PHYS 1443 – Section 004
Lecture #4

Thursday, Sept. 4, 2014

Dr. Jaehoon Yu

- One Dimensional Motion
  - Motion under constant acceleration
  - One dimensional Kinematic Equations
  - How do we solve kinematic problems?
  - Falling motions
- Motion in two dimensions
  - Coordinate system
  - Vector and scalars, their operations

Today's homework is homework #3, due 11pm, Thursday, Sept. 11!!

Announcements

- Quiz #2
  - Beginning of the class coming Thursday, Sept. 11
  - Covers CH1.1 through what we learn Tuesday, Sept. 9
  - Mixture of multiple choice and free response problems
  - Bring your calculator but DO NOT input formula into it!
    - Your phones or portable computers are NOT allowed as a replacement!
  - You can prepare a one 8.5x11.5 sheet (front and back) of **handwritten** formulae and values of constants for the exam
    - None of the parts of the solutions of any problems
    - No derived formulae, derivations of equations or word definitions!
    - No additional formulae or values of constants will be provided!
- First term exam moved from Tuesday, Sept. 23 to **Thursday, Sept. 25**! Please make a note!

Special Project #2 for Extra Credit

- Show that the trajectory of a projectile motion is a parabola!!
  - 20 points
  - Due: Thursday, Sept. 11
  - You MUST show full details of your OWN computations to obtain any credit
    - Beyond what was covered in this lecture note and in the book!

Displacement, Velocity, Speed & Acceleration

Displacement

$$\boxed{\Delta x \equiv x_f - x_i}$$

Average velocity

$$\boxed{v_x \equiv \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}}$$

Average speed

$$\boxed{v \equiv \frac{\text{Total Distance Traveled}}{\text{Total Time Spent}}}$$

Instantaneous velocity

$$\boxed{v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

Instantaneous speed

$$\boxed{|v_x| = \left| \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \right| = \left| \frac{dx}{dt} \right|}$$

Average acceleration

$$a_x \equiv \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t}$$

Instantaneous acceleration

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

Example for Acceleration

- Velocity, v_x, is express in: $v_x(t) = \left(40 - 5t^2\right) m/s$
- Find the average acceleration in time interval, t=0 to t=2.0s

$$v_{xi}(t_i = 0) = 40(m/s)$$

$$v_{xf}(t_f = 2.0) = \left(40 - 5\times 2.0^2\right) = 20(m/s)$$

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{\Delta v_x}{\Delta t} = \frac{20 - 40}{2.0 - 0} = -10(m/s^2)$$

- Find instantaneous acceleration at any time t and t=2.0s

Instantaneous Acceleration at any time

$$a_x(t) \equiv \frac{dv_x}{dt} = \frac{d}{dt}\left(40 - 5t^2\right)$$

$$= -10t\left(m/s^2\right)$$

Instantaneous Acceleration at any time t=2.0s

$$a_x(t = 2.0)$$

$$= -10\times(2.0)$$

$$= -20(m/s^2)$$

Example for Acceleration

- Position is express in: $x = 4 - 27t + t^3 \left(m\right)$
- Find the particle's velocity function v(t) and the acceleration function a(t).

$$v_x(t) = \frac{dx}{dt} = \frac{d}{dt}\left(4 - 27t + t^3\right) = -27 + 3t^2\left(m/s\right)$$

$$a_x(t) = \frac{d^2x}{dt^2} = \frac{dv_x}{dt} = \frac{d}{dt}\left(-27 + 3t^2\right) = +6t\left(m/s^2\right)$$

•Find the average acceleration between t=2.0s and t=4.0s

$$v_x(t=2) = -27 + 3t^2 = -27 + 3\cdot\left(2\right)^2 = -15\left(m/s\right)$$

$$v_x(t=4) = -27 + 3t^2 = -27 + 3\cdot\left(4\right)^2 = +21\left(m/s\right)$$

$$\overline{a}_x = \frac{v_x(t=4) - v_x(t=2)}{4-2} = \frac{21 - \left(-15\right)}{2} = \frac{36}{2} = +18\left(m/s^2\right)$$

•Find the average velocity between t=2.0s and t=4.0s

Check point on Acceleration

- Determine whether each of the following statements is correct!
- When an object is moving in a constant velocity ($v=v_0$), there is no acceleration ($a=0$)
  - Correct! The velocity does not change as a function of time.
  - There is no acceleration when an object is not moving!
- When an object is moving with an increasing speed, the sign of the acceleration is always positive ($a>0$).
  - Incorrect! The sign is negative if the object is moving in negative direction!
- When an object is moving with a decreasing speed, the sign of the acceleration is always negative ($a<0$)
  - Incorrect! The sign is positive if the object is moving in negative direction!
- In all cases, the sign of the velocity is always positive, unless the direction of the motion changes.
  - Incorrect! The sign depends on the direction of the motion.
- Is there any acceleration if an object moves in a constant speed but changes its direction?

The answer is YES!!

