# PHYS 3313 – Section 001
# Lecture #19

*Monday, Nov. 18, 2013*
*Dr. Jaehoon Yu*

- Orbital Angular Momentum Quantum Number
- Magnetic Quantum Number
- The Zeeman Effect
- Intrinsic Spin
- Electron Energy Levels
- Selection Rules
- Probability Distributions
- Statistics

# Announcements

- Homework #7
  - CH7 end of chapter problems: 7, 8, 9, 12, 17 and 29
  - Due on Monday, Nov. 25, in class
- Quiz number 4
  - At the beginning of the class Monday, Nov. 25
  - Covers CH7 and what we finish this Wednesday
- Reading assignments
  - Entire CH8 (in particular CH8.1), CH9.4 and CH9.7
- Class is cancelled on Wednesday, Nov. 27

| Group Number | Reseasrch Group Members | Research Topic | Presentation Date and Order |
|---|---|---|---|
| 1 | Z.Citty, S. Lagerson, K. McElvain, J. Vellarreal | Michelson-Morley Experiment | #6, Dec. 2 |
| 2 | W. Brown, C. Hair, R. Reyes, H. Zapata | The Photo-Electric Effect | #2, Dec. 2 |
| 3 | R. Clark, M. Kruse, C. Nguyen, B. Watson | The Unification of Electromagnetic and Weak Forces | #3, Dec. 2 |
| 4 | J. Bolton, J. Day, B. Nuar, | Discovery of Electron | #6, Dec. 4 |
| 5 | J. Bowerman, C. McNutt, M. Obiang, E. Perez | The property of molecules - the Brownian Motions | #4, Dec. 2 |
| 6 | N. Boseman, V. Hopkins, S. Moorman, S. Moriaty | Black-body Radiation | #1, Dec. 4 |
| 7 | E. Bainglass, J. Chavez, K. Izuagbe, | Rutherford Scattering | #3, Dec. 4 |
| 8 | E. Blomberg, E. Duran, J. Grandinatti, R. Loew | The Discovery of Radioactivity | #1, Dec. 2 |
| 9 | P. Conlin, J. Guevara, F. Islam, A. Nelson | Special Relativity | #5, Dec. 2 |
| 10 | G. Brown, G. Collier, B. Ferguson, R. Subramaniam | Compton Effect | #2, Dec. 4 |
| 11 | K. Brackney, C. Dunn, S. Schroeder, S. Sheladia | Super-Conductivity | #4, Dec. 4 |
| 12 | A. Farrar, C. Jay, C. Smith, J. Umphress | The discovery of the Higgs particle | #5, Dec. 4 |

# Research Project Report

1. Must contain the following at the minimum
   - Original theory or Original observation
   - Experimental proofs or Theoretical prediction + subsequent experimental proofs
   - Importance and the impact of the theory/experiment
   - Conclusions
2. Each member of the group writes a 10 (max) page report, including figures
   - 10% of the total grade
   - Can share the theme and facts but you must write your own!
   - Text of the report must NOT be a copy
   - ***<u>Due Mon., Dec. 2, 2013</u>***

# Research Presentations

- Each of the 10 research groups makes a 10min presentation
  - 8min presentation + 2min Q&A
  - All presentations must be in power point
  - I must receive all final presentation files by 8pm, Sunday, Dec. 1
    - No changes are allowed afterward
  - The representative of the group makes the presentation followed by all group members' participation in the Q&A session
- Date and time:
  - In class Monday, Dec. 2 or in class Wednesday, Dec. 4
- Important metrics
  - Contents of the presentation: 60%
    - Inclusion of all important points as mentioned in the report
    - The quality of the research and making the right points
  - Quality of the presentation itself: 15%
  - Presentation manner: 10%
  - Q&A handling: 10%
  - Staying in the allotted presentation time: 5%
  - Judging participation and sincerity: 5%

# Solution of the Angular and Azimuthal Equations

- The radial wave function *R* and the spherical harmonics *Y* determine the probability density for the various quantum states.
- Thus the total wave function $\psi(r,\theta,\phi)$ depends on *n*, $\ell$, and $m_\ell$. The wave function can be written as

$$\psi_{nlm_l}(r,\theta,\phi) = R_{nl}(r)Y_{lm_l}(\theta,\phi)$$

# Orbital Angular Momentum Quantum Number $\ell$

- It is associated with the $R(r)$ and $f(\theta)$ parts of the wave function.
- Classically, the orbital angular momentum $\vec{L} = \vec{r} \times \vec{p}$ with $L = mv_{orbital}r$.
- $\ell$ is related to the magnitude of $L$ by $L = \sqrt{l(l+1)}\hbar$.
- In an $\ell = 0$ state, $L = \sqrt{0(1)}\hbar = 0$.

