PHYS 1443 – Section 004
Lecture #3

Tuesday, Sept. 2, 2014

Dr. Jaehoon Yu

- Estimates
- Dimensional Analysis
- Fundamentals of kinematics
- Concepts of kinematic quantities
  - Displacement
  - Average Velocity and Speed
  - Instantaneous velocity and speed
  - Acceleration

Announcements

- Quiz results
  - Class average: 41.6/58
    - Equivalent to 71.7/100
  - Top score: 57/58
- Some Homework tips
  - When inputting answers to the Quest homework system
    - Unless the problem explicitly asks for significant figures, input as many digits as you can
    - The Quest is dumb. So it does not know about anything other than numbers
  - More details are http://web4.cns.utexas.edu/quest/support/student/#Num
- Physics department colloquium
  - 4pm every Wednesday in SH101
  - Tomorrow is the faculty research expo

Physics Department

The University of Texas at Arlington

COLLOQUIUM

Physics Faculty Research Expo #1

Wednesday September 3, 2014
4:00 p.m. Rm. 101SH

SPEAKERS:

Dr. Wei Chen
"Nanomaterials for Cancer Treatment and Sensing Technology"

Dr. Kaushik De
"The Search Continues From the God Particle to Dark Matter"

Dr. Ping Liu
"Nanoscale Magnets"

Dr. Ramon Lopez
"Space Physics and Physics Education"

Dr. Zdzislaw Musielak
"My Research Projects"

Dr. Mingwu Jin
''Medical Imaging; From Structures to Functions

Refreshments will be served at 3:30 p.m. in the Physics Lounge

Reminder: Special Project #1 for Extra Credit

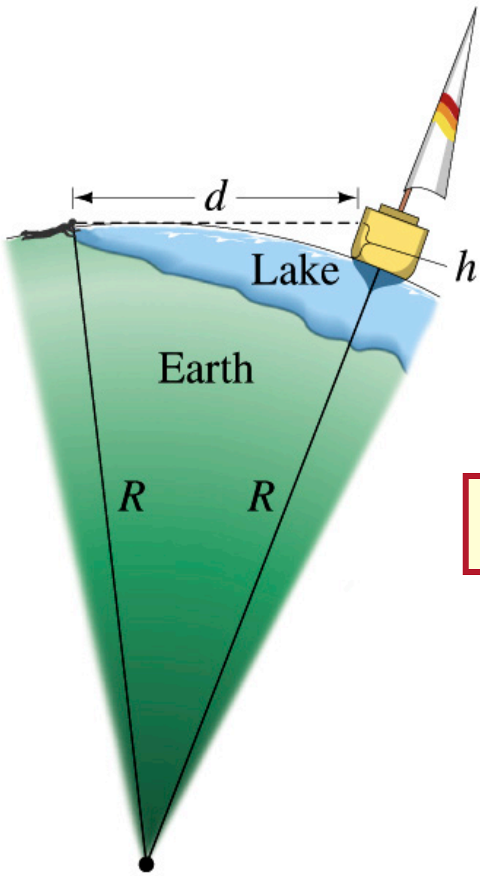
- Find the solutions for $yx^2-zx+v=0$ → 5 points
  - X is the unknown variable, and y, z and v are constant coefficients!
  - You cannot just plug into the quadratic equations!
  - You must show a complete algebraic process of obtaining the solutions!
- Derive the kinematic equation $v^2 = v_0^2 + 2a(x - x_0)$ from first principles and the known kinematic equations → 10 points
- You must **<u>show your OWN work in detail</u>** to obtain the full credit
  - Must be in much more detail than in this lecture note!!!
  - Please do not copy from the lecture note or from your friends. You will all get 0!
- Due Thursday, Sept. 4

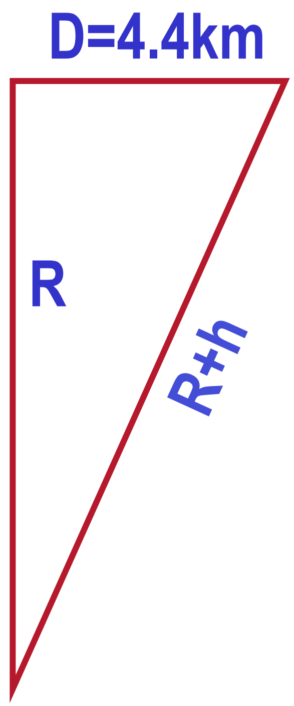
Estimates & Order-of-Magnitude Calculations

- Estimate = Approximation
  - Useful for rough calculations to determine the necessity of higher precision
  - Usually done under certain assumptions
  - Might require modification of assumptions, if higher precision is necessary
- Order of magnitude estimate: Estimates done to the precision of 10s or exponents of 10s;
  - Three orders of magnitude: 10^3=1,000
  - Round up for Order of magnitude estimate; $8x10^7 \sim 10^8$
  - Similar terms: "Ball-park-figures", "guesstimates", etc

Example for Estimates

Estimate the radius of the Earth using triangulation as shown in the picture when d=4.4km and h=1.5m.