One Dimensional Motion

- Let's focus on the simplest case: acceleration is a constant ($a=a_0$)
- Using the definitions of average acceleration and velocity, we can derive equations of motion (description of motion, velocity and position as a function of time)

$$\bar{a}_x = a_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$a_x = \frac{v_{xf} - v_{xi}}{t} \Rightarrow v_{xf} = v_{xi} + a_x t$$

For constant acceleration, average velocity is a simple numeric average

$$\bar{v}_x = \frac{v_{xi} + v_{xf}}{2} = \frac{2v_{xi} + a_x t}{2} = v_{xi} + \frac{1}{2}a_x t$$

$$\bar{v}_x = \frac{x_f - x_i}{t_f - t_i}$$ (If $t_f=t$ and $t_i=0$) $$\bar{v}_x = \frac{x_f - x_i}{t} \Rightarrow x_f = x_i + \bar{v}_x t$$

Resulting Equation of Motion becomes

$$x_f = x_i + \bar{v}_x t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

Kinematic Equations of Motion on a Straight Line Under **Constant Acceleration**

$$v_{xf}(t) = v_{xi} + a_x t$$

Velocity as a function of time

$$x_f - x_i = \frac{1}{2}\bar{v}_x t = \frac{1}{2}\left(v_{xf} + v_{xi}\right)t$$

Displacement as a function of velocities and time

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

Displacement as a function of time, velocity, and acceleration

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Velocity as a function of Displacement and acceleration

You may use different forms of Kinematic equations, depending on the information given to you in specific physical problems!!

How do we solve a problem using a kinematic formula under constant acceleration?

- Identify what information is given in the problem.
  - Initial and final velocity?
  - Acceleration?
  - Distance, initial position or final position?
  - Time?
- Convert the units of all quantities to SI units to be consistent.
- Identify what the problem wants
- Identify which kinematic formula is **most appropriate and easiest** to solve for what the problem wants.
  - Frequently multiple formulae can give you the answer for the quantity you are looking for. ➔ Do not just use any formula but use the one that can be easiest to solve.
- Solve the equations for the quantity or quantities wanted.

Example

Suppose you want to design an air-bag system that can protect the driver in a head-on collision at a speed 100km/hr (~60miles/hr). Estimate how fast the air-bag must inflate to effectively protect the driver. Assume the car crumples upon impact over a distance of about 1m. How does the use of a seat belt help the driver?

How long does it take for the car to come to a full stop? ⇒ As long as it takes for it to crumple.

The initial speed of the car is $$v_{xi} = 100km/h = \frac{100000m}{3600s} = 28m/s$$

We also know that $v_{xf} = 0m/s$ and $x_f - x_i = 1m$

Using the kinematic formula $$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

The acceleration is $$a_x = \frac{v_{xf}^2 - v_{xi}^2}{2\left(x_f - x_i\right)} = \frac{0-\left(28m/s\right)^2}{2\times 1m} = -390m/s^2$$

Thus the time for air-bag to deploy is $$t = \frac{v_{xf} - v_{xi}}{a} = \frac{0-28m/s}{-390m/s^2} = 0.07s$$

Check point for conceptual understanding

- Which of the following equations for positions of a particle as a function of time can the four kinematic equations applicable?
- (a) $x = 3t - 4$
- (b) $x = -5t^3 + 4t^2 + 6$
- (c) $x = 2/t^2 - 4/t$
- (d) $x = 5t^2 - 3$
- What is the key here? Finding which equation gives a constant acceleration using its definition!
- Yes, you are right! The answers are (a) and (d)!

Falling Motion

- Falling motion is a motion under the influence of the gravitational pull (gravity) only; Which direction is a freely falling object moving?

  Yes, down to the center of the earth!!

  - A motion under constant acceleration
  - All kinematic formula we learned can be used to solve for falling motions.
- Gravitational acceleration is inversely proportional to the distance between the object and the center of the earth
- The magnitude of the gravitational acceleration is $g=9.80 m/s^2$ on the surface of the earth, most of the time.
- The direction of gravitational acceleration is **<u>ALWAYS</u>** toward the center of the earth, which we normally call (-y); where up and down direction are indicated as the variable "y"
- Thus the correct denotation of gravitational acceleration on the surface of the earth is $g=-9.80 m/s^2$ when +y points upward

Example for Using 1D Kinematic Equations on a Falling object

Stone was thrown straight upward at t=0 with +20.0m/s initial velocity on the roof of a 50.0m tall building,

What is the acceleration in this motion? $\boxed{g=-9.80m/s^2}$

(a) Find the time the stone reaches at the maximum height.

What is so special about the maximum height? V=0

$$v_f = v_{yi} + a_y t = +20.0 - 9.80t = 0.00m/s \quad \xrightarrow{\text{Solve for t}} \quad t = \frac{20.0}{9.80} = 2.04s$$

(b) Find the maximum height.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 50.0 + 20 \times 2.04 + \frac{1}{2} \times (-9.80) \times (2.04)^2$$

$$= 50.0 + 20.4 = 70.4(m)$$

Example of a Falling Object cnt'd

(c) Find the time the stone reaches back to its original height.

$$t = 2.04 \times 2 = 4.08s$$

(d) Find the velocity of the stone when it reaches its original height.

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 4.08 = -20.0(m/s)$$

(e) Find the velocity and position of the stone at t=5.00s.

Velocity

$$v_{yf} = v_{yi} + a_y t = 20.0 + (-9.80) \times 5.00 = -29.0(m/s)$$

Position

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$= 50.0 + 20.0 \times 5.00 + \frac{1}{2} \times (-9.80) \times (5.00)^2 = +27.5(m)$$