PHYS 3313 – Section 001
Lecture #21

Monday, Nov. 25, 2013
Dr. Jaehoon Yu

- Equipartition Theorem
- Classical and Quantum Statistics
- Fermi-Dirac Statistics
- Liquid Helium

Announcements

- Reminder: Research materials
  - Presentation submission via e-mail to Dr. Yu by 8pm this Sunday, Dec. 1
  - Research papers due in class Monday, Dec. 2
- Final exam:
  - Date and time: 11am – 1:30pm, Monday, Dec. 9, in SH125
  - Comprehensive exam covering CH1.1 to CH9.7 + appendices 3 – 7
  - BYOF: one handwritten, letter size, front and back
    - No derivations or solutions of any problems allowed!
- Reading assignments
  - CH9.6 and CH9.7
- Class is cancelled this Wednesday, Nov. 27
- Colloquium today at 4pm in SH101

Reminder: Research Project Report

1. Must contain the following at the minimum
   – Original theory or Original observation
   – Experimental proofs or Theoretical prediction + subsequent experimental proofs
   – Importance and the impact of the theory/experiment
   – Conclusions
2. Each member of the group writes a 10 (max) page report, including figures
   – 10% of the total grade
   – Can share the theme and facts but you must write your own!
   – Text of the report must be your original!
   – ***<u>Due Mon., Dec. 2, 2013</u>***

Group Number	Reseasrch Group Members	Research Topic	Presentation Date and Order
1	Z.Citty, S. Lagerson, K. McElvain, J. Vellarreal	Michelson-Morley Experiment	#6, Dec. 2
2	W. Brown, C. Hair, R. Reyes, H. Zapata	The Photo-Electric Effect	#2, Dec. 2
3	R. Clark, M. Kruse, C. Nguyen, B. Watson	The Unification of Electromagnetic and Weak Forces	#3, Dec. 2
4	J. Bolton, J. Day, B. Nuar,	Discovery of Electron	#6, Dec. 4
5	J. Bowerman, C. McNutt, M. Obiang, E. Perez	The property of molecules - the Brownian Motions	#4, Dec. 2
6	N. Boseman, V. Hopkins, S. Moorman, S. Moriaty	Black-body Radiation	#1, Dec. 4
7	E. Bainglass, J. Chavez, K. Izuagbe,	Rutherford Scattering	#3, Dec. 4
8	E. Blomberg, E. Duran, J. Grandinatti, R. Loew	The Discovery of Radioactivity	#1, Dec. 2
9	P. Conlin, J. Guevara, F. Islam, A. Nelson	Special Relativity	#5, Dec. 2
10	G. Brown, G. Collier, B. Ferguson, R. Subramaniam	Compton Effect	#2, Dec. 4
11	K. Brackney, C. Dunn, S. Schroeder, S. Sheladia	Super-Conductivity	#4, Dec. 4
12	A. Farrar, C. Jay, C. Smith, J. Umphress	The discovery of the Higgs particle	#5, Dec. 4

Research Presentations

- Each of the 10 research groups makes a 10min presentation
  - 8min presentation + 2min Q&A
  - All presentations must be in power point
  - I must receive all final presentation files by 8pm, Sunday, Dec. 1
    - No changes are allowed afterward
  - The representative of the group makes the presentation followed by all group members' participation in the Q&A session
- Date and time:
  - In class Monday, Dec. 2 or in class Wednesday, Dec. 4
- Important metrics
  - Contents of the presentation: 60%
    - Inclusion of all important points as mentioned in the report
    - The quality of the research and making the right points
  - Quality of the presentation itself: 15%
  - Presentation manner: 10%
  - Q&A handling: 10%
  - Staying in the allotted presentation time: 5%
  - Judging participation and sincerity: 5%

Equipartition Theorem

- The formula for average kinetic energy 3kT/2 works for monoatomic molecule what is it for diatomic molecule?
- Consider oxygen molecule as two oxygen atoms connected by a massless rod ➔ This will have both translational and rotational energy
- How much rotational energy is there and how is it related to temperature?
- **Equipartition Theorem**:
  - *In equilibrium a mean energy of ½ kT per molecule is associated with each independent quadratic term in the molecule's energy.*
  - Each independent phase space coordinate: ***degree of freedom***
  - Essentially the mean energy of a molecule is *½ kT *NDoF*