Pythagorian theorem

$$(R+h)^2 \approx d^2 + R^2$$

$$R^2 + 2hR + h^2 \approx d^2 + R^2$$

Solving for R

$$R \approx \frac{d^2 - h^2}{2h}$$

$$= \frac{(4400m)^2 - (1.5m)^2}{2 \times 1.5m}$$

$$= 6500km$$

Real R=6380km

Dimensions of Units and Dimensional Analysis

- An extremely useful concept in solving physical problems
- Good to write physical laws in mathematical expressions
- No matter what units are used the base quantities are the same
  - *Length* (distance) is length whether meter or inch is used to express the size: Usually denoted as *[L]*
  - The same is true for *Mass ([M])* and *Time ([T])*
  - One can say "Dimension of Length, Mass or Time"
  - Dimensions are treated as algebraic quantities: Can perform two algebraic operations; multiplication or division

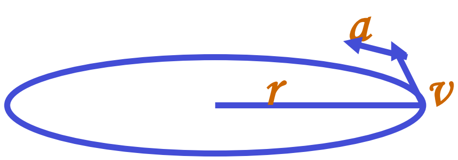
Dimensions of units and Dimensional Analysis cnt'd

- One can use dimensions only to check the validity of one's expression: Dimensional analysis
  - Eg: Speed $[v] = [L]/[T]=[L][T^{-1}]$
    - *Distance (L) traveled by a car running at the speed V in time T*
      - $L = V*T = [L/T]*[T]=[L]$
- More general expression of dimensional analysis is using exponents: eg. $[v]=[L^nT^m] =[L][T^{-1}]$ *where* $n = 1$ *and* $m = -1$

Examples

- Show that the expression $[v] = [at]$ is dimensionally correct
  - Speed: $[v]$ =[L]/[T]
  - Acceleration: $[a]$ =[L]/[T]^2
  - Thus, $[at]$ = (L/T^2)xT=LT$^{(-2+1)}$ =LT^{-1} =[L]/[T]= $[v]$

•Suppose the acceleration a of a circularly moving particle with speed v and radius r is proportional to r^n and v^m. What are n and m?



$$a = kr^n v^m$$

Dimensionless constant

Length

Speed

$$L^1T^{-2} = (L)^n\left(\frac{L}{T}\right)^m = L^{n+m}T^{-m}$$

$$-m = -2 \Rightarrow \boldsymbol{m = 2}$$

$$n + m = n + 2 = 1 \Rightarrow \boldsymbol{n = -1}$$

$$a = kr^{-1}v^2 = \frac{v^2}{r}$$

Some Fundamentals

- Kinematics: Description of Motion without understanding the cause of the motion
- Dynamics: Description of motion accompanied with understanding the cause of the motion
- Vector and Scalar quantities:
  - Scalar: Physical quantities that require magnitude but no direction
    - Speed, length, mass, height, volume, area, magnitude of a vector quantity, etc
  - Vector: Physical quantities that require both magnitude and direction
    - Velocity, Acceleration, Force, Momentum
    - It does not make sense to say “I ran with velocity of 10miles/hour.”
- Objects can be treated as point-like if their sizes are smaller than the scale in the problem
  - Earth can be treated as a point like object (or a particle)in celestial problems
    - Simplification of the problem (The first step in setting up to solve a problem…)
  - Any other examples?

Some More Fundamentals

- Motions: Can be described as long as the position is known at any given time (or position is expressed as a function of time)
  - Translation: Linear motion along a line
  - Rotation: Circular or elliptical motion
  - Vibration: Oscillation
- Dimensions of geometry
  - 0 dimension: A point
  - 1 dimension: Linear drag of a point, resulting in a line ➔ Motion in one-dimension is a motion on a line
  - 2 dimension: Linear drag of a line resulting in a surface
  - 3 dimension: Perpendicular Linear drag of a surface, resulting in a stereo object

Displacement, Velocity and Speed

One dimensional displacement is defined as:

$$\Delta x \equiv x_f - x_i$$

A vector quantity

Displacement is the difference between initial and final poisitions of the motion and is a vector quantity. How is this different than distance?

Unit? **m**

The average velocity is defined as: $$v_{x-avg} \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \equiv \frac{\text{Displacement}}{\text{Elapsed Time}}$$

Unit? **m/s**

A vector quantity

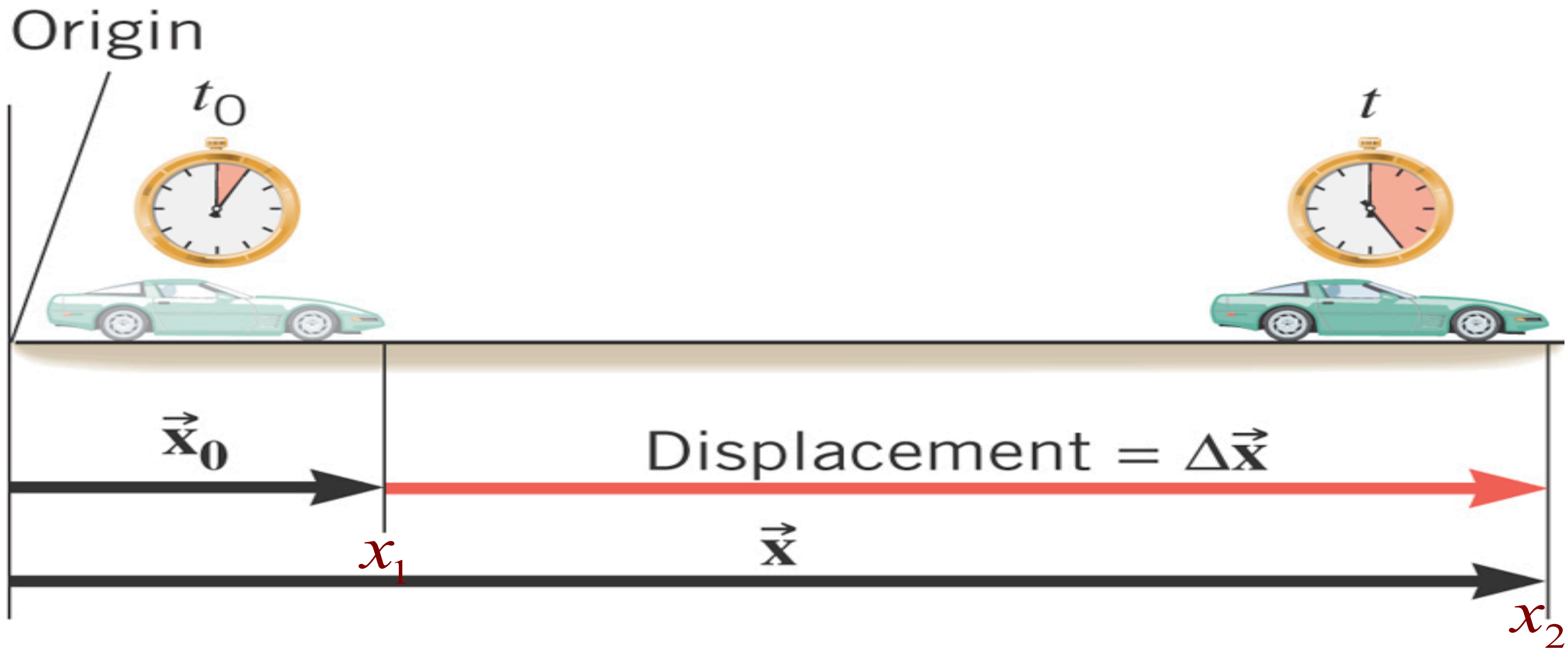
Displacement per unit time in the interval throughout the motion