It disagrees with Bohr's semi-classical "planetary" model of electrons orbiting a nucleus $L = n\hbar$.

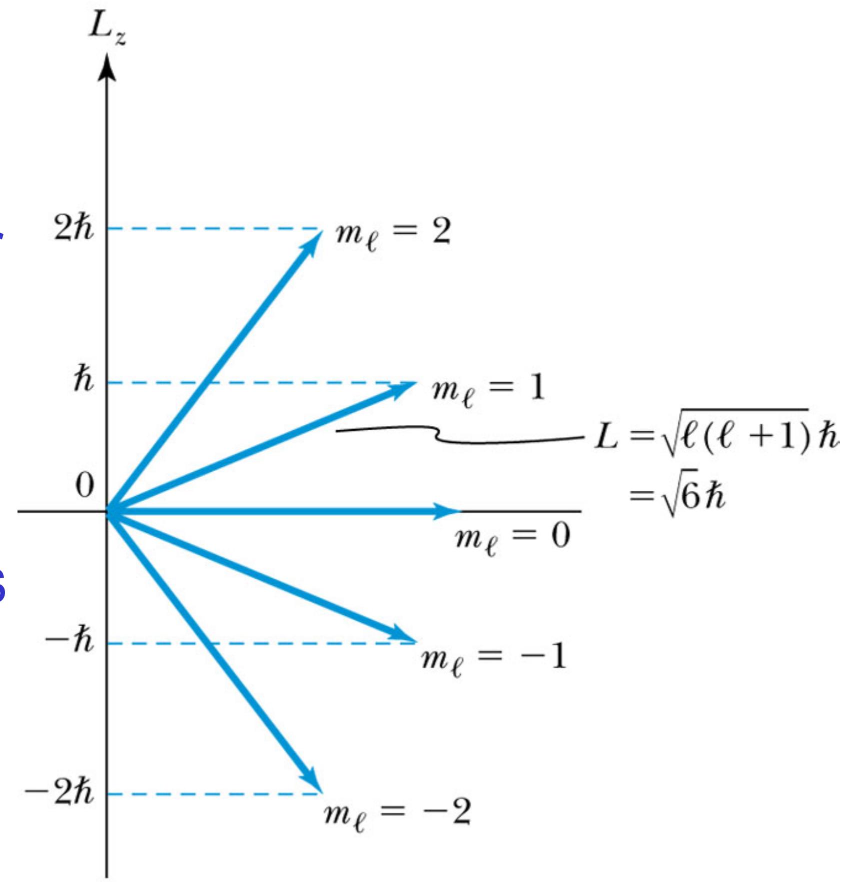
# Orbital Angular Momentum Quantum Number $\ell$

- Certain energy level is **degenerate** with respect to $\ell$ when the energy is independent of $\ell$.
- Use letter names for the various $\ell$ values
  - $\ell$ = 0 1 2 3 4 5 . . .
  - Letter = *s* *p* *d* *f* *g* *h* . . .
- Atomic states are referred by their *n* and $\ell$
  - **s**=**s**harp, **p**=**p**rincipal, **d**=**d**iffuse, **f** =**f**undamental, then alphabetical
- A state with *n* = 2 and $\ell$ = 1 is called the 2*p* state
  - Is 2d state possible?
- The boundary conditions require $n > \ell$

# Magnetic Quantum Number $m_\ell$

- The angle ϕ is a measure of the rotation about the z axis.
- The solution for $g(\phi)$ specifies that $m_\ell$ is an integer and related to the *z* component of *L*.

$$L_z = m_l \hbar$$




- The relationship of *L*, $L_z$, ℓ, and $m_\ell$ for ℓ = 2.
- $L = \sqrt{l(l+1)}\hbar = \sqrt{6}\hbar$ is fixed.
- Because $L_z$ is quantized, only certain orientations of $\vec{L}$ are possible and this is called **space quantization**.
- $m_\ell$ is called the magnetic moment since z axis is chosen customarily along the direction of magnetic field.