Equipartition Theorem

- In a monoatomic ideal gas, each molecule has

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right)$$

- There are three degrees of freedom.
- Mean kinetic energy is $3\left(\tfrac{1}{2}kT\right) = \tfrac{3}{2}kT$
- In a gas of *N* helium molecules, the total internal energy is

$$U = N\bar{E} = \tfrac{3}{2}NkT$$

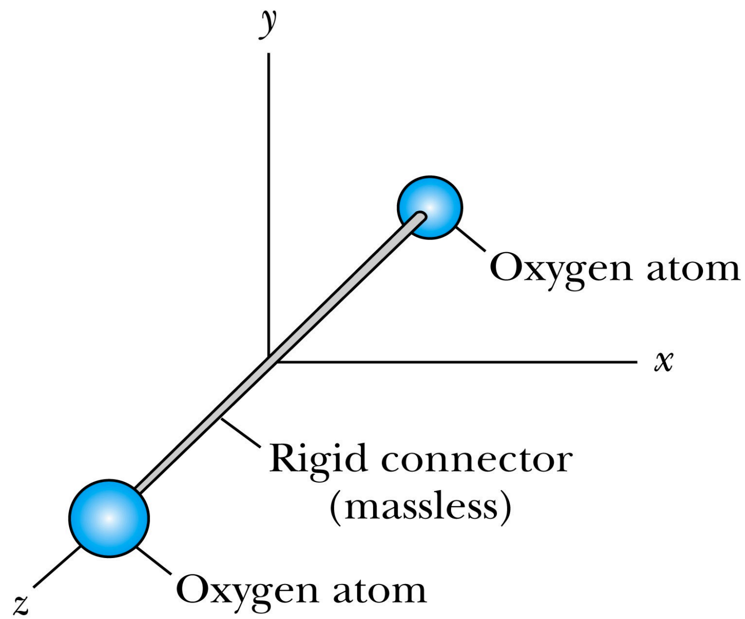
- The heat capacity at constant volume is $C_V = \dfrac{\partial U}{\partial T} = \tfrac{3}{2}Nk$
- For the heat capacity for 1 mole,

$$c_V = \tfrac{3}{2}N_A k = \tfrac{3}{2}R = 12.5\,\mathrm{J/K}$$

- using the ideal gas constant R = 8.31 J/K.

The Rigid Rotator Model

- For diatomic gases, consider the rigid rotator model.



- The molecule has rotational E only when it rotates about *x* or *y* axis.
- The corresponding rotational energies are $\frac{1}{2}I_x\omega_x^2$ and $\frac{1}{2}I_y\omega_y^2$
- There are five degrees of freedom (three translational and two rotational) ➔ resulting in mean energy of 5kT/2 per molecule according to equi-partition principle (C_V=5R/2)

Table of Measured Gas Heat Capacities

Table 9.1 Molar Heat Capacities for Selected Gases at 15°C and 1 Atmosphere

Gas	c_V (J/K)	c_V/R
Ar	12.5	1.50
He	12.5	1.50
CO	20.7	2.49
H_2	20.4	2.45
HCl	21.4	2.57
N_2	20.6	2.49
NO	20.9	2.51
O_2	21.1	2.54
Cl_2	24.8	2.98
CO_2	28.2	3.40
CS_2	40.9	4.92
H_2S	25.4	3.06
N_2O	28.5	3.42
SO_2	31.3	3.76

