PHYS 3313 – Section 001
Lecture #15

Monday, Nov. 4, 2013

Dr. Jaehoon Yu

- Finite Potential Well
- Penetration Depth
- Degeneracy
- Simple Harmonic Oscillator
- Barriers and Tunneling

Announcements

- Reminder: Homework #5
  - CH6 end of chapter problems: 3, 5, 11, 14, 22, and 26
  - Due on this Wednesday, Nov. 6 in class
- Quiz #3
  - At the beginning of the class this Wed., Nov. 6
  - Covers CH5.6 to what we cover today (CH6.6?)
  - Prepare your own formula sheet
- Research paper template is posted onto the research link
  - Deadline for research paper submission is Monday, Dec. 2!!
- Colloquium coming week
  - 4pm Monday, Nov. 4, SH101, Dr. Yujie Ding of Lehigh Univ., Double extra credit
  - 4pm Wednesday, Nov. 6, SH101, Dr. David Nygren of Lorentz Berkeley National Laboratory, Triple extra credit

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Dr. Yujie J. Ding

Department of Electrical and Computer Engineering
Lehigh University

Candidate for Faculty Position:
The Richard N. Claytor Distinguished Professorship in Optics

Monday November 4, 2013 at 4:00 pm in Room 101 SH

Parametric Generation of Terahertz Waves: from Saturation of Conversion Efficiency to Compact and Portable Sources for Variety of Applications

Abstract:

We have achieved the record-high photon conversion efficiency of 40% based on the parametric generation of high-power, tunable and monochromatic terahertz (THz) waves. Meanwhile, we have made a great progress on designing, assembling, and testing of dual-frequency solid-state lasers. Using these lasers, we have implemented portable and compact THz sources. We have significantly scaled up output powers through solid-state laser engineering. By replacing an active Q-switch with a passive Q-switch, we have investigated the parametric generation of the THz waves. We have explored intracavity generation of THz waves by using a nonlinear composite as an output coupler of an infrared laser. These novel configurations allow us to substantially reduce the dimension of the THz source and to realize a monolithic THz source. We have generated multiple THz frequencies and achieved interference effects using multiple infrared frequencies from solid-state lasers and dual signals and dual idlers from coupled optical parametric oscillators. We have investigated enhancements of THz output powers by exploiting polariton resonances, surface-emitting geometry, and extremely large internal electric fields in nitride heterostructures and nanostructures. Besides THz sources, photonics and applications, we have achieved single-photon detection at the communication wavelengths, reached the level well above the threshold for laser cooling based on anti-Stokes photoluminescence, restored distorted images based on phase conjugation, implemented coupled optical parametric oscillators which exhibit significantly reduced phase noises and have rich applications from quantum communications to biological imaging, and investigated novel nanostructures and nanodevices.

Refreshments will be served at 3:30p.m in the PhysicS lounge room 108SH

Physics Department

The University of Texas at Arlington

SPECIAL COLLOQUIUM

National Academy of Sciences Member

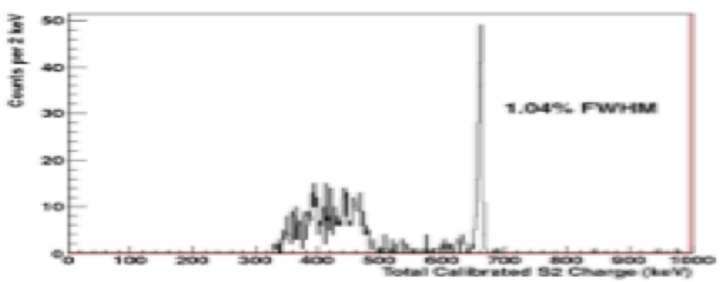
Dr. David Nygren

Physics Division, Lawrence Berkeley National Laboratory

4:00 pm Wednesday, November 6, 2013, Room 101 SH

Abstract:

The first detection of single ionizing events occurred more than 100 years ago when Ernest Rutherford and Hans Geiger succeeded in recording individual alpha-particles from radon decay using a gas-filled detector and an electrometer. Remarkably diverse and useful innovations followed, and continue to emerge even today. Thus, gas-filled detectors are the exemplary evolutionary survivors in nuclear and particle physics experimental technique. Although this ample record has many interesting chapters, I will focus on my favorite topics within this humble corner of the quest to understand our universe. The evolution of these devices is interesting not only for their substantial contributions to scientific progress, but also for what was, surprisingly, overlooked as technology evolved.



