 28.
- You MUST have your own answers!

The Schrödinger Wave Equation

- Erwin Schrödinger and Werner Heinsenberg proposed quantum theory in 1920
  - The two proposed very different forms of equations
  - Heinserberg: Matrix based framework
  - Schrödinger: Wave mechanics, similar to the classical wave equation
- Paul Dirac and Schrödinger later on proved that the two give identical results
- The probabilistic nature of quantum theory is contradictory to the direct cause and effect seen in classical physics and makes it difficult to grasp!

The Schrödinger Wave Equation

- The Schrödinger wave equation in its time-dependent form for a particle of energy *E* moving in a potential *V* in one dimension is

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}+V\Psi(x,t)$$

- The extension into three dimensions is

$$i\hbar\frac{\partial\Psi}{\partial t}=-\frac{\hbar^2}{2m}\left(\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2}\right)+V\Psi(x,y,z,t)$$

- where $i=\sqrt{-1}$ is an imaginary number

Ex 6.1: Wave equation and Superposition

The wave equation must be linear so that we can use the superposition principle to. Prove that the wave function in Schrodinger equation is linear by showing that it is satisfied for the wave equation $\Psi(x,t)=a\Psi_1(x,t)+b\Psi_2(x,t)$ where a and b are constants and $\Psi_1(x,t)$ and $\Psi_2(x,t)$ describe two waves each satisfying the Schrodinger Eq.

$$\Psi = a\Psi_1 + b\Psi_2 \qquad i\hbar\frac{\partial\Psi_1}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi_1}{\partial x^2} + V\Psi_1 \qquad i\hbar\frac{\partial\Psi_2}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi_2}{\partial x^2} + V\Psi_2$$

$$\frac{\partial\Psi}{\partial t} = \frac{\partial}{\partial t}\left(a\Psi_1 + b\Psi_2\right) = a\frac{\partial\Psi_1}{\partial t} + b\frac{\partial\Psi_2}{\partial t}$$

$$\frac{\partial\Psi}{\partial x} = \frac{\partial}{\partial x}\left(a\Psi_1 + b\Psi_2\right) = a\frac{\partial\Psi_1}{\partial x} + b\frac{\partial\Psi_2}{\partial x} \Rightarrow \frac{\partial^2\Psi}{\partial x^2} = \frac{\partial}{\partial x}\left(a\frac{\partial\Psi_1}{\partial x} + b\frac{\partial\Psi_2}{\partial x}\right) = a\frac{\partial^2\Psi_1}{\partial x^2} + b\frac{\partial^2\Psi_2}{\partial x^2}$$

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi \xrightarrow{\text{Rearrange terms}} i\hbar\frac{\partial\Psi}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} - V\Psi = \left(i\hbar\frac{\partial}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - V\right)\Psi = 0$$

$$i\hbar\frac{\partial\Psi}{\partial t} = i\hbar\left(a\frac{\partial\Psi_1}{\partial t} + b\frac{\partial\Psi_2}{\partial t}\right) = -\frac{\hbar^2}{2m}\left(a\frac{\partial^2\Psi_1}{\partial x^2} + b\frac{\partial^2\Psi_2}{\partial x^2}\right) + V\left(a\Psi_1 + b\Psi_2\right)$$

$$a\left(i\hbar\frac{\partial\Psi_1}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2\Psi_1}{\partial x^2} - V\Psi_1\right) = -b\left(i\hbar\frac{\partial\Psi_2}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2\Psi_2}{\partial x^2} - V\Psi_2\right) = 0$$

General Solution of the Schrödinger Wave Equation

- The general form of the solution of the Schrödinger wave equation is given by:

$$\Psi(x,t) = Ae^{i(kx-\omega t)} = A\left[\cos(kx-\omega t) + i\sin(kx-\omega t)\right]$$

- which also describes a wave propagating in the *x* direction. In general the amplitude may also be complex. *This is called the* ***wave function of the particle****.*
- The wave function is also **not** restricted to being real. Only the physically measurable quantities (or **observables**) must be real. These include the probability, momentum and energy.