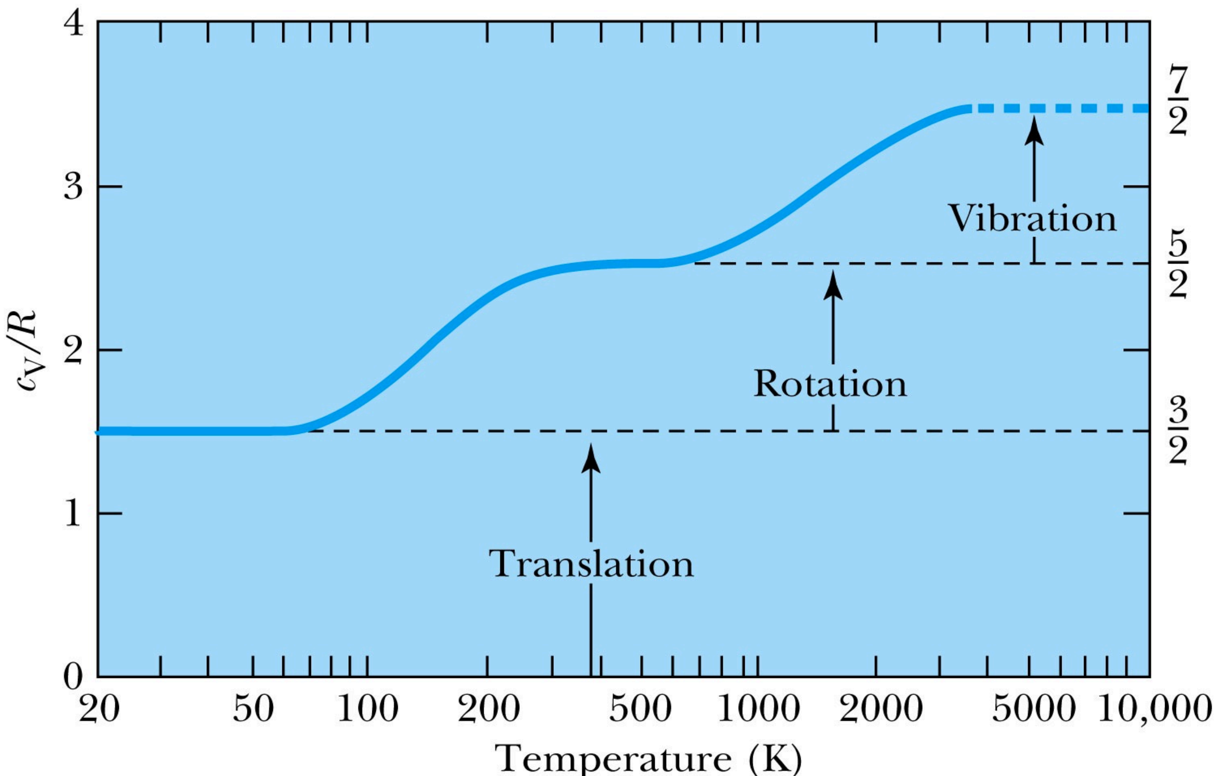
Equipartition Theorem

- Most the mass of an atom is confined to a nucleus whose magnitude is smaller than the whole atom.
  - I_z is smaller than I_x and I_y.
  - Only rotations about *x* and *y* contributes to the energy
- In some circumstances it is better to think of atoms connected to each other by a massless spring.
- The vibrational kinetic energy is $\frac{1}{2}m\left(dr/dt\right)^2$
- There are seven degrees of freedom (three translational, two rotational, and two vibrational). ➔ 7kT/2 per molecule
- While it works pretty well, the simple assumptions made for equi-partition principle, such as massless connecting rod, is not quite sufficient for detailed molecular behaviors

Molar Heat Capacity

- The heat capacities of diatomic gases are also temperature dependent, indicating that the different degrees of freedom are "turned on" at different temperatures.

Example of H_2



Classical and Quantum Statistics

- In gas, particles are so far apart, they do not interact substantially & are free→ even if they collide, they can be considered as elastic and do not affect the mean values
- If molecules, atoms, or subatomic particles are in the liquid or solid state, the **Pauli exclusion principle*** prevents two particles with identical quantum states from sharing the same space → limits available energy states in quantum systems
  - Recall there is no restriction on particle energies in classical physics.
- This affects the overall distribution of energies

*Pauli Exclusion Principle: No two electrons in an atom may have the same set of quantum numbers (n, l, m_l, m_s).

Classical Distributions

- Rewrite Maxwell speed distribution in terms of energy.

$$F(v)dv = 4\pi C \exp\left(-\beta m v^2/2\right) v^2 dv = F(E)dE$$

  - Probability for finding a particle between speed v and v+dv
- For a monoatomic gas the energy is all translational kinetic energy.

$$E = \tfrac{1}{2} m v^2$$

$$dE = mv\, dv$$

$$dv = \frac{dE}{mv} = \frac{dE}{m\sqrt{2E/m}} = \frac{dE}{\sqrt{2mE}}$$

- where

$$F(E) = \frac{8\pi C}{\sqrt{2}m^{3/2}} \exp(-\beta E) E^{1/2}$$

Classical Distributions

- Boltzmann showed that the statistical factor exp($-\beta E$) is a characteristic of any classical system.
  - regardless of how quantities other than molecular speeds may affect the energy of a given state
- **Maxwell-Boltzmann factor** for classical system:

$$F_{MB} = A\exp(-\beta E)$$

- The energy distribution for classical system:

$$n(E) = g(E)F_{MB}$$

- $n(E)\,dE$: the number of particles with energies between E and $E + dE$
- $g(E)$, the **density of states**, is the number of states available per unit energy
- F_{MB}: the relative probability that an energy state is occupied at a given temperature

Quantum Distributions

- Identical particles cannot be distinguished if their wave functions overlap significantly
  - Characteristic of indistinguishability is what makes quantum statistics different from classical statistics.
- Consider two distinguishable particles in two different energy states with the same probability (0.5 each)
- The possible configurations are

E1	**E2**
A, B	
A	B
B	A
	A, B

- Since the four states are equally likely, the probability of each state is one-fourth (0.25).