Energy spectrum measured for ^{137}Cs γ-rays (662 keV) with a high-pressure xenon gas TPC, relevant to the search for neutrino-less double-beta decay in ^{136}Xe. This appears to be the best energy resolution ever obtained in a xenon-based detector. This result also implies several important benefits for a direct detection WIMP search – including the possibility of directional sensitivity to the expected "WIMP wind" in a massive detector.

A special reception will be served at 3:30p.m in the Physics Lounge

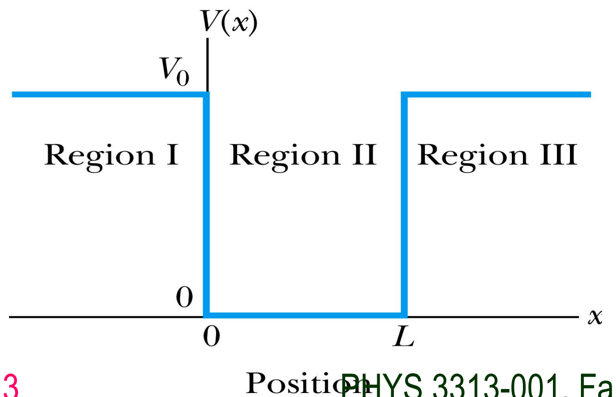
Reminder: Special project #5

- Show that the Schrodinger equation becomes Newton's second law. (15 points)
- Deadline Monday, Nov. 11, 2013
- You MUST have your own answers!

Finite Square-Well Potential

- The finite square-well potential is $V(x)=\begin{cases}V_0 & x\le 0,\\ 0 & 0<x<L\\ V_0 & x\ge L\end{cases}$
- The Schrödinger equation outside the finite well in regions I and III is
$-\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2}=E-V_0$ for regions I and III, or using $\alpha^2=2m(V_0-E)/\hbar^2$

  yields $\frac{d^2\psi}{dx^2}=\alpha^2\psi$. The solution to this differential has exponentials of the form $e^{\alpha x}$ and $e^{-\alpha x}$. In the region x > L, we reject the positive exponential and in the region x < 0, we reject the negative exponential. Why?



$$\psi_I(x)=Ae^{\alpha x} \quad \text{region I}, x<0$$
$$\psi_{III}(x)=Ae^{-\alpha x} \quad \text{region III}, x>L$$

This is because the wave function should be 0 as x→infinity.

Finite Square-Well Solution

- Inside the square well, where the potential *V* is zero and the particle is free, the wave equation becomes $\frac{d^2\psi}{dx^2} = -k^2\psi$ where $k = \sqrt{2mE/\hbar^2}$
- Instead of a sinusoidal solution we can write

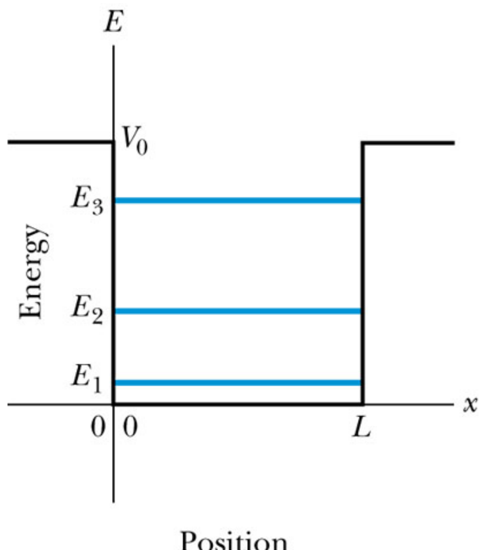
$$\psi_{II}(x) = Ce^{ikx} + De^{-ikx} \quad \text{region II, } 0 < x < L$$