# Magnetic Quantum Number $m_\ell$

- Quantum mechanics allows $\vec{L}$ to be quantized along only one direction in space and because of the relationship $L^2 = L_x^2 + L_y^2 + L_z^2$, once a second component is known, the third component will also be known. ➔ violation of uncertainty principle
  - One of the three components, such as $L_z$, can be known clearly but the other components will not be precisely known
- Now, since we know there is no preferred direction,

$$\langle L_x^2 \rangle = \langle L_y^2 \rangle = \langle L_z^2 \rangle$$

- We expect the average of the angular momentum components squared to be: $\langle L^2 \rangle = 3\langle L_z^2 \rangle = \frac{3}{2l+1}\sum_{m_l=-l}^{+l} m_l^2\hbar^2 = l(l+1)\hbar^2$

# Magnetic Effects on Atomic Spectra—The Normal Zeeman Effect

- A Dutch physicist Pieter Zeeman showed as early as 1896 that the spectral lines emitted by atoms in a magnetic field split into multiple energy levels. It is called the **Zeeman effect**.

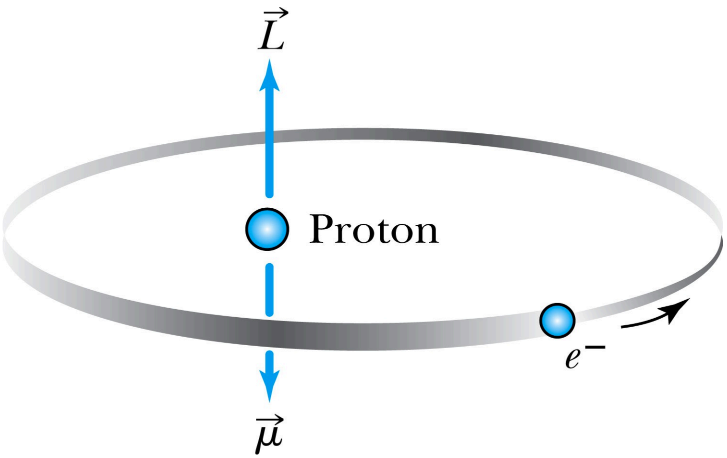
**The Normal Zeeman effect**:

- A spectral line of an atom is split into **three** lines.
- Consider the atom to behave like a small magnet.
- The current loop has a magnetic moment $\mu = IA$ and the period $T = 2\pi r / v$. If an electron can be considered as orbiting a circular current loop of $I = dq / dt$ around the nucleus, we obtain

$$\mu = IA = qA/T = \pi r^2(-e)/(2\pi r/v) = -erv/2 = -\frac{e}{2m}mrv = -\frac{e}{2m}L$$

- $\vec{\mu} = -\frac{e}{2m}\vec{L}$ where $L = mvr$ is the magnitude of the orbital angular momentum

# The Normal Zeeman Effect




- Since there is no magnetic field to align them, $\vec{\mu}$ points in random directions.
- The dipole has a potential energy

$$V_B = -\vec{\mu} \cdot \vec{B}$$

- The angular momentum is aligned with the magnetic moment, and the torque between $\vec{\mu}$ and $\vec{B}$ causes a precession of $\vec{\mu}$.

$$\mu_z = \frac{e}{2m} L_z = \frac{e\hbar}{2m} m_l = -\mu_B m_l$$

Where **$\mu_B = e\hbar / 2m$** is called the **Bohr magneton**.

- $\vec{\mu}$ cannot align exactly in the z direction and has only certain allowed quantized orientations.

$$\vec{\mu} = -\frac{\mu_B \vec{L}}{\hbar}$$

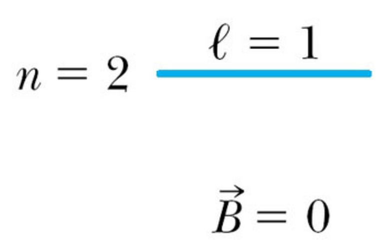
# The Normal Zeeman Effect

- The potential energy is quantized due to the magnetic quantum number $m_\ell$.

$$V_B = -\mu_z B = +\mu_B m_l B$$

- When a magnetic field is applied, the 2*p* level of atomic hydrogen is split into three different energy states with the electron energy difference of $\Delta E = \mu_B B \, \Delta m_\ell$.