Quantum Distributions

- If the two particles are indistinguishable:

State 1	State 2
XX	
X	X
	XX

- There are only three possible configurations
- Thus the probability of each is one-third (~0.33).
- Because some particles do not obey the Pauli exclusion principle, two kinds of quantum distributions are needed.
- **Fermions**: Particles with half-spins (1/2) that **obey** the Pauli principle.
  - Examples? Electron, proton, neutron, any atoms or molecules with odd number of fermions
- **Bosons**: Particles with zero or integer spins that do **NOT obey** the Pauli principle.
  - Examples? Photon, force mediators, pions, any atoms or molecules with even number of fermions

Quantum Distributions

- **Fermi-Dirac** distribution: $n(E) = g(E)F_{FD}$

  where $$F_{FD} = \frac{1}{B_{FD}\exp(\beta E)+1}$$

- **Bose-Einstein** distribution: $n(E) = g(E)F_{BE}$

  where $$F_{BE} = \frac{1}{B_{BE}\exp(\beta E)-1}$$

- B_i (i = FD or BE) is the normalization factor.
- Both distributions reduce to the classical Maxwell-Boltzmann distribution when B_i exp(βE) is much greater than 1.
  - the Maxwell-Boltzmann factor A exp($-\beta E$) is much less than 1.
  - In other words, the probability that a particular energy state will be occupied is much less than 1!

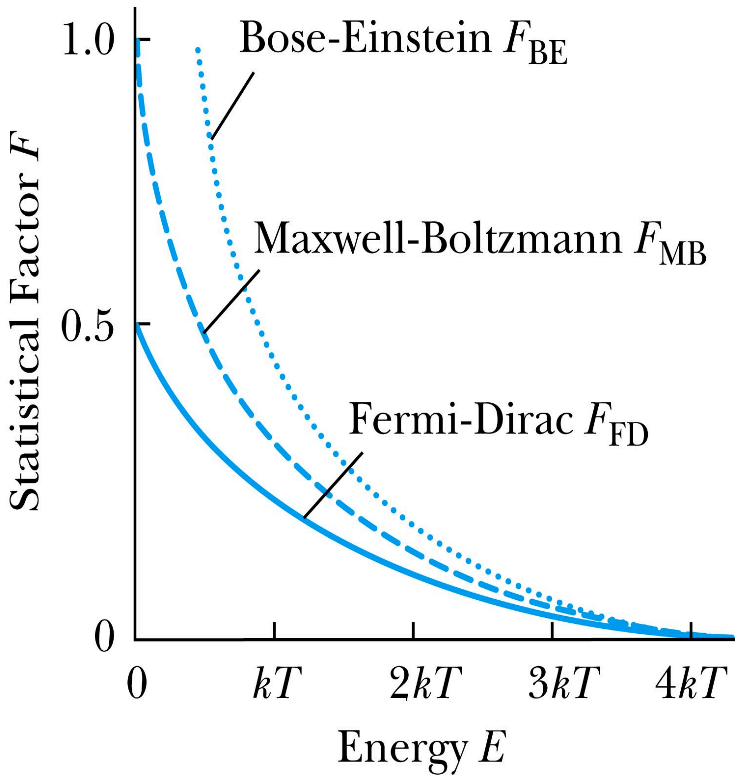
Summary of Classical and Quantum Distributions

Table 9.2 Classical and Quantum Distributions

Distributors	Properties of the Distribution	Examples	Distribution Function
Maxwell-Boltzmann	Particles are identical but distinguishable	Ideal gases	$F_{\mathrm{MB}} = A\exp(-\beta E)$
Bose-Einstein	Particles are identical and indistinguishable with integer spin	Liquid ^{4}He, photons	$F_{\mathrm{BE}} = \dfrac{1}{B_{\mathrm{BE}}\exp(\beta E) - 1}$
Fermi-Dirac	Particles are identical and indistinguishable with half-integer spin	Electron gas (free electrons in a conductor)	$F_{\mathrm{FD}} = \dfrac{1}{B_{\mathrm{FD}}\exp(\beta E) + 1}$

