PHYS 3313 – Section 001
Lecture #20

Wednesday, Nov. 20, 2013
Dr. Jaehoon Yu

- Historical Overview
- Maxwell Velocity Distribution
- Equipartition Theorem

Announcements

- Reminder: Homework #7
  - CH7 end of chapter problems: 7, 8, 9, 12, 17 and 29
  - Due coming Monday, Nov. 25, in class
- Reminder: Quiz number 4
  - At the beginning of the class coming Monday, Nov. 25
  - Covers CH7 and what we finish today
- Reminder: Research materials
  - Presentation submission via e-mail to Dr. Yu by 8pm Sunday, Dec. 1
  - Research papers due in class Monday, Dec. 2
- Final exam:
  - Comprehensive exam covering CH1.1 to what we cover Monday, Nov. 25 + appendices 3 – 7
  - BYOF: one handwritten, letter size, front and back
- Reading assignments
  - Entire CH8 (in particular CH8.1), CH9.4 and CH9.7
- Class is cancelled next Wednesday, Nov. 27
- Colloquium today: Dr. B. Franklin of Baylor U.

Physics Department
The University of Texas at Arlington
COLLOQUIUM

Resummation in Quantum Field Theory: QCD for the LHC and Quantum Gravity

Dr. Bennie Franklin Leon Ward

Baylor University

Distinguished Professor of Physics

4:00 pm Wednesday November 20, 2013 in room 101 SH

Abstract:

We present the current status of the application of our approach to exact amplitude-based resummation in quantum field theory to two areas of investigation: the precision QCD calculations as needed for LHC physics and the resummed quantum gravity realization of Feynman's formulation of Einstein's theory of general relativity. We discuss recent results as they relate to experimental observations. There is reason for optimism in the attendant comparisons of theory and experiment.

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Refreshments will be served at 3:30p.m in the Physics lounge

Why is statistical physics necessary?

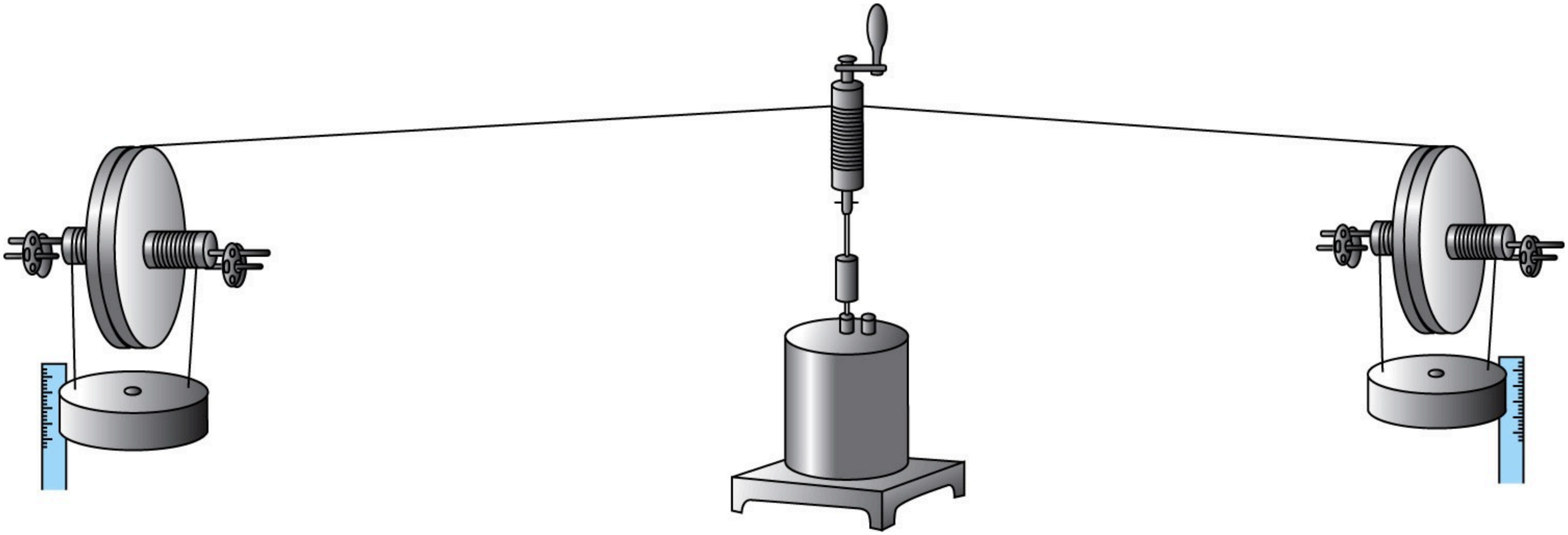
- Does physics perceive inherent uncertainty and indeterminism since everything is probabilistic?
- Statistical physics is necessary since
  - As simple problems as computing probability of coin tosses is complex, so it is useful to reduce it to statistical terms
  - When the number of particles gets large, it is rather impractical to describe the motion of individual particle than describing the motion of a group of particles
  - Uncertainties are inherent as Heisenberg's uncertainty principle showed and are of relatively large scale in atomic and subatomic level
- Statistical physics necessary for atomic physics and the description of solid states which consists of many atoms