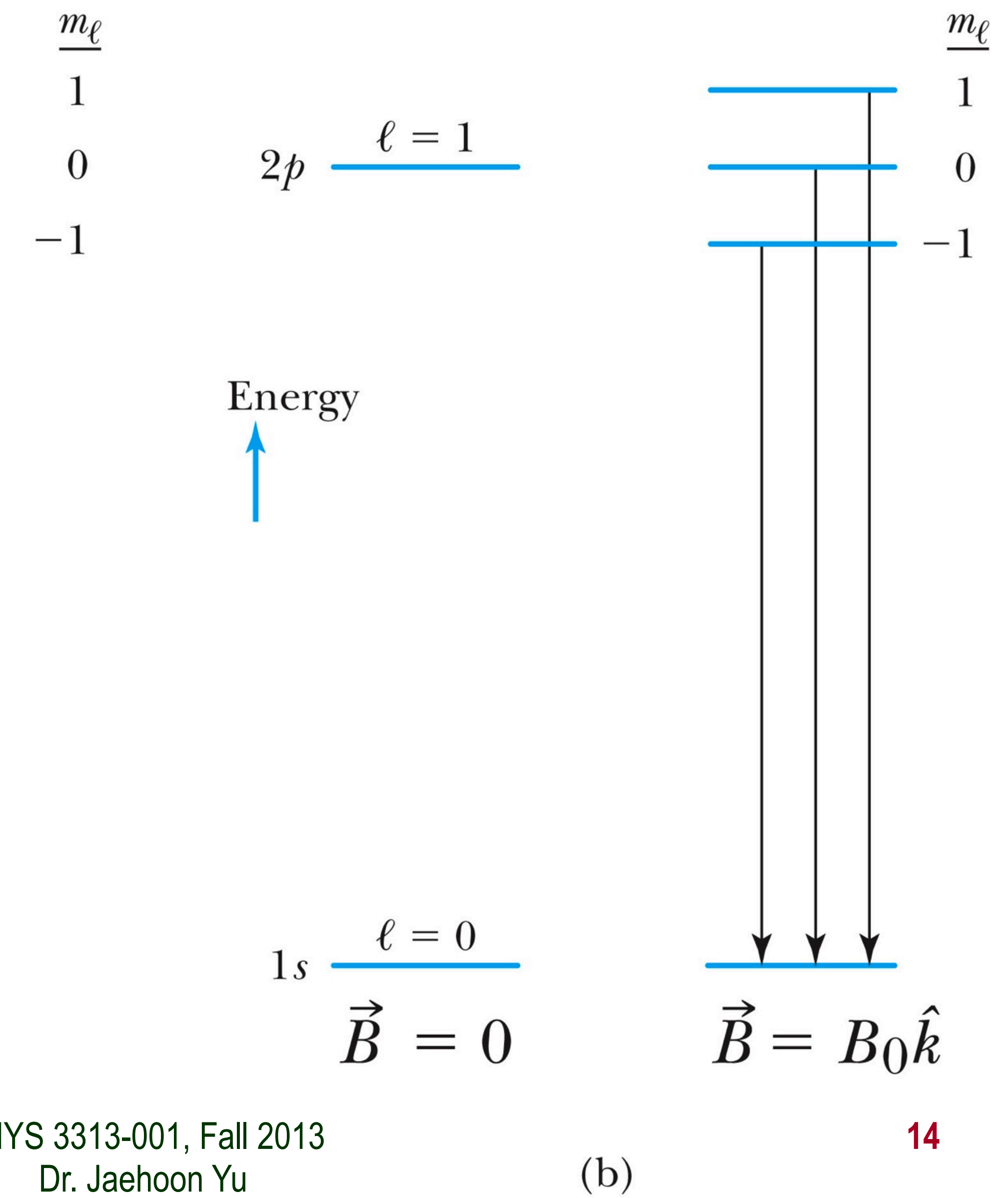
| $m_\ell$ | Energy |
|---|---|
| 1 | $E_0 + \mu_B B$ |
| 0 | $E_0$ |
| −1 | $E_0 - \mu_B B$ |