The average speed is defined as: $$v_{avg} \equiv \frac{\text{Total Distance Traveled}}{\text{Total Elapsed Time}}$$

Unit? **m/s**

A scalar quantity

Can someone tell me what the difference between speed and velocity is?



What is the displacement? $\Delta x = x_2 - x_1$

How much is the elapsed time? $\Delta t = t - t_0$

What is the average velocity? $v_{x-avg} = \dfrac{x_2 - x_1}{t - t_0} = \dfrac{\Delta x}{\Delta t}$

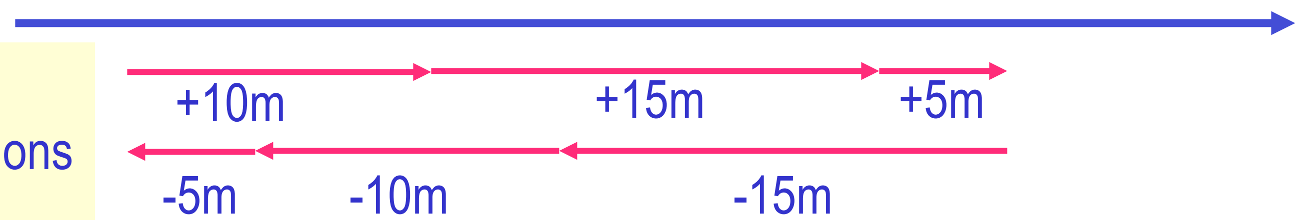
What is the average speed? $v_{avg} = \dfrac{|x_2 - x_1|}{t - t_0} = \dfrac{|\Delta x|}{\Delta t}$

Difference between Speed and Velocity

- Let's take a simple one dimensional translational motion that has many segments:

Let's call this line as X-axis

Let's have a couple of motions in a total time interval of 20 sec.



Total Displacement: $\Delta x \equiv x_f - x_i = x_i - x_i = 0(m)$

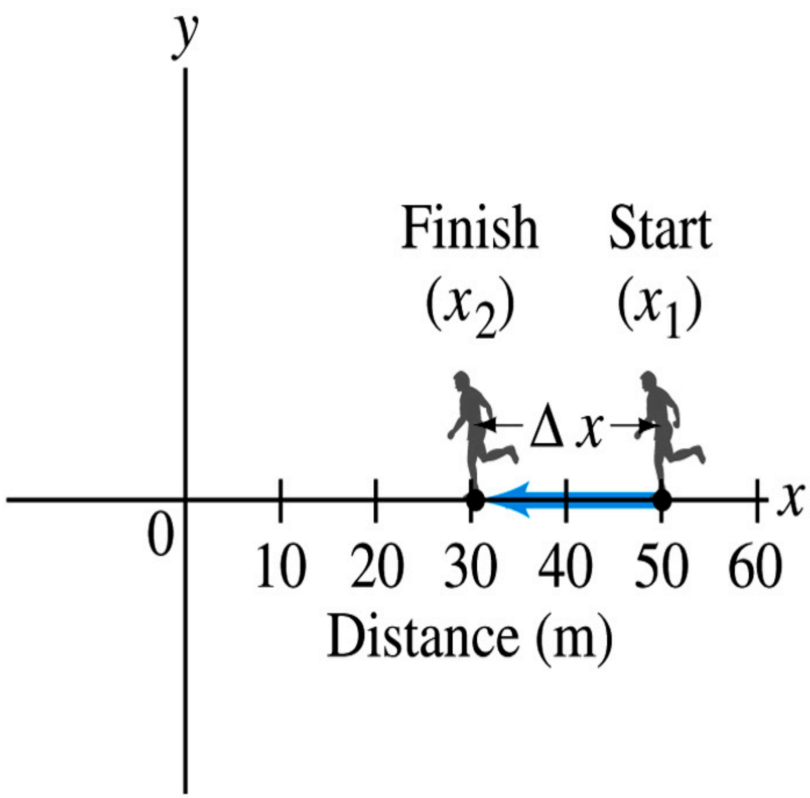
Average Velocity: $v_x \equiv \dfrac{x_f - x_i}{t_f - t_i} = \dfrac{\Delta x}{\Delta t} = \dfrac{0}{20} = 0(m/s)$

Total Distance Traveled: $D = 10 + 15 + 5 + 15 + 10 + 5 = 60(m)$

Average Speed: $v_{avg} \equiv \dfrac{\text{Total Distance Traveled}}{\text{Total Time Interval}} = \dfrac{60}{20} = 3(m/s)$

Example

The position of a runner as a function of time is plotted as moving along the x axis of a coordinate system. During a 3.00-s time interval, the runner's position changes from x_1=50.0m to x_2=30.5 m, as shown in the figure. What was the runner's average velocity? What was the average speed?