Historical Overview

- **Statistics and probability:** New mathematical methods developed to understand the Newtonian physics through 18th and 19th centuries.
- **Lagrange** around 1790 and **Hamilton** around 1840 added significantly to the computational power of Newtonian mechanics.
- **Pierre-Simon de Laplace** (1749-1827)
  - Had a view that it is possible to have a perfect knowledge of the universe
  - Can predict the future and the past to the beginning of the universe
  - He told Napoleon that the hypothesis of God is not necessary
  - He made major contributions to the theory of probability
- **Benjamin Thompson (Count Rumford):** Put forward the idea of heat as merely the motion of individual particles in a substance but not well accepted
- **James Prescott Joule:** Demonstrated experimentally the mechanical equivalence of heat and energy

Joule's experiment

- Showed deterministically the equivalence of heat and energy
- Dropping weights that turns the paddles in the water and measuring the change of water temperature



Historical Overview

- **James Clark Maxwell**
  - Brought the mathematical theories of probability and statistics to bear on the physical thermodynamics problems
  - Showed that distributions of an ideal gas can be used to derive the observed macroscopic phenomena
  - His electromagnetic theory succeeded to the statistical view of thermodynamics
- **Einstein:** Published a theory of Brownian motion, a theory that supported the view that atoms are real
- **Bohr:** Developed atomic and quantum theory

Maxwell's Velocity Distributions

- Laplace claimed that it is possible to know everything about an ideal gas by knowing the position and velocity precisely
- There are six parameters—the position (x, y, z) and the velocity (v_x, v_y, v_z)—per molecule to know the position and instantaneous velocity of an ideal gas.
- These parameters make up 6D **phase space**
- The velocity components of the molecules are more important than positions because the energy of a gas should depend only on the velocities
- Define a **velocity distribution function** = the probability of finding a particle with velocity between $\vec{v}$ and $\vec{v}+d^3\vec{v}$ where $d^3\vec{v}=dv_x dv_y dv_z$

Maxwell's Velocity Distributions

- Maxwell proved that the probability distribution function is proportional to $\exp\left(-\frac{1}{2}mv^2/kT\right)$
  Therefore $f\left(\vec{v}\right)d^3\vec{v} = C\exp\left(-\frac{1}{2}\beta m v^2\right)d^3\vec{v}$.
  where C is a proportionality constant and $\beta \equiv (kT)^{-1}$.
- Since $v^2 = v_x^2 + v_y^2 + v_z^2$,

$$f\left(\vec{v}\right)d^3\vec{v} = C\exp\left[-\frac{1}{2}\beta m\left(v_x^2 + v_y^2 + v_z^2\right)\right]d^3\vec{v}$$