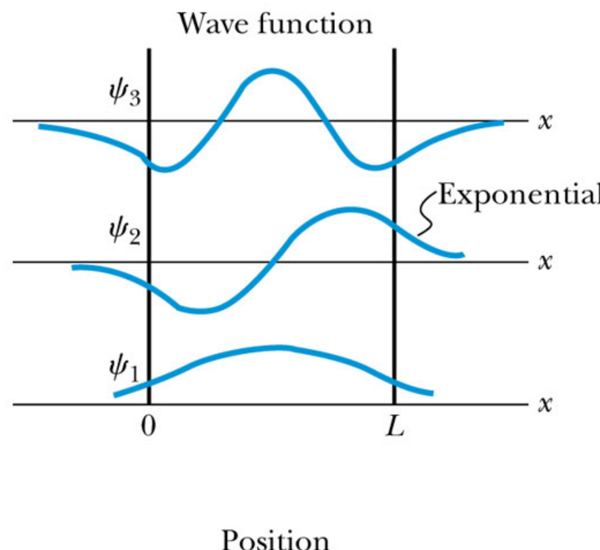
- The boundary conditions require that

$$\psi_I = \psi_{II} \text{ at } x = 0 \text{ and } \psi_I = \psi_{III} \text{ at } x = L$$

and the wave function must be smooth where the regions meet.

- Note that the wave function is nonzero outside of the box.
- Non-zero at the boundary either..
- What would the energy look like?





Penetration Depth

- The penetration depth is the distance outside the potential well where the probability significantly decreases. It is given by

$$\delta x \approx \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m\left(V_0 - E\right)}}$$

- It should not be surprising to find that the penetration distance that violates classical physics is proportional to Planck's constant.

Three-Dimensional Infinite-Potential Well

- The wave function must be a function of all three spatial coordinates.
- We begin with the conservation of energy $E = K + V = \frac{p^2}{2m} + V$
- Multiply this by the wave function to get

$$E\psi = \left(\frac{p^2}{2m} + V\right)\psi = \frac{p^2}{2m}\psi + V\psi$$

- Now consider momentum as an operator acting on the wave function. In this case, the operator must act twice on each dimension. Given:

$$p^2 = p_x^2 + p_y^2 + p_z^2 \qquad \hat{p}_x\psi = -i\hbar\frac{\partial\psi}{\partial x} \quad \hat{p}_y\psi = -i\hbar\frac{\partial\psi}{\partial y} \quad \hat{p}_z\psi = -i\hbar\frac{\partial\psi}{\partial z}$$

- The three dimensional Schrödinger wave equation is

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) + V\psi = E\psi$$

Rewrite

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

Ex 6.10: Expectation values inside a box

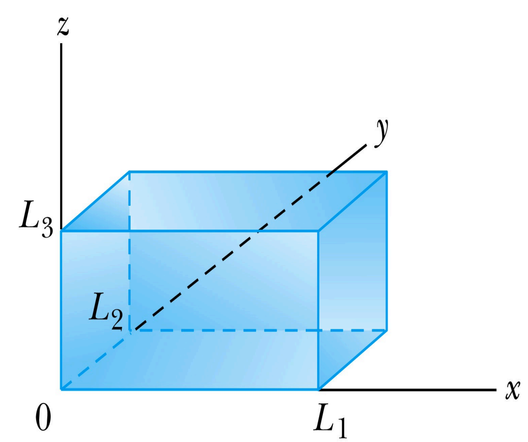
Consider a free particle inside a box with lengths L_1, L_2 and L_3 along the x, y, and z axes, respectively, as shown in the Figure. The particle is constrained to be inside the box. Find the wave functions and energies. Then find the ground energy and wave function and the energy of the first excited state for a cube of sides L.

What are the boundary conditions for this situation?

Particle is free, so x, y and z wave functions are independent from each other!

Each wave function must be 0 at the wall! Inside the box, potential V is 0.

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \Rightarrow -\frac{\hbar^2}{2m}\nabla^2\psi = E\psi$$



A reasonable solution is

$$\psi(x,y,z) = A\sin(k_1x)\sin(k_2y)\sin(k_3z)$$

Using the boundary condition

$$\psi = 0 \text{ at } x = L_1 \Rightarrow k_1L_1 = n_1\pi \Rightarrow k_1 = n_1\pi/L_1$$

So the wave numbers are $k_1 = \frac{n_1\pi}{L_1}$ $\quad k_2 = \frac{n_2\pi}{L_2}$ $\quad k_3 = \frac{n_3\pi}{L_3}$

Ex 6.10: Expectation values inside a box

Consider a free particle inside a box with lengths L_1, L_2 and L_3 along the x, y, and z axes, respectively, as shown in Fire. The particle is constrained to be inside the box. Find the wave functions and energies. Then find the round energy and wave function and the energy of the first excited state for a cube of sides L.