- Displacement:

$$\Delta x \equiv x_f - x_i = x_2 - x_1 = 30.5 - 50.0 = -19.5(m)$$

- Average Velocity:

$$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \frac{-19.5}{3.00} = -6.50(m/s)$$

- Average Speed:

$$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Interval}}$$

$$= \frac{50.0 - 30.5}{3.00} = \frac{+19.5}{3.00} = +6.50(m/s)$$

Example of a segmented motion

You drive a beat-up pickup truck along a straight road for 8.4km at 70km/h, at which point the truck runs out of gasoline and stops. Over the next 30min, you walk another 2.0km farther along the road to a gas station.

(a) what is your overall displacement from the beginning of your drive to your arrival at the station?

You can split this motion in two segments

Segment 1: from the start to the point where the truck ran out of gas, Δx_1 $\quad \Delta x_1 \equiv x_f - x_i = +8.4km$

Segment 2: from the point where you started walking to the gas station, Δx_2 $\quad \Delta x_2 \equiv x_f - x_i = +2.0km$

Overall displacement Δx is $\Delta x = \Delta x_1 + \Delta x_2 = +8.4 + (+2.0) = 10.4(km)$

$$= 10400(m)$$

Example

You drive a beat-up pickup truck along a straight road for 8.4km at 70km/h, at which point the truck runs out of gasoline and stops. Over the next 30min, you walk another 2.0km farther along the road to a gas station.

(b) What is the time interval Δt from the beginning of your drive to your arrival at the station?

Figure out the time interval in each segments of the motion and add them together.

We know what the interval for the second segment is: Δt_2=30min

What is the interval for the first segment, Δt_1?

Use the definition of average velocity and solve it for Δt_1

$$v_{x1} = \frac{\Delta x_1}{\Delta t_1} \quad \Rightarrow \Delta t_1 = \frac{\Delta x_1}{v_{x1}} = \frac{8.4}{70} = 0.12(hr) = 7.2(\text{min})$$

Add the two time intervals for total time interval Δt

$$\Delta t = \Delta t_1 + \Delta t_2 = 7.2 + 30 = 37(\text{min})$$
$$= 37 \times 60 = 2200(s)$$

Example

You drive a beat-up pickup truck along a straight road for 8.4km at 70km/h, at which point the truck runs out of gasoline and stops. Over the next 30min, you walk another 2.0km farther along the road to a gas station.

(c) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the station?

Using the definition of the average velocity, just simply divide the overall displacement by the overall time interval:

$$v_x = \frac{\Delta x}{\Delta t} = \frac{\Delta x_1 + \Delta x_1}{\Delta t_1 + \Delta t_1} = \frac{10.4(km)}{37(\min)} = \frac{10400(m)}{2200(s)} = 4.7(m/s)$$

What is the average speed?

What is the average velocity and average speed in km/h?

Instantaneous Velocity and Speed

- Can average quantities tell you the detailed story of the whole motion? **NO!!**
- Instantaneous velocity is defined as:
  - What does this mean?
    - Displacement in an infinitesimal time interval
    - Average velocity over a very, very short amount of time

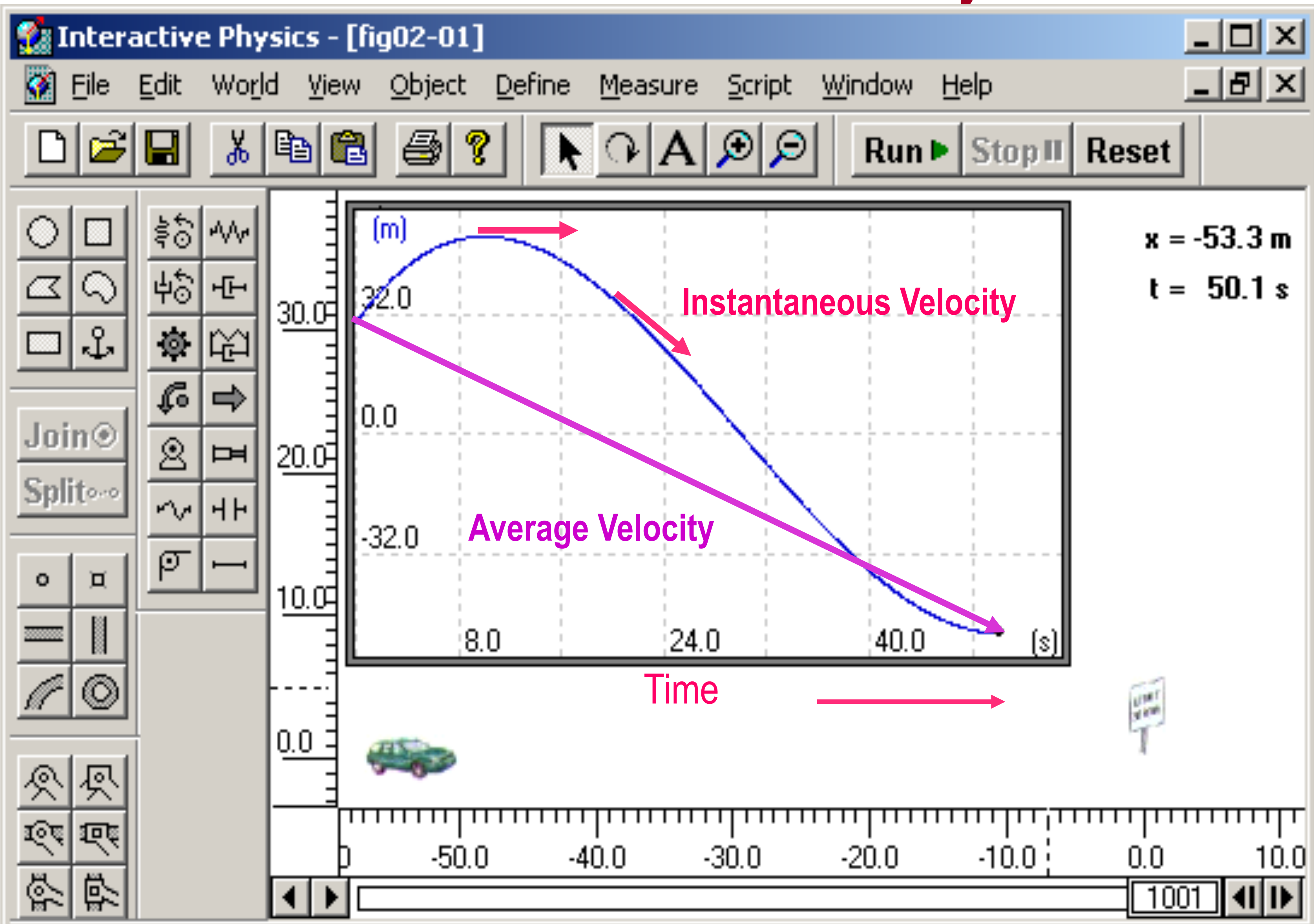
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