Ex 6.2: Solution for Wave Equation

Show that $Ae^{i(kx-\omega t)}$ satisfies the time-dependent Schrodinger wave Eq.

$$\Psi = Ae^{i(kx-\omega t)} \Rightarrow \frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial t}\left(Ae^{i(kx-\omega t)}\right) = -iA\omega e^{i(kx-\omega t)} = -i\omega\Psi$$

$$\frac{\partial \Psi}{\partial x} = \frac{\partial}{\partial x}\left(Ae^{i(kx-\omega t)}\right) = iAke^{i(kx-\omega t)} = ik\Psi$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{\partial}{\partial x}(ik\Psi) = ik\frac{\partial}{\partial x}(\Psi) = ik\left(iAke^{i(kx-\omega t)}\right) = -Ak^2e^{i(kx-\omega t)} = -k^2\Psi$$

$$i\hbar\frac{\partial \Psi}{\partial t} = i\hbar(-i\omega\Psi) = \hbar\omega\Psi = -\frac{\hbar^2}{2m}\left(-k^2\Psi\right) + V\Psi$$

$$\left(\hbar\omega - \frac{\hbar^2k^2}{2m} - V\right)\Psi = 0$$

$$= \left(E - \frac{p^2}{2m} - V\right) = 0$$

The Energy: $E = hf = h\left(\frac{\omega}{2\pi}\right) = \hbar\omega$

The wave number: $k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{2\pi p}{h} = \frac{p}{\hbar}$ $\Rightarrow$ The momentum: $p = \hbar k$

From the energy conservation: $E = K + V = \frac{p^2}{2m} + V$ $\Rightarrow$ $E - \frac{p^2}{2m} - V = 0$

So $Ae^{i(kx-\omega t)}$ is a good solution and satisfies Schrodinger Eq.

Ex 6.3: Bad Solution for Wave Equation

Determine Ψ(x,t)=Asin(kx-ωt) is an acceptable solution for the time-dependent Schrodinger wave Eq.

$$\Psi = A\sin(kx-\omega t) \Rightarrow \frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial t}\left(A\sin(kx-\omega t)\right) = -A\omega\cos(kx-\omega t)$$

$$\frac{\partial \Psi}{\partial x} = \frac{\partial}{\partial x}\left(A\sin(kx-\omega t)\right) = kA\cos(kx-\omega t)$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{\partial}{\partial x}\left(kA\cos(kx-\omega t)\right) = -k^2 A\sin(kx-\omega t) = -k^2\Psi$$

$$i\hbar\left(-\omega\cos(kx-\omega t)\right) = -\frac{\hbar^2}{2m}\left(-k^2\sin(kx-\omega t)\right) + V\sin(kx-\omega t)$$

$$-i\hbar\omega\cos(kx-\omega t) = \left(\frac{\hbar^2 k^2}{2m}+V\right)\sin(kx-\omega t)$$

$$\Rightarrow -iE\cos(kx-\omega t) = \left(\frac{p^2}{2m}+V\right)\sin(kx-\omega t)$$

This is not true in all x and t. So Ψ (x,t)=Asin(kx-ωt) is not an acceptable solution for Schrodinger Eq.

Normalization and Probability

- The probability $P(x)\,dx$ of a particle being between x and $X + dx$ was given in the equation

$$P(x)dx = \Psi^*(x,t)\Psi(x,t)dx$$

- Here Ψ^* denotes the complex conjugate of Ψ
- The probability of the particle being between x_1 and x_2 is given by

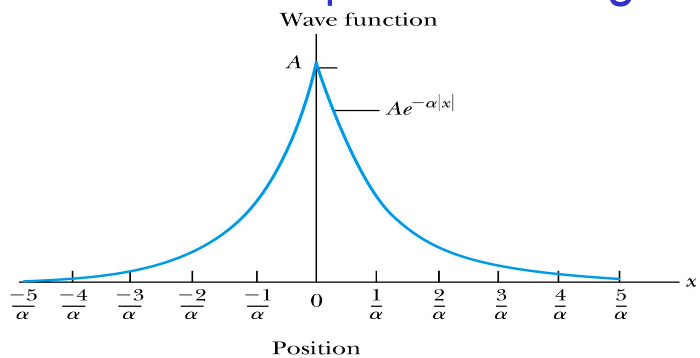
$$P = \int_{x_1}^{x_2} \Psi^* \Psi \, dx$$