The energy can be obtained through the Schrödinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi = E\psi$$

$$\frac{\partial\psi}{\partial x} = \frac{\partial}{\partial x}\left(A\sin(k_1x)\sin(k_2y)\sin(k_3z)\right) = k_1A\cos(k_1x)\sin(k_2y)\sin(k_3z)$$

$$\frac{\partial^2\psi}{\partial x^2} = \frac{\partial^2}{\partial x^2}\left(A\sin(k_1x)\sin(k_2y)\sin(k_3z)\right) = -k_1^2A\sin(k_1x)\sin(k_2y)\sin(k_3z) = -k_1^2\psi$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi = \frac{\hbar^2}{2m}\left(k_1^2+k_2^2+k_3^2\right)\psi = E\psi$$

$$E = \frac{\hbar^2}{2m}\left(k_1^2+k_2^2+k_3^2\right) = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2}\right)$$

What is the ground state energy?

$E_{1,1,1}$ when $n_1=n_2=n_3=1$, how much?

When are the energies the same for different combinations of n_i?

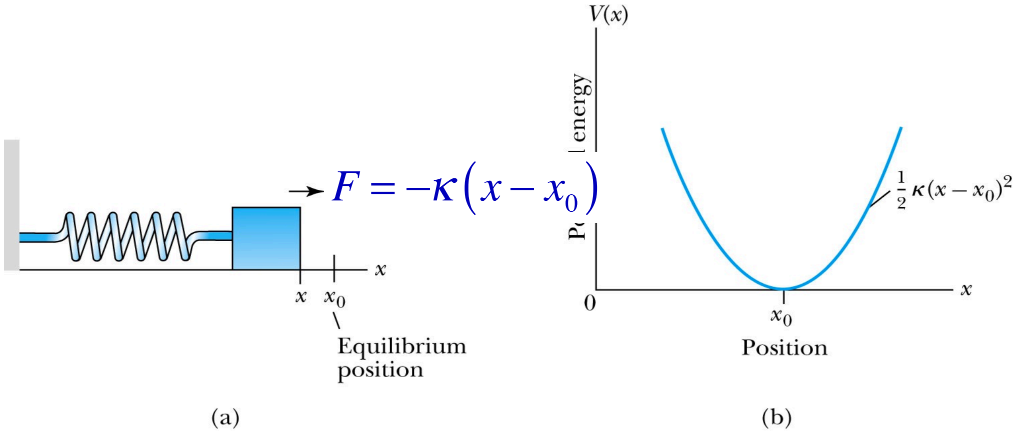
Degeneracy*

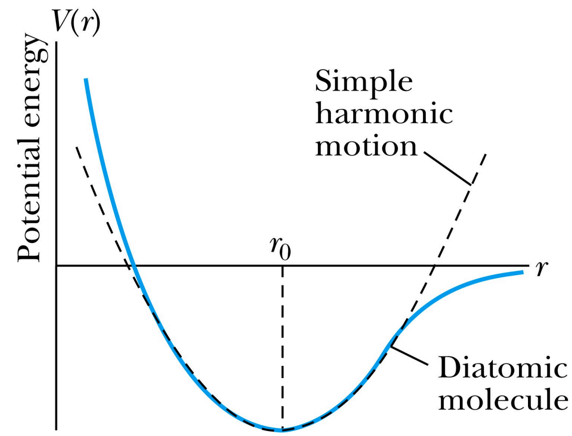
- Analysis of the Schrödinger wave equation in three dimensions introduces three quantum numbers that quantize the energy.
- A quantum state is degenerate when there is more than one wave function for a given energy.
- Degeneracy results from particular properties of the potential energy function that describes the system. A perturbation of the potential energy, such as the spin under a B field, can remove the degeneracy.

***Mirriam-webster: having two or more states or subdivisions having two or more states or subdivisions**

The Simple Harmonic Oscillator

- Simple harmonic oscillators describe many physical situations: springs, diatomic molecules and atomic lattices.