- Rewrite this as the product of three factors (i.e. probability density).

$$g\left(v_x\right)dv_x = C'\exp\left(-\frac{1}{2}\beta m v_x^2\right)dv_x$$

$$g\left(v_y\right)dv_y = C'\exp\left(-\frac{1}{2}\beta m v_y^2\right)dv_y$$

$$g\left(v_z\right)dv_z = C'\exp\left(-\frac{1}{2}\beta m v_z^2\right)dv_z$$

$$f\left(\vec{v}\right)d^3\vec{v} = Cg\left(v_x\right)g\left(v_y\right)g\left(v_z\right)dv_x dv_y dv_z$$

The solution

- Since the probability is 1 when integrated over entire space, we obtain

$$\int_{-\infty}^{+\infty} g(v_x)\,dv_x = C'\left(\frac{2\pi}{\beta m}\right)^{1/2} = 1 \quad \xrightarrow{\text{Solve for C'}} \quad C' = \left(\frac{\beta m}{2\pi}\right)^{1/2}$$

- Thus $g(v_x)\,dv_x = \sqrt{\frac{\beta m}{2\pi}}\exp\left(-\frac{1}{2}\beta m v_x^2\right)dv_x$

- The average velocity in x direction is

$$\bar{v}_x = \int_{-\infty}^{+\infty} v_x g(v_x)\,dv_x = C'\int_{-\infty}^{+\infty} v_x \exp\left(-\frac{1}{2}\beta m v_x^2\right)dv_x = 0$$

- The average of the square of the velocity in x direction is

$$\overline{v_x^2} = C'\int_{-\infty}^{+\infty} v_x^2 \exp\left(-\frac{1}{2}\beta m v_x^2\right)dv_x = 2C'\int_{0}^{+\infty} v_x^2 \exp\left(-\frac{1}{2}\beta m v_x^2\right)dv_x$$

$$= \sqrt{\frac{\beta m}{2\pi}}\frac{\sqrt{\pi}}{2}\left(\frac{2}{\beta m}\right)^{3/2} = \frac{1}{\beta m} = \frac{kT}{m}$$

- Where T is the absolute temperature (temp in C+273), m is the molecular mass and k is the Boltzman constant $k = 1.38\times10^{-23}\,J/K$

Maxwell Velocity Distribution

- The results for the *x*, *y*, and *z* velocity components are identical.
- The mean translational kinetic energy of a molecule:

$$\overline{K} = \overline{\frac{1}{2}mv^2} = \frac{1}{2}m\left(\overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}\right) = \frac{1}{2}m\left(\frac{3kT}{m}\right) = \frac{3}{2}kT$$

- Purely statistical consideration is a good evidence of the validity of this statistical approach to thermodynamics.
- Note no dependence of the formula to the mass!!

Ex 9.1: Molecule Kinetic Energy

Compute the mean translational KE of (a) a single ideal gas molecule in eV and (b) a mol of ideal gas in J at room temperature 20°C.

$$(a)\overline{K}=\frac{3}{2}kT=\frac{3}{2}\left(1.38\times10^{-23}\right)\cdot\left(273+20\right)=6.07\times10^{-21}\left(J\right)=$$

$$=0.038\left(eV\right)\approx\frac{1}{25}\left(eV\right)$$

$$(b)\overline{K}=\left(\frac{3}{2}kT\right)N_A=\left[\frac{3}{2}\left(1.38\times10^{-23}\right)\cdot\left(273+20\right)\right]\cdot6.02\times10^{23}=$$

$$=6.07\times10^{-21}\cdot6.02\times10^{23}\left(J\right)=3650\left(J\right)$$

What is the mean translational KE of 1kg of steam at 1atm at 100°C, assuming an ideal gas? Water molecule is 18g/mol.