Quantum Distributions

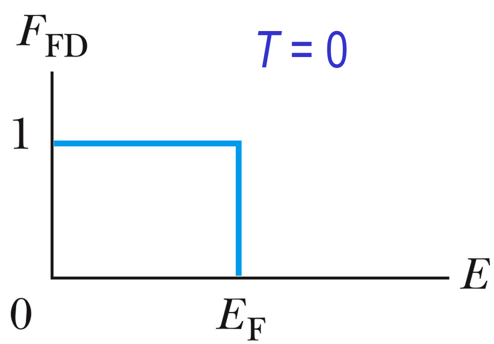
- The normalization constants for the distributions depend on the physical system being considered.
- Because bosons **do not obey the Pauli exclusion principle**, more bosons can fill lower energy states.
- Three graphs coincide at high energies – the classical limit.
- Maxwell-Boltzmann statistics may be used in the classical limit.

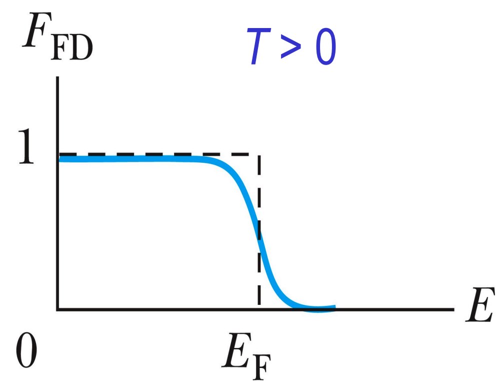


Fermi-Dirac Statistics

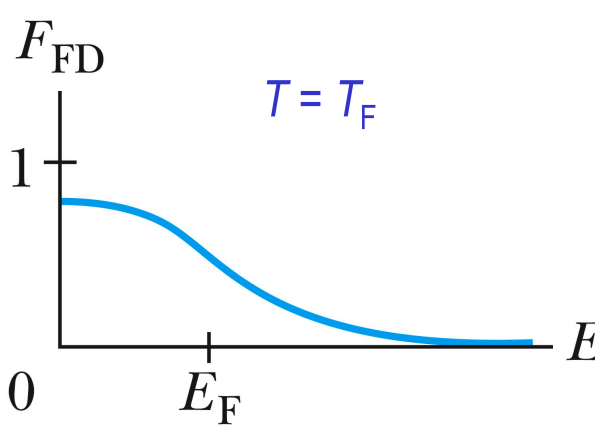
- This is most useful for electrical conduction
- The normalization factor B_{FD} $B_{FD} = \exp(-\beta E_F)$
  - *Where* E_F is called the **Fermi energy**.
- The Fermi-Dirac Factor becomes $F_{FD} = \dfrac{1}{\exp\left[\beta(E - E_F)\right] + 1}$
- When $E = E_F$, the exponential term is 1. ➔ $F_{FD} = 1/2$
- In the limit as T → 0, $F_{FD} = \begin{cases} 1 \text{ for } E < E_F \\ 0 \text{ for } E > E_F \end{cases}$
- At $T = 0$, fermions occupy the lowest energy levels available to them
  - Since they cannot all fill the same energy due to Pauli Exclusion principle, they will fill the energy states up to Fermi Energy
- Near $T = 0$, there is little a chance that the thermal agitation will kick a fermion to an energy greater than E_F.

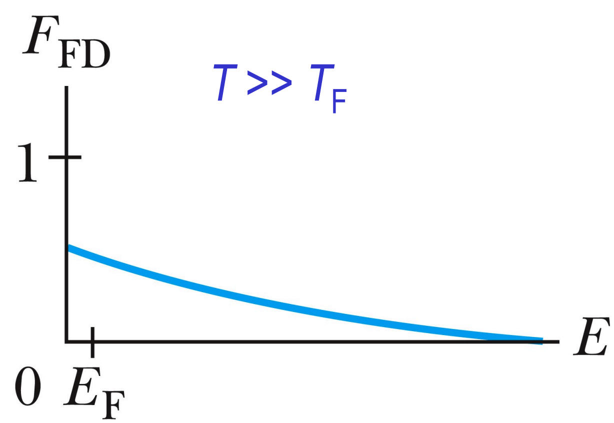
Fermi-Dirac Statistics





- As the temperature increases from $T = 0$, the Fermi-Dirac factor "smears out", and more fermions jump to higher energy level above Fermi energy
- We can define **Fermi temperature**, defined as $T_F \equiv E_F / k$