- Consider the Taylor expansion of a potential function:

$$V(x) = V_0 + V_1(x - x_0) + \frac{1}{2}V_2(x - x_0)^2 + \cdots$$

The minimum potential at $x=x_0$, so $dV/dx=0$ and $V_1=0$; and the zero potential $V_0=0$, we have

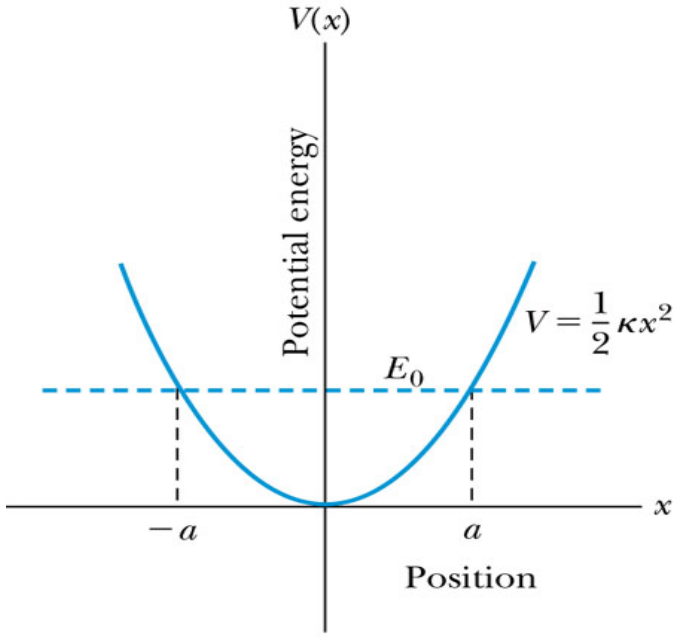
$$V(x) = \frac{1}{2}V_2(x - x_0)^2$$

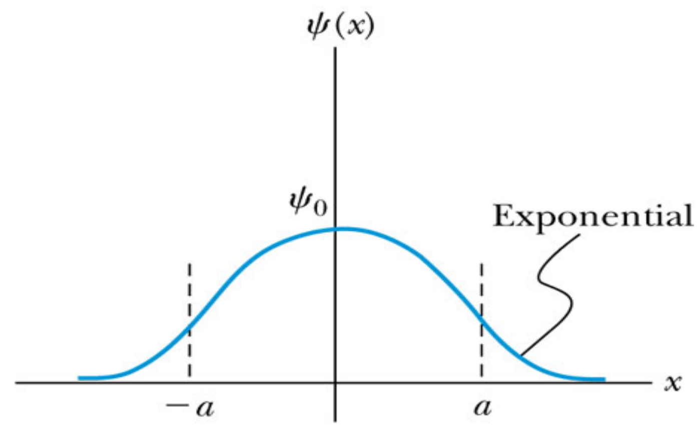
Substituting this into the wave equation:

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}\left(E - \frac{\kappa x^2}{2}\right)\psi = \left(-\frac{2m}{\hbar^2}E + \frac{m\kappa x^2}{\hbar^2}\right)\psi$$

Let $\alpha^2 = \frac{m\kappa}{\hbar^2}$ and $\beta = \frac{2mE}{\hbar^2}$ which yields $\frac{d^2\psi}{dx^2} = (\alpha^2 x^2 - \beta)\psi$.