•Instantaneous speed is the size (magnitude) of the velocity vector:

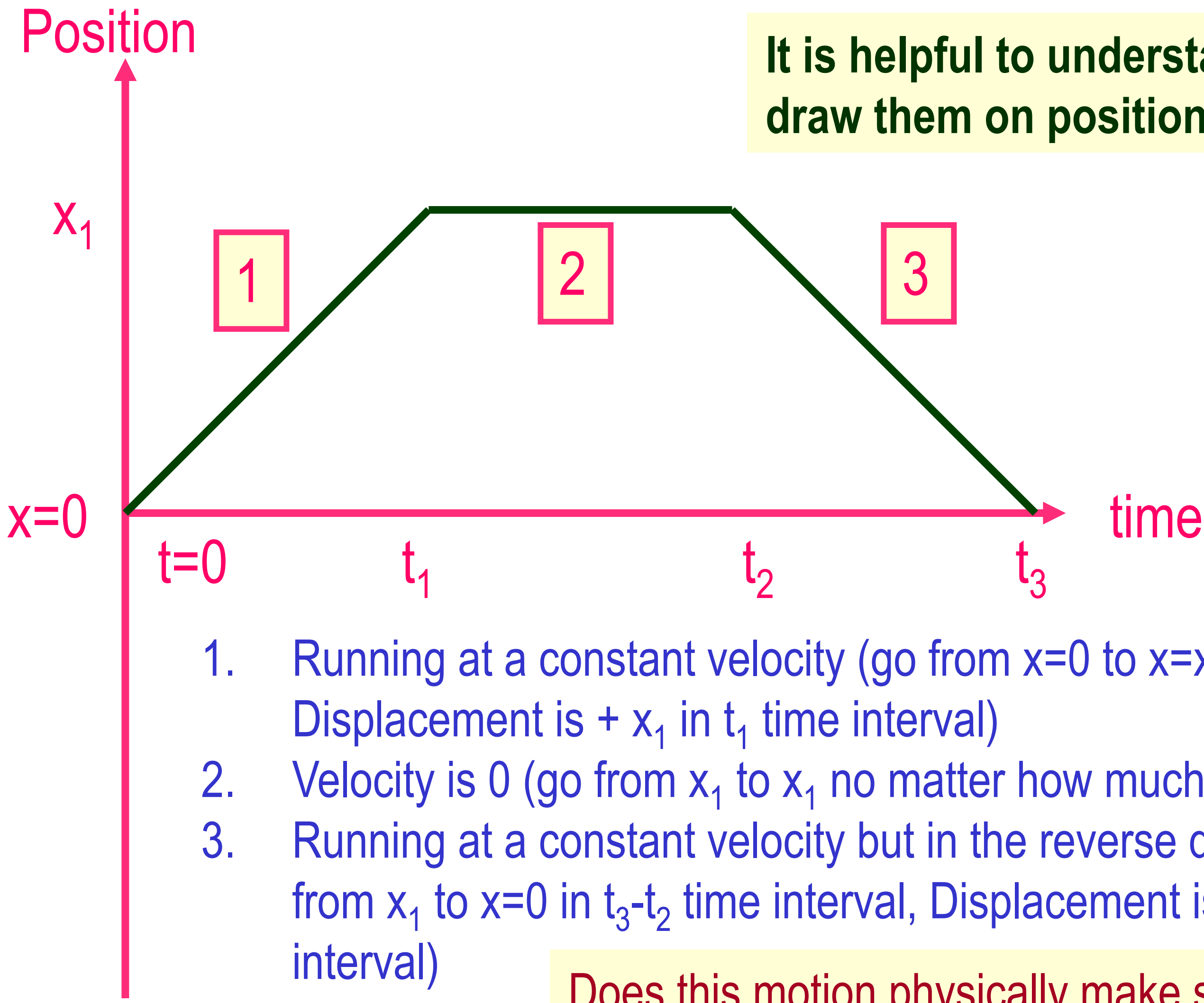
$$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right| = \left| \frac{dx}{dt} \right|$$