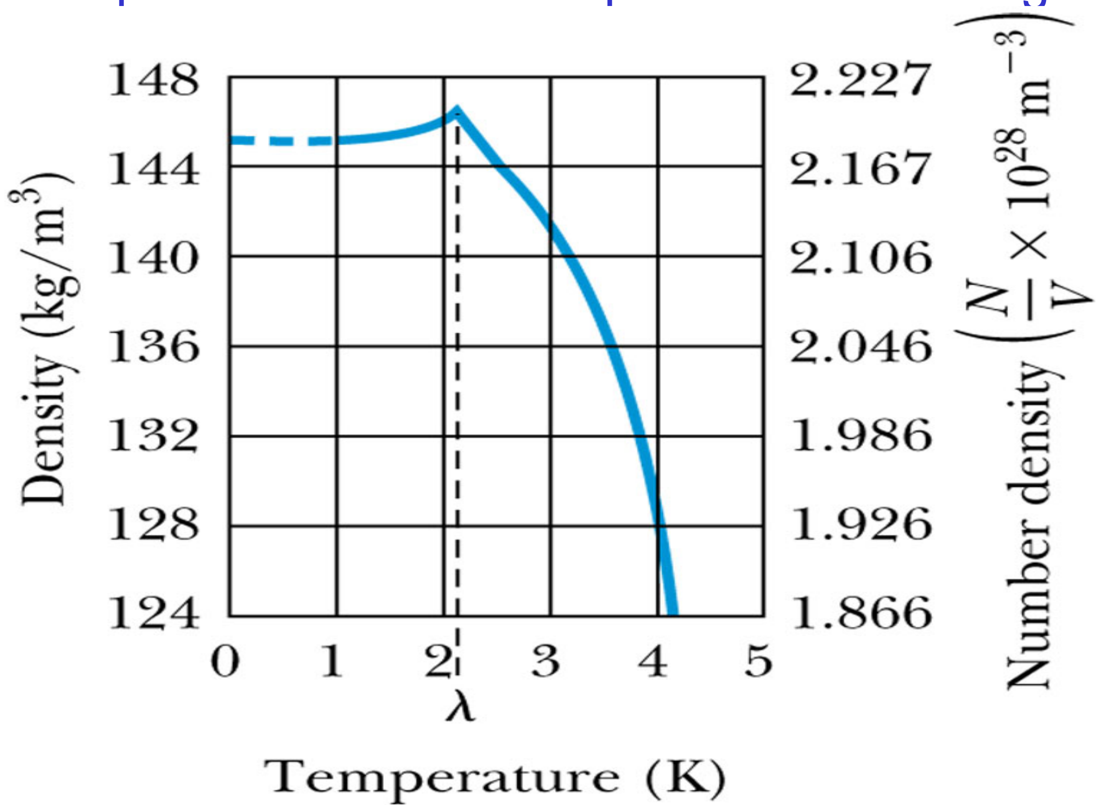




- When $T >> T_F$, F_{FD} approaches a simple decaying exponential

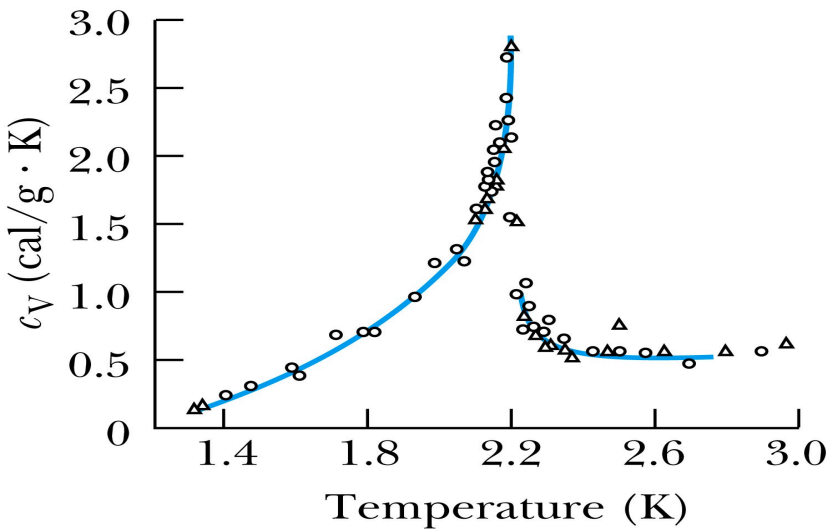
Liquid Helium

- Has the lowest boiling point of any element (4.2 K at 1 atmosphere pressure) and has no solid phase at normal pressure
- Helium is so light and has high speed and so escapes outside of the Earth atmosphere ➔ Must be captured from underground