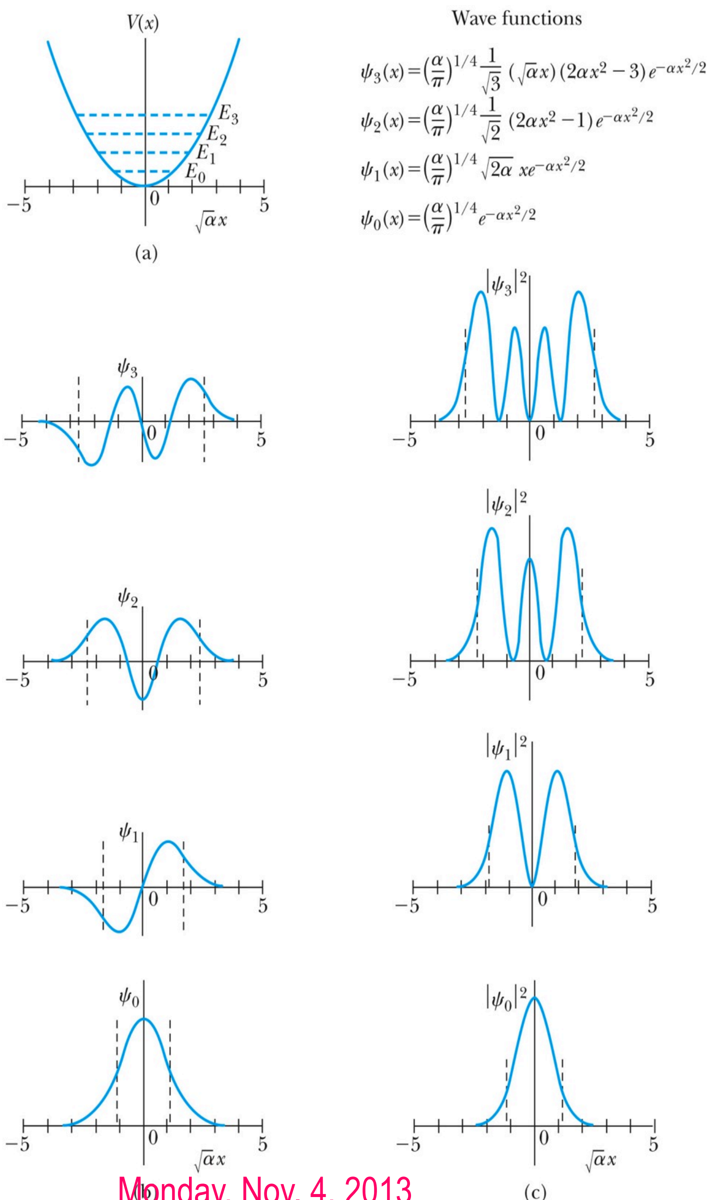
Parabolic Potential Well

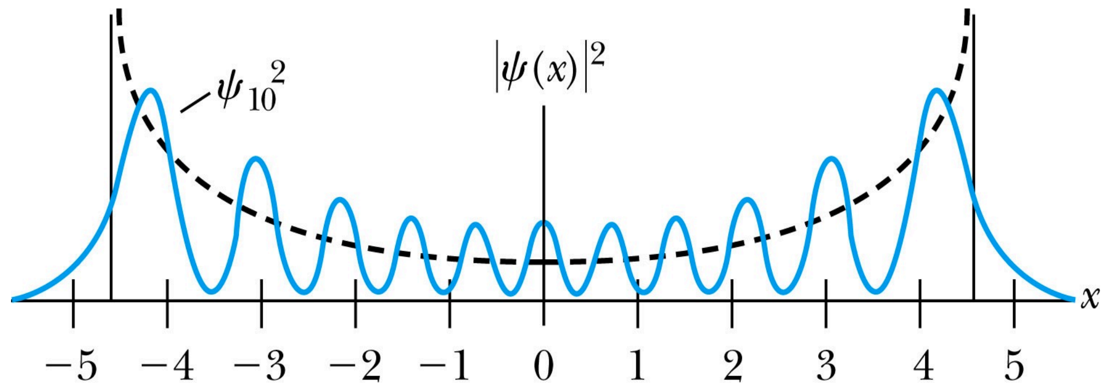




- If the lowest energy level is zero, this violates the uncertainty principle.
- The wave function solutions are $\psi_n = H_n(x)e^{-\alpha x^2/2}$ where $H_n(x)$ are Hermite polynomials of order *n*.
- In contrast to the particle in a box, where the oscillatory wave function is a sinusoidal curve, in this case the oscillatory behavior is due to the polynomial, which dominates at small *x*. The exponential tail is provided by the Gaussian function, which dominates at large *x*.

Analysis of the Parabolic Potential Well





- The energy levels are given by

$$E_n = \left(n + \frac{1}{2}\right)\hbar\sqrt{\kappa/m} = \left(n + \frac{1}{2}\right)\hbar\omega$$

- The zero point energy is called the Heisenberg limit:

$$E_0 = \frac{1}{2}\hbar\omega$$

- Classically, the probability of finding the mass is greatest at the ends of motion's range and smallest at the center (that is, proportional to the amount of time the mass spends at each position).
- Contrary to the classical one, the largest probability for this lowest energy state is for the particle to be at the center.