*Magnitude of Vectors are Expressed in absolute values

Instantaneous Velocity



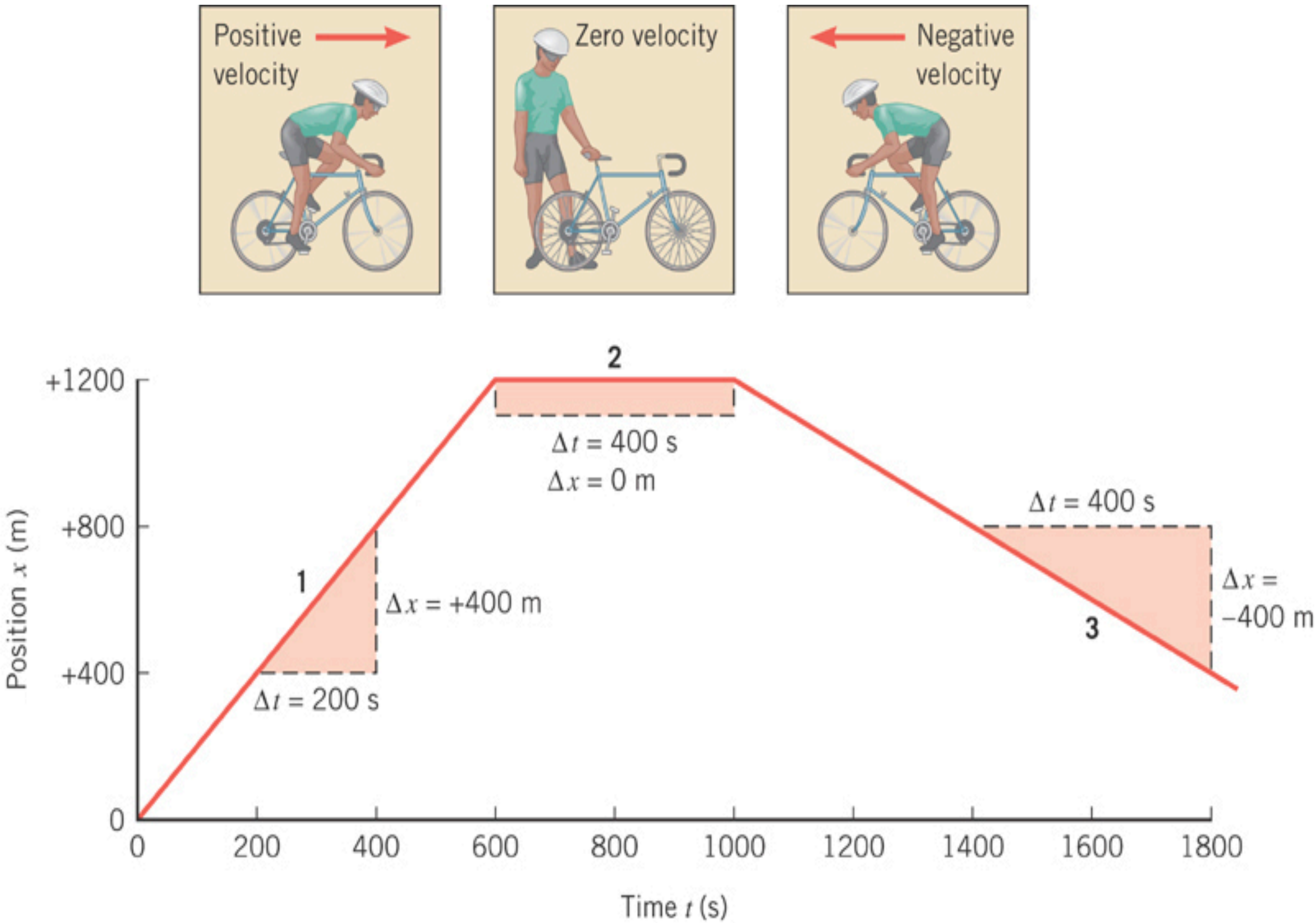
Position vs Time Plot



It is helpful to understand motions to draw them on position vs time plots.

1. Running at a constant velocity (go from $x=0$ to $x=x_1$ in t_1, Displacement is $+ x_1$ in t_1 time interval)
2. Velocity is 0 (go from x_1 to x_1 no matter how much time changes)
3. Running at a constant velocity but in the reverse direction as 1. (go from x_1 to $x=0$ in t_3-t_2 time interval, Displacement is $- x_1$ in t_3-t_2 time interval)

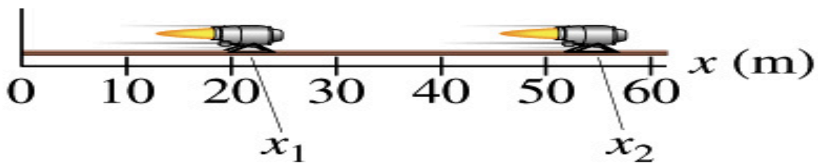
Does this motion physically make sense?

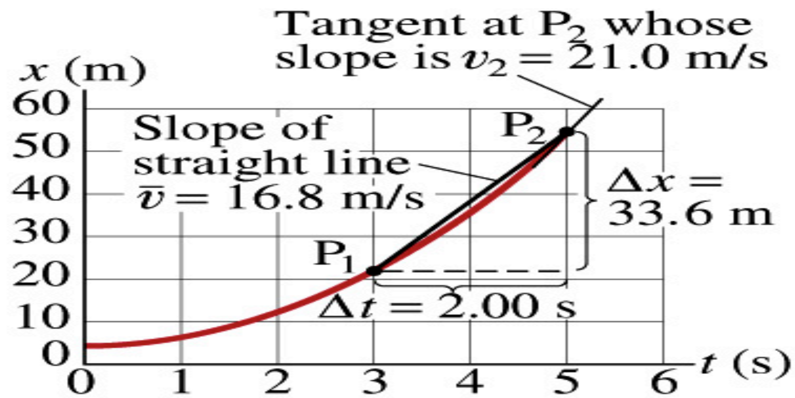


Positive velocity
Zero velocity
Negative velocity
2
+1200
$\Delta t = 400$ s
$\Delta x = 0$ m
$\Delta t = 400$ s
+800
1
$\Delta x = +400$ m
$\Delta x =$
-400 m
3
+400
$\Delta t = 200$ s
Position x (m)
0
0
200
400
600
800
1000
1200
1400
1600
1800
Time t (s)

Example

A jet engine moves along a track. Its position as a function of time is given by the equation $x=At^2+B$ where A=2.10m/s^2 and B=2.80m.