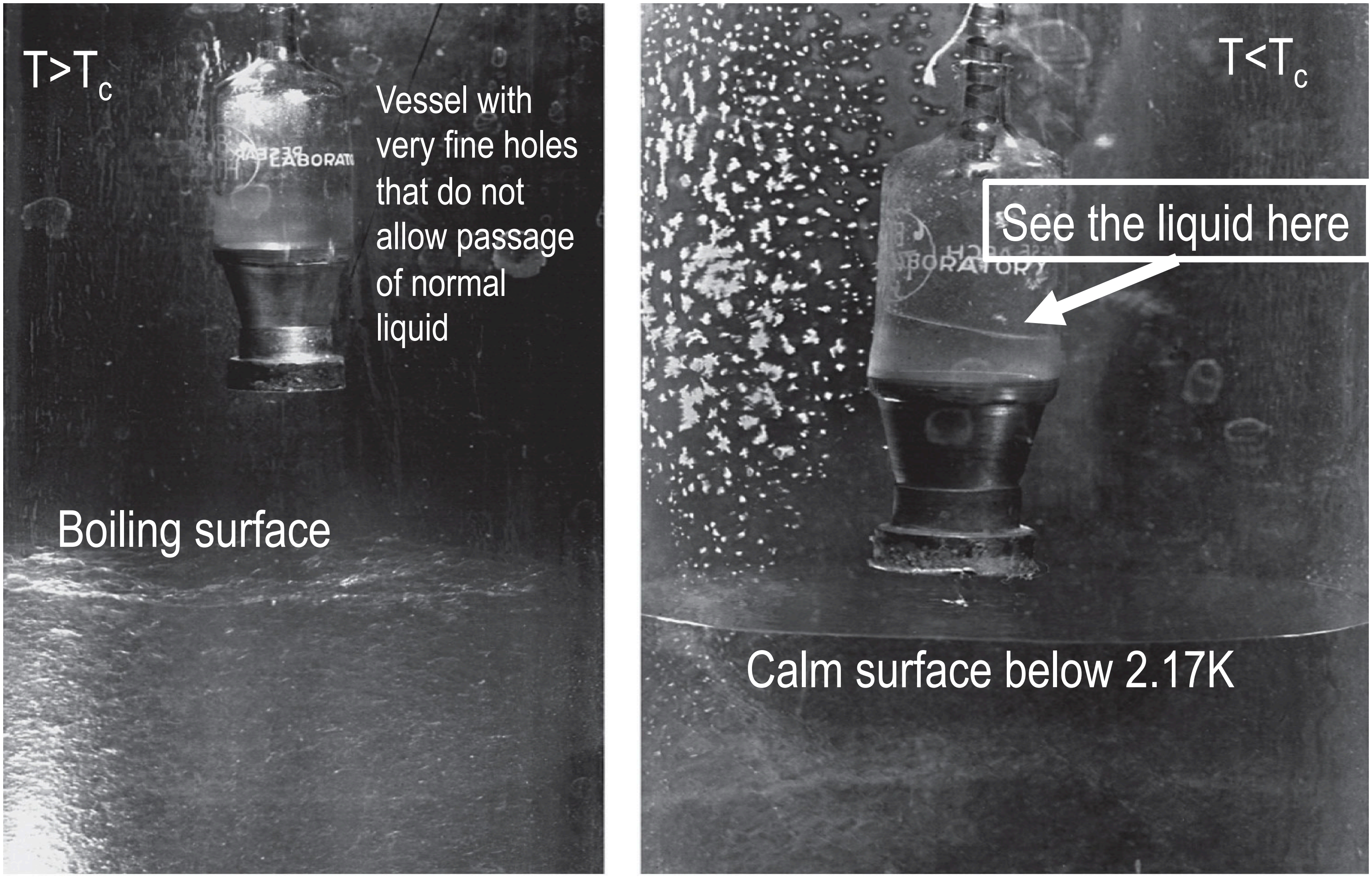
Liquid Helium

The specific heat of liquid helium as a function of temperature



- The temperature at about 2.17 K is referred to as the ***critical temperature (T_c), transition temperature*, or the *lambda point*.**
- As the temperature is reduced from 4.2 K toward the lambda point, the liquid boils vigorously. At 2.17 K the boiling suddenly stops.
- What happens at 2.17 K is a transition from the **normal phase** to the **superfluid phase**.