- So split is into a total of $2\ell+1$ energy states

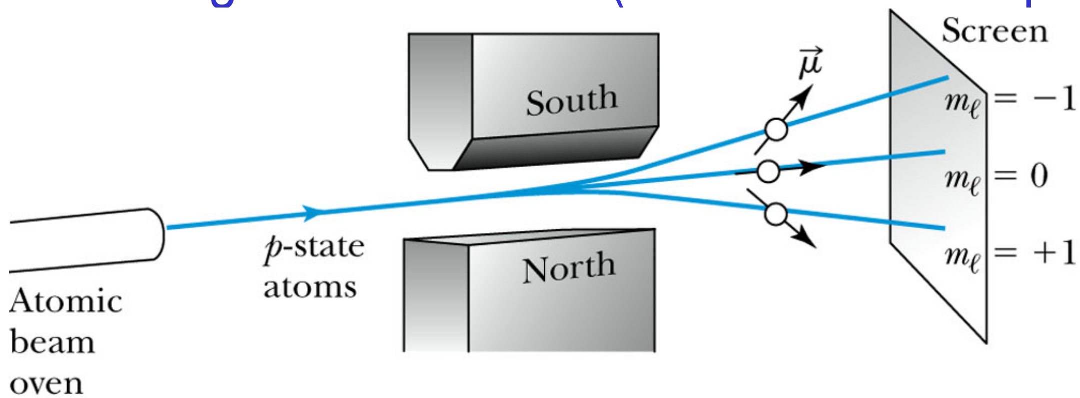
# The Normal Zeeman Effect

- A transition from 1s to 2p
- A transition from 2*p* to 1*s*



# The Normal Zeeman Effect

- An atomic beam of particles in the $\ell$ = 1 state pass through a magnetic field along the *z* direction. (Stern-Gerlach experiment)




- $V_B = -\mu_z B$
- $F_z = -\left(dV_B/dz\right) = \mu_z\left(dB/dz\right)$
- The $m_\ell$ = +1 state will be deflected down, the $m_\ell$ = −1 state up, and the $m_\ell$ = 0 state will be undeflected. ➔ saw only 2 with silver atom
- If the space quantization were due to the magnetic quantum number $m_\ell$, the number of $m_\ell$ states is always odd at $(2\ell + 1)$ and should have produced an odd number of lines.

# Intrinsic Spin

- In 1920, to explain spectral line splitting of Stern-Gerlach experiment, Wolfgang Pauli proposed the forth quantum number assigned to electrons
- In 1925, Samuel Goudsmit and George Uhlenbeck in Holland proposed that *the* ***electron must have an intrinsic angular momentum*** and therefore a magnetic moment.
- Paul Ehrenfest showed that the surface of the spinning electron should be moving faster than the speed of light to obtain the needed angular momentum!!
- In order to explain experimental data, Goudsmit and Uhlenbeck proposed that the electron must have an **intrinsic spin quantum number** $s = 1/2$.

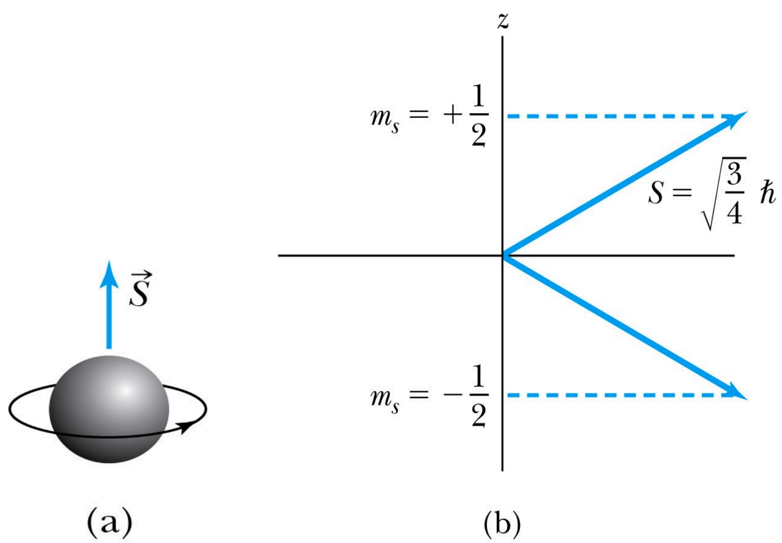
# Intrinsic Spin

- The spinning electron reacts similarly to the orbiting electron in a magnetic field. (Dirac showed that this is necessary due to special relativity..)
- We should try to find $L$, $L_z$, $\ell$, and $m_\ell$.
- The **magnetic spin quantum number** $m_s$ has only two values, $m_s = \pm½$.