(a) Determine the displacement of the engine during the interval from t_1=3.00s to t_2=5.00s.

$$x_1 = x_{t_1=3.00} = 2.10\times(3.00)^2 + 2.80 = 21.7m \qquad x_2 = x_{t_2=5.00} = 2.10\times(5.00)^2 + 2.80 = 55.3m$$

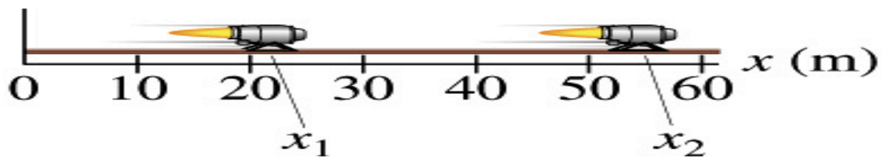
Displacement is, therefore:

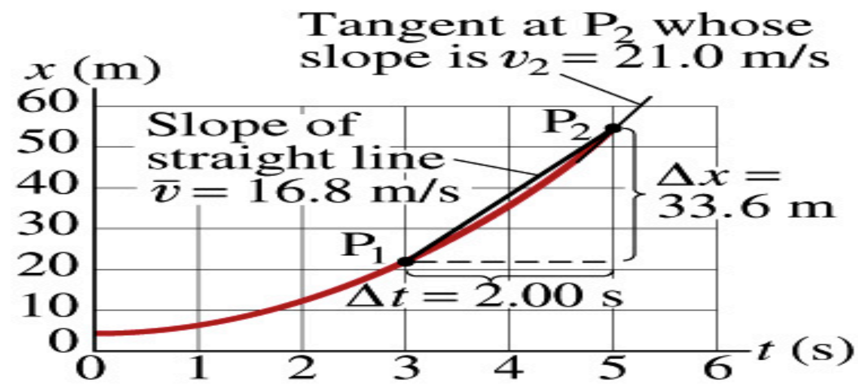
$$\Delta x = x_2 - x_1 = 55.3 - 21.7 = +33.6(m)$$

(b) Determine the average velocity during this time interval.

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{33.6}{5.00-3.00} = \frac{33.6}{2.00} = 16.8(m/s)$$

Example cont'd





(c) Determine the instantaneous velocity at $t=t_2=5.00$s.

Calculus formula for derivative $\boxed{\frac{d}{dt}\left(Ct^n\right)=nCt^{n-1}}$ and $\boxed{\frac{d}{dt}\left(C\right)=0}$

The derivative of the engine's equation of motion is

$$v_x = \lim_{\Delta t \to 0}\frac{\Delta x}{\Delta t} = \frac{dx}{dt} = \frac{d}{dt}\left(At^2+B\right) = 2At$$

The instantaneous velocity at t=5.00s is

$$v_x\left(t=5.00s\right) = 2A\times 5.00 = 2.10\times 10.0 = 21.0\left(m/s\right)$$

Definitions of Displacement, Velocity and Speed

Displacement $$\Delta x \equiv x_f - x_i$$

Average velocity $$v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

Average speed $$v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}$$

Instantaneous velocity $$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous speed $$|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right| = \left| \frac{dx}{dt} \right|$$

Acceleration

Change of velocity in time (what kind of quantity is this?) $\boxed{\text{Vector}}$

- Definition of the average acceleration:

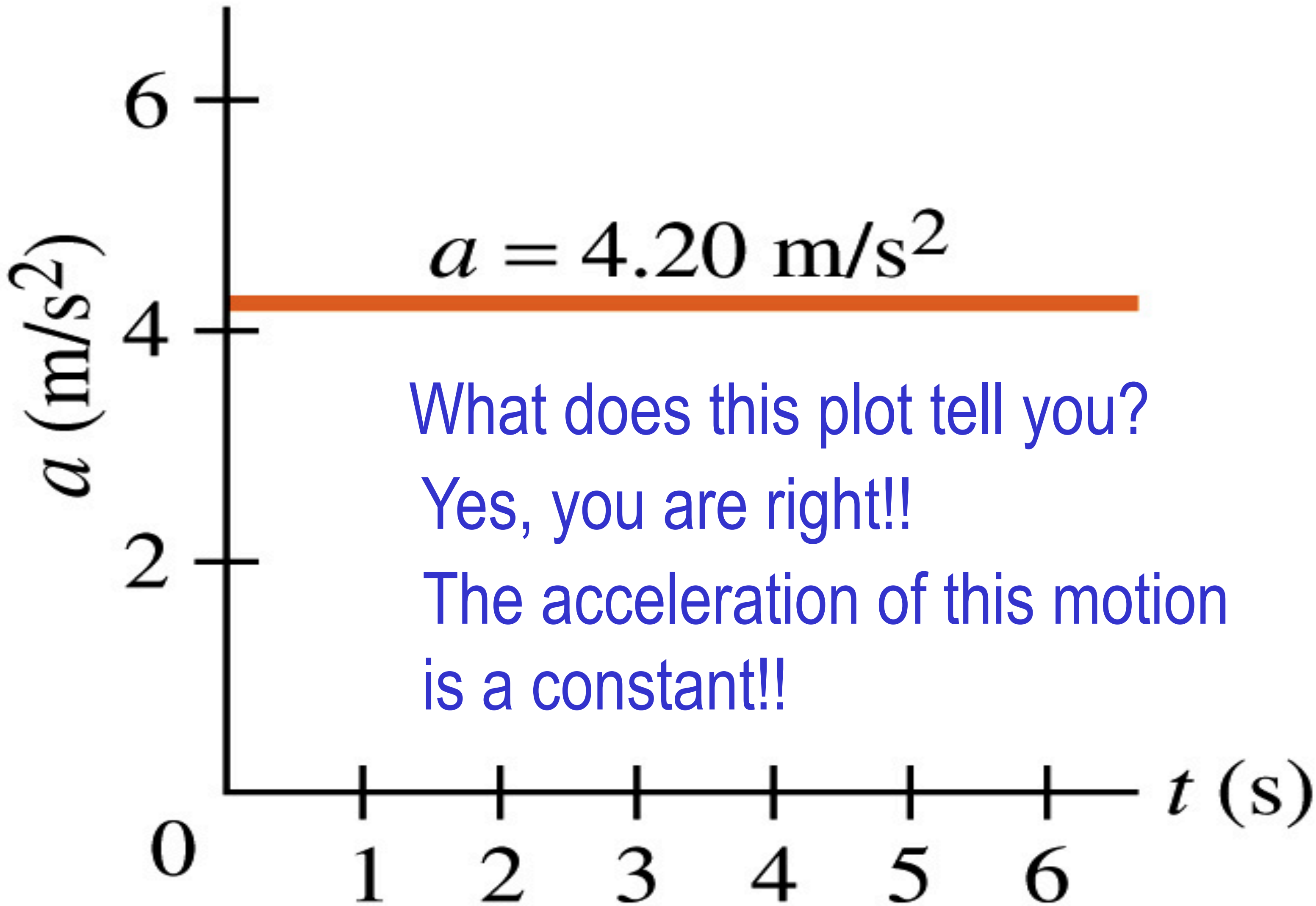
$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} \quad \text{analogous to} \quad v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

- Definition of the instantaneous acceleration:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \quad \text{analogous to} \quad \boxed{v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

- In calculus terms: The slope (derivative) of the velocity vector with respect to time or the change of slopes of position as a function of time

Acceleration vs Time Plot



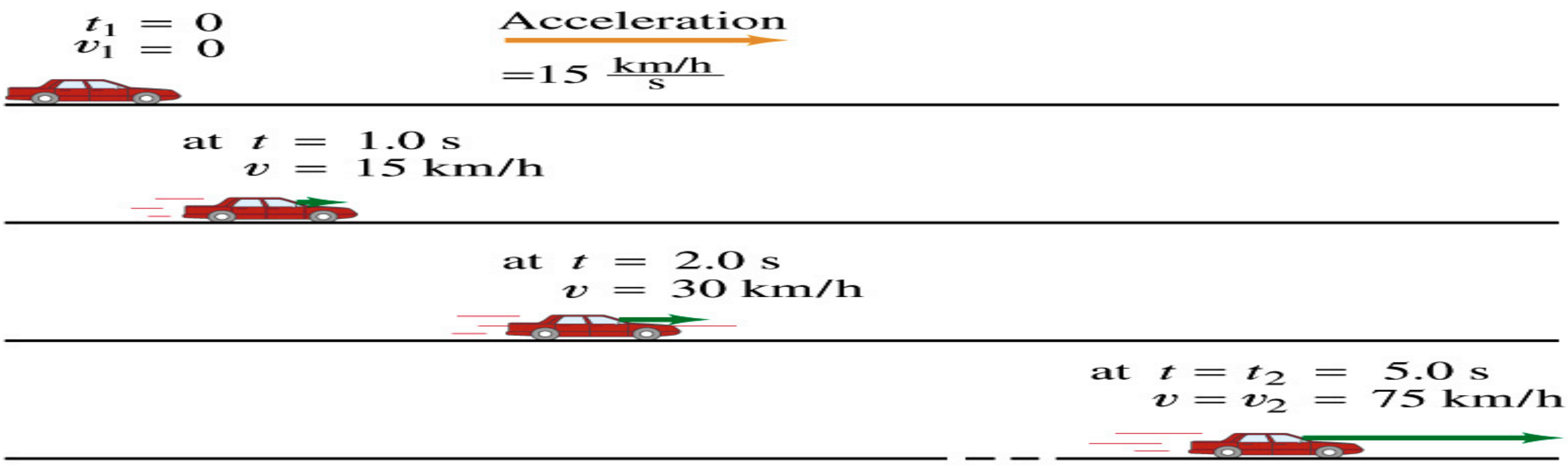
What does this plot tell you?

Yes, you are right!!

The acceleration of this motion is a constant!!

Example

A car accelerates along a straight road from rest to 75km/h in 5.0s.



What is the magnitude of its average acceleration?

$$v_{xi} = 0\ m/s$$

$$v_{xf} = \frac{75000m}{3600s} = 21\ m/s$$

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} = \frac{21-0}{5.0} = \frac{21}{5.0} = 4.2(m/s^2)$$

$$= \frac{4.2 \times (3600)^2}{1000} = 5.4 \times 10^4 (km/h^2)$$

Few Confusing Things on Acceleration

- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Is there any acceleration when an object is not moving?
- When an object is moving faster as time goes on, ($v=v(t)$), acceleration is positive ($a>0$).
  - Incorrect, since the object might be moving in negative direction initially
- When an object is moving slower as time goes on, ($v=v(t)$), acceleration is negative ($a<0$)
  - Incorrect, since the object might be moving in negative direction initially
- In all cases, velocity is positive, unless the direction of the movement changes.
  - Incorrect, since the object might be moving in negative direction initially
- Is there acceleration if an object moves in a constant speed but changes direction?

The answer is YES!!