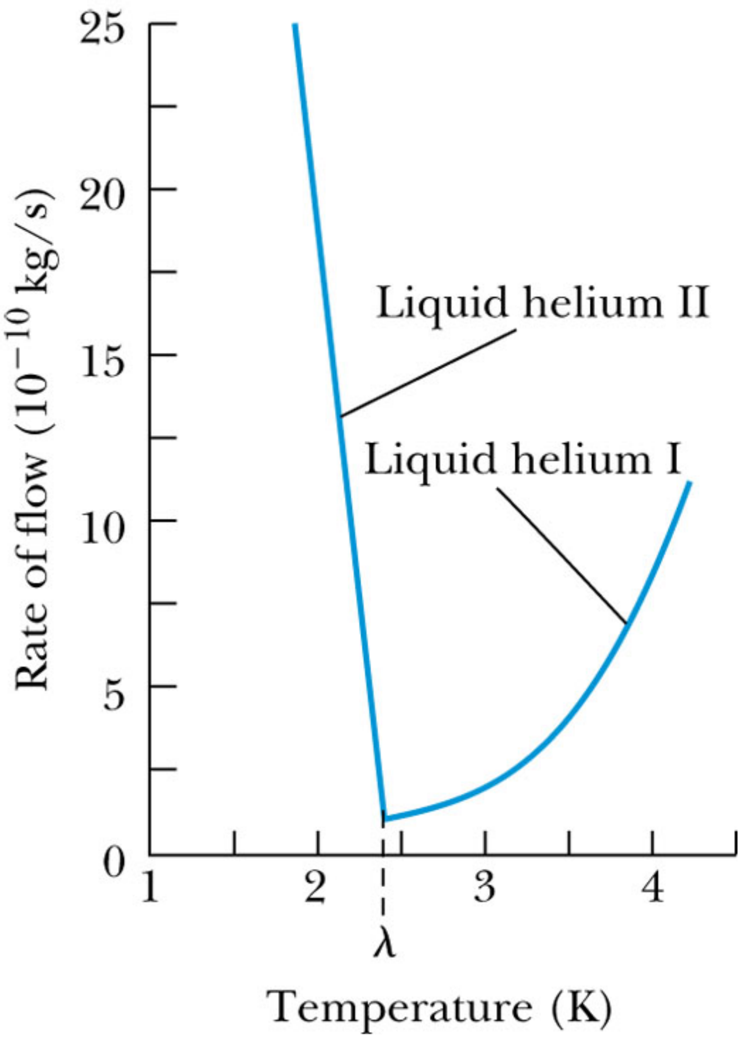
He Transition to Superfluid State



(a) (b)

Liquid Helium

- The rate of flow increases dramatically as the temperature is reduced because the superfluid has a low viscosity.
- **Creeping film** – formed when the viscosity is very low
- But when the viscosity is measured through the drag on a metal surface, He behaves like a normal fluid
  ➔ Contradiction!!



Liquid Helium

- Fritz London claimed (1938) that liquid helium below the lambda point is a mixture of superfluid and normal fluid.
  - As the temperature approaches absolute zero, the superfluid approaches 100% superfluid.
- The fraction of helium atoms in the superfluid state:

$$F = 1 - \left(\frac{T}{T_c} \right)^{3/2}$$

- Superfluid liquid helium (^{4}He) is referred to as a **Bose-Einstein condensation**.
  - ^{4}He is a boson thus it is not subject to the Pauli exclusion principle
  - all particles are in the same quantum state

Bose-Einstein Condensation in Gases

- BE condensation in liquid has been accomplished but gas condensation state hadn't been until 1995
- The strong Coulomb interactions among gas particles made it difficult to obtain the low temperatures and high densities needed to produce the BE condensate.
- Finally success was achieved by E. Cornell and C. Weiman in Boulder, CO, with Rb (at 20nK) and W. Kettle at MIT on Sodium (at 20μK) ➔ Awarded of Nobel prize in 2001
- The procedure
  - Laser cool their gas of ^{87}Rb atoms to about 1 mK.
  - Used a magnetic trap to cool the gas to about 20 nK, driving away atoms with higher speeds and keeping only the low speed ones
  - At about 170 nK, Rb gas went through a transition, resulting in very cold and dense state of gas
- Possible application of BEC is an atomic laser but it will take long time..