- The wave function must also be normalized so that the probability of the particle being somewhere on the x axis is 1.

$$\int_{-\infty}^{+\infty} \Psi^*(x,t)\Psi(x,t)dx = 1$$

Ex 6.4: Normalization

Consider a wave packet formed by using the wave function that $Ae^{-\alpha|x|}$, where A is a constant to be determined by normalization. Normalize this wave function and find the probabilities of the particle being between 0 and $1/\alpha$, and between $1/\alpha$ and $2/\alpha$.



$$\Psi = Ae^{-\alpha|x|}$$

Probability density

$$\int_{-\infty}^{+\infty} \Psi^* \Psi dx = \int_{-\infty}^{+\infty} \left(Ae^{-\alpha|x|}\right)^* \left(Ae^{-\alpha|x|}\right) dx = \int_{-\infty}^{+\infty} \left(A^* e^{-\alpha|x|}\right)\left(Ae^{-\alpha|x|}\right) dx =$$

$$= \int_{-\infty}^{+\infty} A^2 e^{-2\alpha|x|} dx = 2\int_{0}^{+\infty} A^2 e^{-2\alpha|x|} dx = \left.\frac{2A^2}{-2\alpha} e^{-2\alpha|x|}\right|_0^{+\infty} = 0 + \frac{A^2}{\alpha} = 1$$

$$A = \sqrt{\alpha}$$

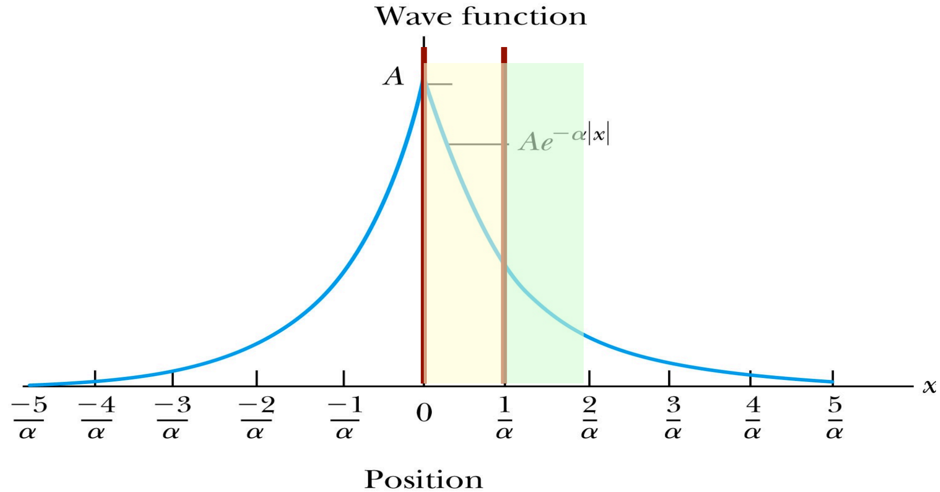
Normalized Wave Function

$$\Psi = \sqrt{\alpha}\, e^{-\alpha|x|}$$

Ex 6.4: Normalization, cont'd

Using the wave function, we can compute the probability for a particle to be with 0 to $1/\alpha$ and $1/\alpha$ to $2/\alpha$.

$$\Psi = \sqrt{\alpha}\, e^{-\alpha|x|}$$



For 0 to $1/\alpha$:

$$P = \int_0^{1/\alpha} \Psi^*\Psi dx = \int_0^{1/\alpha} \alpha e^{-2\alpha x} dx = \frac{\alpha}{-2\alpha} e^{-2\alpha x}\Big|_0^{1/\alpha} = -\frac{1}{2}\left(e^{-2} - 1\right) \approx 0.432$$

For $1/\alpha$ to $2/\alpha$:

$$P = \int_{1/\alpha}^{2/\alpha} \Psi^*\Psi dx = \int_{1/\alpha}^{2/\alpha} \alpha e^{-2\alpha x} dx = \frac{\alpha}{-2\alpha} e^{-2\alpha x}\Big|_{1/\alpha}^{2/\alpha} = -\frac{1}{2}\left(e^{-4} - e^{-2}\right) \approx 0.059$$

How about $2/\alpha$:to ∞?