The electron's spin will be either "up" or "down" and can never be spinning with its magnetic moment $\mu_s$ exactly along the z axis.

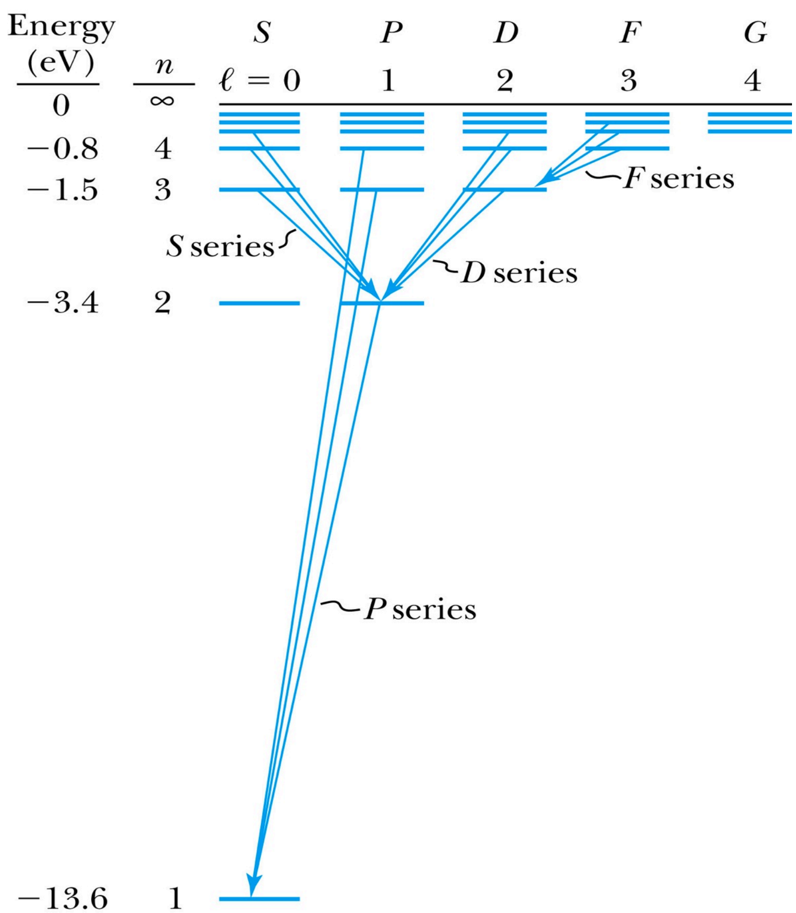
For each state of the other quantum numbers, there are two spins values

The **intrinsic spin angular momentum** vector $\left|\vec{S}\right| = \sqrt{s(s+1)}\hbar = \sqrt{3/4}\hbar$



# Energy Levels and Electron Probabilities

- For hydrogen, the energy level depends on the principle quantum number *n*.




- In ground state an atom cannot emit radiation. It can absorb electromagnetic radiation, or gain energy through inelastic bombardment by particles.

# Selection Rules

- We can use the wave functions to calculate transition probabilities for the electron to change from one state to another.

**Allowed transitions**: Electrons absorbing or emitting photons can change states when $\Delta\ell = \pm 1$. (Evidence for the photon carrying one unit of angular momentum!)

$$\Delta n = \text{anything}$$

$$\Delta\ell = \pm 1$$

$$\Delta m_\ell = 0, \pm 1$$

**Forbidden transitions**: Other transitions possible but occur with much smaller probabilities when $\Delta\ell \neq \pm 1$.

# Probability Distribution Functions

- We must use wave functions to calculate the probability distributions of the electrons.
- The "position" of the electron is spread over space and is not well defined.
- We may use the radial wave function $R(r)$ to calculate radial probability distributions of the electron.
- The probability of finding the electron in a differential volume element $d\tau$ is

$$dP = \psi^*(r,\theta,\phi)\psi(r,\theta,\phi)d\tau$$

# Probability Distribution Functions

- The differential volume element in spherical polar coordinates is

$$d\tau = r^2 \sin\theta dr d\theta d\phi$$

Therefore,

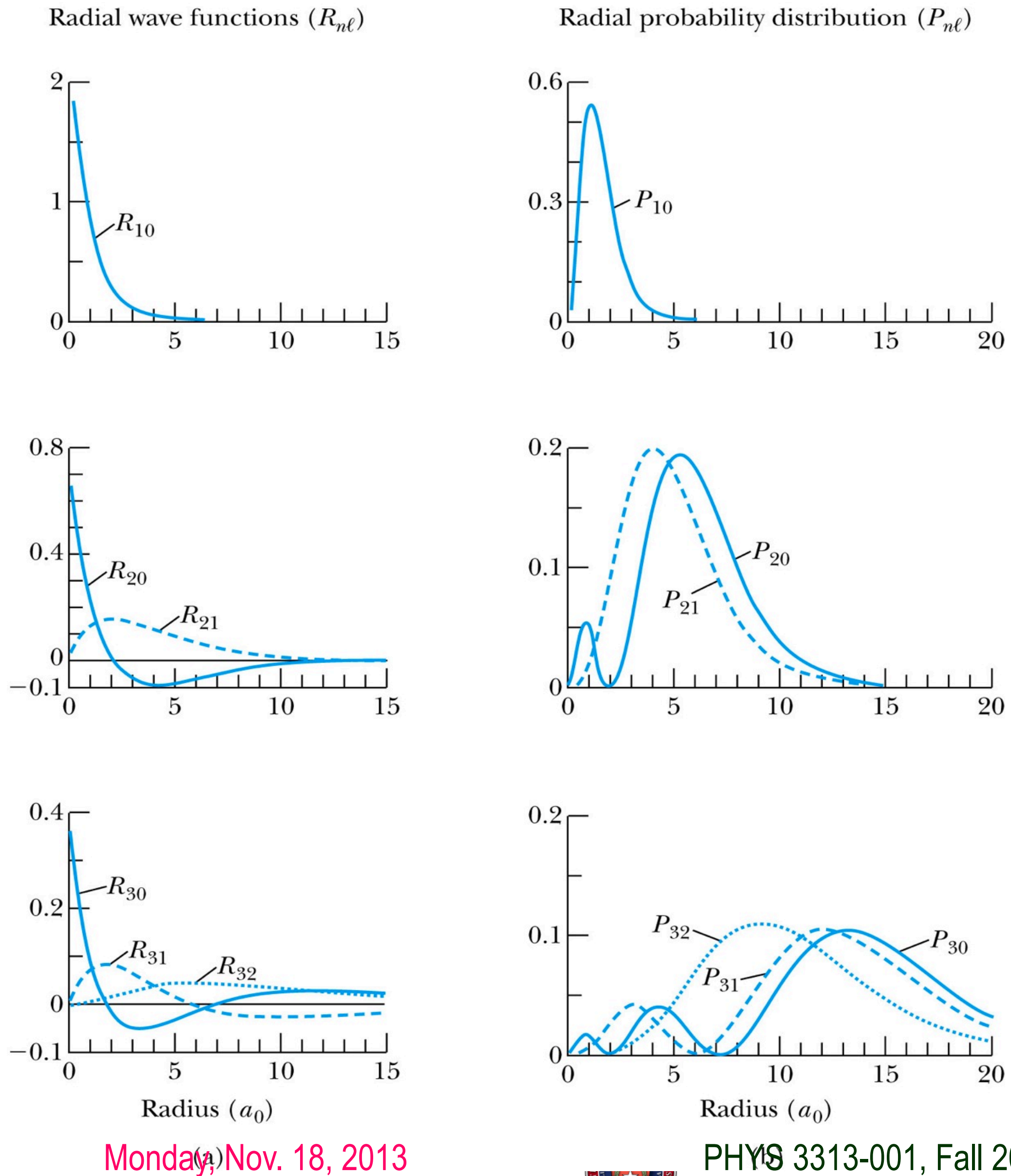
$$P(r)dr = r^2 R^*(r)R(r)dr \int_0^{\pi} |f(\theta)|^2 \sin\theta d\theta \int_0^{2\pi} g(\phi)d\phi$$

- We are only interested in the radial dependence.

$$P(r)dr = r^2 |R(r)|^2 dr$$

- The radial probability density is $P(r) = r^2|R(r)|^2$ and it depends only on *n* and *l*.

# Probability Distribution Functions




- *R*(*r*) and *P*(*r*) for the lowest-lying states of the hydrogen atom

# Probability Distribution Functions

- The probability density for the hydrogen atom for three different electron states